Energy, knowledge, and demo-economic development in the long run: a unified growth model

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Abstract: This article provides a knowledge-based and energy-centered unified growth model of the economic transition from limited to sustained growth. In an overlapping generation framework, we introduce final energy as a production factor of a composite final good sector, along with human capital, a learning-by-doing technology, and a Schumpeterian technology. Final energy results from a CES aggregation of energy inputs that come from renewable (biomass, wind, water) and exhaustible (coal, oil, gas) primary resources. The production of those inputs also requires human capital along with specific learning-by-doing and Schumpeterian technologies. Furthermore, with an endogenous sequence of *General Purpose Technologies* (GPTs), we explicitly feature pure technological externalities that foster the efficiency of both learning-by-doing and R&D-based technological progress. This setting allows us to distinguish two economic regimes: (i) a pre-modern organic regime dominated by limited growth in per capita output, high fertility, low levels of human capital, technological progress generated by learning-by-doing, and rare GPT arrivals; and (ii) a modern fossil regime characterized by sustained growth of per capita output, low fertility, high levels of human capital, technological progress generated by profit-motivated R&D, and increasingly frequent GPT arrivals. Most importantly, these economic, technological and demographic regimes' changes are associated with an energy transition. This transition results from the endogenous shortage of renewable resources availability and the arrival of new GPTs, which redirect technological progress towards the exploitation of previously unprofitable exhaustible energy carriers. Calibrations of the model are currently in progress and will allow a simulation of the historical experience of England for the period 1560-2010. In a second step, we plan to reiterate these simulations to compare the different trajectories of Western Europe and Eastern Asia.

Key Words: Unified Growth Theory; Useful Knowledge; Energy Transition; Demography.

JEL Classification: J13, J24, N10, O31, O40.

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1 Introduction

1.1 The need for a Unified Growth Theory and its development so far

Many agree with Galor's arguments to justify the need for a unified analytical framework able to explain both the occurrence of the *Great Divergence* and its persistence over time. Indeed, for so long as the economic take-off encountered by some countries two hundred years ago remains a mystery, confidence in modern economic growth theories can only be fragile. Moreover, the factors that prompted the take-off of the currently developed economies must be identified to allow a comprehensive understanding of the obstacles preventing current developing countries from reaching a state of sustained economic growth (Galor, 2005, p.176). Traditionally, the many structural changes occurring in an economy transitioning from limited to sustained growth have been studied in two-sectors models (agricultural vs. manufacturing sectors, or more generally traditional vs. modern sectors). Yet, all these models necessarily require exogenous shocks on prices, productivities, mortality rates, or preferences in order to generate a transition from limited to sustained economic growth (e.g., Hansen and Prescott, 2002; Mourmouras and Rangazas, 2009).

Galor and Weil (2000) proposed the first model able to deliver a purely endogenous transition from limited to sustained growth. This benchmark Galor-Weil model fostered further researches which now form a *Unified Growth Theory* (UGT). Different mechanisms have been proposed to explain the economic take-off process, such as: the scale effect of population on technological change in Galor and Weil (2000), Yakita (2010), Galindev (2011), Fröling (2011), and Strulik et al. (2013); the Darwinian selection of child quality-oriented individuals in Galor and Moav (2002); the Darwinian selection of entrepreneurial-oriented individuals in Galor and Michalopoulos (2012); the improvements in gender equality in Lagerlöf (2003); the decreasing demand for child labor in Doepke (2004); the decreasing child mortality rate and consequent improvement in life expectancy at birth in Cervellati and Sunde (2005); the improvement of health (but not longevity) in Hazan and Zoabi (2006); the increasing productivity of agriculture in Strulik and Weisdorf (2008); the increasing size of markets in Desmet and Parente (2012); and the increase of general knowledge in O'Rourke et al. (2013). An important feature of UGT is that the transition from a rather stagnant Malthusian regime towards sustained modern growth appears as the inevitable outcome of the growth process itself. As a corollary, in all these models it is possible to observe differential timing and magnitudes of take-off across countries as the result of deep-rooted factors (of a biogeographical, cultural, institutional, or contingent nature), but a country cannot be locked in a stagnation trap because the take-off is inevitable by construction.[1]

1.2 Missing perspective, goal, and organization of the article

It makes no doubts that the numerous contributions to Unified Growth Theory have shed new light on the process of long-term economic growth, from both theoretical and empirical perspectives. However, it is precisely the path-breaking significance of the benchmark model of Galor and Weil (2000) that might have occulted other dimensions of the shift towards modern economic growth. In particular, a key issue usually overlooked in economic growth theories concerns the role of energy. And indeed, to the best of our knowledge, energy is absent from all unified growth models of the literature apart from Fröling (2011). Accordingly, all these models are supposed to explain

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[1] Nguyen Dao and Dávila (2013) argue that this is mostly because technological losses are not possible in unified growth models, whereas in reality technology must not only be acquired but maintained too. History indeed provides many examples of technology losses due to geographical, cultural, or political reasons (see Diamond (1997, pp.257-258, pp.312-313), and Morris (2010, pp.413-417) respectively).

the Industrial Revolution without appealing to the role of energy, and in particular the associated energy transition towards fossil fuels. This view contrasts with the work of many economic historians, such as Pomeranz (2000), Allen (2009), Kander et al. (2013), and Wrigley (2016), who place a great emphasis on the role of coal to explain the early economic take-off of England towards sustained economic growth (see Section 2.3.2).[2]

In Fröling (2011)'s model, energy services are a constant elasticity of substitution (CES) aggregate of coal and biomass (with an elasticity of substitution of three). Knowledge enhances the productivity of coal in producing energy services, but it cannot augment the productivity of biomass, which contradicts historical evidence (Kander and Stern, 2014). The aggregate energy service (made of biomass and coal) is then combined with labor and land in a Cobb-Douglas function to produce the final output good. It is assumed that another stock of knowledge enhances total factor productivity (TFP) in the production of final output, which implies that TFP growth augments energy services, labor, and land at the same rate in final production. Such an hypothesis hardly cope with historical facts regarding relative productivities of production factors. Similarly, the model assumes a constant overall allocation to research and development (R&D) over time, and a constant split of this allocation between final and energy research sectors. The addition of these different drawbacks might explain that the global simulations of Fröling (2011)' model have difficulty to fit with historical data.

Taking advantages of recent advances in unified growth theories, the present article aims at providing a unified growth model able to better take into account the crucial role of energy for the transition from limited to sustained growth. It thus contributes to reconcile economic growth theory with historical facts regarding the role of energy emphasized by historical economists. Section 2 presents several empirical facts regarding the relation of useful knowledge, demography, and energy with the economic take-off towards modernity. Based on these insights, Section 3 develops a knowledge-based and energy-centered unified growth model. The balanced growth path of this model is analyzed in Section 4. Calibrations of the model to the historical experiences of England, Western Europe and Eastern Asia are performed in Section 5. Finally, a summary of the contributions of this article is given in Section 6, along with recommendations for future research.

2 Useful knowledge, demography, and energy transition

In this section, we briefly survey the literature centered around knowledge, demography and energy to highlight the key role that these variables play in economic growth. Performing this analysis allows us to identify the main building blocks of the theoretical framework presented in Section 3.

2.1 Useful knowledge rather than human capital of general population

2.1.1 The exaggerated role of general human capital

Because human capital is considered central to sustained modern growth, many unified growth theories make human capital also crucial to explain the take-off from limited to sustained growth. Nevertheless, the accuracy of this latter proposition shall be mitigated. Indeed, Mokyr (2011, p.232), notices the weak accomplishment of schooling to build human capital that would be useful to reach a modern regime. According to him, even in the eighteenth-century, public education in Britain was primarily destined to educate "gentlemen in the traditional sense of the word, that is, men without a well-defined occupation" whose "curricula consisted of the classics, languages, and other humanities". On the contrary, Mokyr (2011, p.233) asserts that "the great English

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[2] Several economic historians, such as Debeir et al. (1991), Crosby (2007), and Smil (2017), go further and make energy central to their analysis of the whole history of human societies.

engineers of the Industrial Revolution learned their skills by being apprenticed to able masters, and otherwise were largely self-taught." This latter observation suggests that learning-by-doing used to play a prominent role in pre-modern growth regimes.

Besides, Mokyr (2011, p.239) shows that adult literacy rates in Britain *circa* 1800 were equivalent to those of France and Belgium, and were even lower than those of the Netherlands. Moreover, Mokyr (2011, p.239) asserts that even if Britain rapidly became richer than other countries thanks to its early economic take-off, its ability or willingness to educate its young did not appreciably improved during the first phase of the Industrial Revolution. Accordingly, at the end of the nineteenth century, school enrollment was not higher in Britain compared to countries that experienced delayed takeoffs such as Prussia or France.

Finally, as an unequivocal criticism of the crucial role that most unified growth models assign to general human capital, Mokyr (2011, p.240) adds that, at the time of the British economic take-off, human capital was surely not the result of "an investment process in which the [human capital] rate of return on the margin would be equal to the interest rate." Rather, it might well be that the causal direction was reversed and that "many people decided for noneconomic reasons to educate their children and then discovered that this education imparted economically useful capabilities." He then concludes that "in any event, to the extent that the data available permit us to make inferences, the notion that the Industrial Revolution depended a great deal on human capital as customarily defined is not sustained." To be precise, Mokyr (2011, p.486) emphasizes the importance of schooling, and the resulting improvements of human capital, to explain the second phase of the Industrial Revolution (i.e. after 1850). Nevertheless, given the above arguments, it is clear that an alternative mechanism seems to be missing in the canonical UGT model in order to explain the early take-off of Britain.

2.1.2 The crucial role of useful knowledge

Both theoretical and empirical literatures seem to identify useful knowledge as a more likely cause of the intellectual changes preceding the Industrial Revolution. Hence, eminent scholars, such as Jacob (1997), Goldstone (2009), and Mokyr (2011), attribute much of the credit for the burst of innovations, and accelerated diffusion of best practices after 1750, to the scientific culture of Western Europe and in particular Britain. They argue that Western European societies were particularly dynamic and inclined to see a technological breakthrough in the eighteenth century thanks to the increase, or propagation during the previous two hundred years, of printing books, publishers, scientific societies, university networks, relatively accessible public lectures, and growing day-to-day exchanges between scientists, engineers, and craftmen. More precisely, these authors explain the success of the British Industrial Revolution by changes in the intellectual, social, and institutional background environment. These advances then crystallized in the emergence of a modern science capable of fostering the conversion of ideas and inventions —whatever their geographical origin— into workable innovations that were rapidly transformed into useful technologies able to yield profits to their developers. It is important to understand that all these scholars do not denigrate the many scientific breakthroughs that episodically originated in China and Islamic countries. They rather highlight the earliness of Britain in creating a scientific culture able to transpose useful knowledge into technological change thanks to a favorable institutional environment.[3]

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[3] Lipsey et al. (2005, p.225-289) stress that the roots of mechanistic science in Western Europe lie in the emergence of the appropriate institutions associated with the development of pluralistic societies in the last half of the medieval period that ultimately freed natural philosophers to seek an explanation of the world in terms of mechanical laws. Lipsey et al. (2005) also assert that the absence of early economic takeoff in China and advanced Islamic countries is explained by the failure of these countries to develop anything like modern science because of inappropriate institutions determined, in part, by their monolithic state structures and religious dogmas.

The empirical study of Squicciarini and Voigtländer (2015) is the first to provide systematic evidence for Mokyr's hypothesis about the importance of useful knowledge for industrialization. As a proxy for scientific elites, these authors use *Encyclopédie* subscriber density and show that this measure of "upper-tail knowledge" is strongly associated with other indicators of local scientific activity, both before and after the *Encyclopédie* was printed in the mid-eighteenth century. Squicciarini and Voigtländer (2015) then show that upper-tail knowledge is a strong predictor of city growth after the onset of the French industrialization. Furthermore, by joining data on British patents with a large French firm survey from the 1840s, it appears that scientific elites indeed caused productivity increases in innovative industrial technology which were then associated with economics growth. On the contrary, Squicciarini and Voigtländer (2015) show that literacy levels representing human capital of the general population are associated with development in the cross-section, but they do not predict growth.

Recent unified growth models have taken into account the importance of useful knowledge. Strulik et al. (2013) propose a setting where technological change is initially only due to learning-by-doing *prior* to the apparition of an expanding input variety R&D sector that then fosters sustained economic growth. O'Rourke et al. (2013) introduce a stock of useful general knowledge whose level impacts the cost of innovation in a Schumpeterian R&D sector. As will be shown in Section 3, we build on these two recent articles and explicitly introduce a stock of useful knowledge when developing our model. More precisely, we rely on the innovative theoretical approach of Schaefer et al. (2014) that tracks down the history of technological improvements that cumulate in a stock of useful knowledge. The latter stock then shapes the pattern of *General Purpose Technology* (GPT) arrivals, which are characterized by a maturity level and impact the efficiency of all kinds of technological progress —distinguished between learning-by-doing and profit-motivated R&D as in Strulik et al. (2013) and O'Rourke et al. (2013)—, thus featuring a pure knowledge externalities.

2.2 Demographic transition: choosing among controversial issues

2.2.1 Is there a child quality-quantity trade-off?

A very important feature of unified growth models is to propose an endogenous demographic transition associated with the take-off from limited to sustained economic growth. Most of these models assume that parents perform a conscious trade-off between the number of children they want to have and the level of education they choose for them. If some studies, such as Cáceres-Delphiano (2006) for the USA, Li et al. (2008) for China, and Becker et al. (2010) for Prussia, find the expected negative family-size/child-quality relationship, other empirical studies, such as Angrist et al. (2010) for Israel, Black et al. (2005) for Norway, and Clark and Cummins (2016) for England, find no evidence of such a quality-quantity trade-off. Regarding the emblematic case of Britain on the period 1780–1880, Clark and Cummins (2016) find that family size did not affect education, occupation, longevity, or even wealth. On the wider 1580–1830 period, Wrigley et al. (1997, p.461) suggest that "natural fertility was the norm" in England, so that "small groups may have been practising family limitation, but the reconstitution evidence suggests that such behavior was restricted to a small minority of the population, if present at all". In summary, Acemoglu (2009, p.736) points out that "there is relatively little direct evidence that this [quality-quantity] trade-off is important in general or that it leads to the demographic transition. Other social scientists have suggested social norms, the large declines in mortality starting in the nineteenth century, and the reduced need for child labor as potential factors contributing to the demographic transition."

2.2.2 Choosing two successive mechanisms for the demographic transition

Becker (1981) was the first to formalize a theory relating the quantity-quality trade-off of households to the rise in demand for human capital. But twenty years before, Becker (1960) advanced the much simpler argument that the decline in fertility was a by-product of the increase in income and the associated rise in the opportunity cost of raising children. This theory hinges on the supposition that individuals' preferences reflect an innate bias against child quantity beyond a certain level of income. This mechanism was recently modeled by Strulik et al. (2013). Before them, Jones (2001) used a simplified version of the same approach with a formal representation of the mortality rate, which allowed him to reproduce the fact that mortality rates decrease before fertility rates in countries experiencing a demographic transition.

In the model developed in Section 3, our population module is an adaptation of Strulik et al. (2013)'s formulation to which we have added (i) an impact of the stock of useful knowledge on the efficiency of the human capital production, and (ii) the mortality rate of Jones (2001). As a result, the endogenous demographic transition of our model is triggered by an initial per capita production increase that does not hinge on a conscious quality-quantity trade-off. However, once the economic take-off is established fertility decrease and education expenditure increase as a result of the changing technological environment. Such an approach seems the most appropriate to comply with historical evidence while remaining as neutral as possible on the unresolved debate surrounding both the existence of a conscious child quality-quantity trade-off and the underlying mechanism of this arbitrage.

2.3 Energy and the economic growth process

2.3.1 The misguided reasons for the omission of energy

As already mentioned, apart from Fröling (2011), energy is absent from all unified growth models. Assigning a modest importance to energy in explaining growth is conventionally justified by its small share in national income. Indeed, the so-called 'cost share theorem' implies that, if the aggregate production function is homogeneous of degree one , the output elasticities of production factors equal their income allocation in total GDP. Consequently, GDP elasticities with respect to labor and capital are generally set to 0.7 and 0.3 according to their respective empirical shares of GDP, while energy is usually neglected because its cost usually represents around 5% of the national income. Even when it is considered as a production factor, the output elasticity of energy is set to 0.05, such that labor and capital remain the most important production factors (Denison, 1979). However, it can be argued that this 'cost share theorem' is fallacious for several reasons.

First, by construction, GDP is allocated exclusively to capital and labor payments. Accordingly, energy expenditure is itself only made of capital and labor payments (plus temporary market powers).[4] But the fact that energy expenditures are relatively low in developed economies does not imply that energy *per se* is of no importance for economic growth. This fact was well illustrated by the first energy crisis of 1973, during which a 5% decrease in oil availability induced a 6% loss of GDP in the US, which is much higher than the mere 0.25% that the 'cost-share theorem' predicted.[5] Moreover, energy expenditures used to account for up to 50–70% of national income in pre-industrial low growth economies, and it is only thanks to the use of previously untapped concentrated, and consequently cheap, fossil fuels that this value gradually declined below 10%

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[4] For instance, the price of gasoline is constituted of capital interest, labor payment, and various taxes that are required to extract and refine the crude oil provided free-of-charge by nature.

[5] As a corollary, this event showed that the output elasticity of energy of 0.05 generally presupposed in standard macroeconomics is underestimated, whereas the output elasticities of capital and labor of 0.3 and 0.7 respectively, are overestimated.

(Fizaine and Court, 2016). The small cost share of energy in modern economies is not a sign of its worthlessness, but it might contrarily indicate the crucial importance that concentrated fossil energy has on modern economic growth. Kander et al. (2013, p.7) indeed assert that the "decrease in the cost of energy, at the same time that much greater quantities of it could be supplied, has allowed vast reserves of capital to be employed, delivering other kinds of goods and services rather than covering only basic energetic needs" as it used to be the case during pre-modern times.

Finally, the path-breaking work of Kümmel and Lindenberger (2014) shows that, whenever *hard* technological constraints are taken into account, shadow prices raise factor costs, implying that the cost share theorem no longer holds.[6] As a consequence, pure financial expenditure accounting downplays the role of energy because it does not take into account the interrelation between energy and particular technological developments that have been crucial to generate an expansion of many sectors of the economy (e.g., the design of modern transport systems and the associated suburban habitat have been wholly dependent on the Internal Combustion Engine (ICE) fueled by gasoline, or the electric or gas-fired heating and cooling systems that make domestic and office life bearable in a variety of climates.)

2.3.2 Distinguishing several 'kinds' of energy

In order to understand the importance of energy for the economic process, it is crucial to distinguish between primary, final, and useful energy.[7] Primary energy is present in the environment in the form of natural stocks (coal, oil, gas, uranium) or flows (sun, water, wind, geothermal, wave and tide) that must be converted into secondary energy carriers in order to be usable. Such final energy vectors consists in heat flows, electricity, and solid, liquid or gaseous refined products. Finally, end-use devices allow the conversion of final carriers into useful energy services in the form of motion (i.e., mechanical drive), temperature control, lighting, and information processing. Because technological change affects each conversion step of energy systems with different magnitudes, the prices of primary, final, and useful energies do not evolve similarly. An example of such difference is given in Figure 1, where the average price of primary energy is compared to the average price of useful energy services in Great Britain from 1700 to 2000. As Fouquet (2011) argues, focusing on the price of primary energy rather than the price of useful energy services can lead to flawed reasoning because the former ignore major technological improvements that are developed to provide the latter.

2.3.3 Coal and the transition towards sustained economic growth in Britain

Focusing now on the specific case of England, the central role of coal to explain the early take-off of this economy is obvious for many economic historians. First, it is recognized that, from the

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[6] Besides, Ayres et al. (2013) argue that there are also some *soft* constraints—corresponding to social, financial, organizational, or legal restrictions— that determine additional limits to substitution possibilities between inputs over time.

[7] As repeatedly stressed by scholars such as Ayres and Warr (2009) and Kümmel (2011), what is commonly called energy in economic studies and models is in fact *exergy*. Exergy is the valuable part, or more formally the potentially useful part, of energy (precisely, it is the maximum work that can be done by a system that reversibly approaching its thermodynamic equilibrium). As required by the first law of thermodynamics, energy is conserved in the economic process. On the other hand, the second law of thermodynamics stipulates that exergy is degraded through the functioning of the economic system, since it is composed of multiples irreversible processes that imply entropy creation. Energy enters the economy as a high quality (high exergy content) input in the forms of concentrated solar energy (biomass and water/wind flows), geothermal and tidal potential, fossils fuels, and nuclear energy. Those energy forms are ultimately dissipated into a lower-quality (lower exergy content) heat output that potentially contains zero exergy (and thus zero ability to generate useful work) if its temperature is the same as the broader environment. Hence, it is the exergy content of energy that constitutes a production factor used up in the economic process and not energy *per se*. In the remainder of this article we will stick to the familiar term of energy, even if, strictly speaking, we refer to exergy.

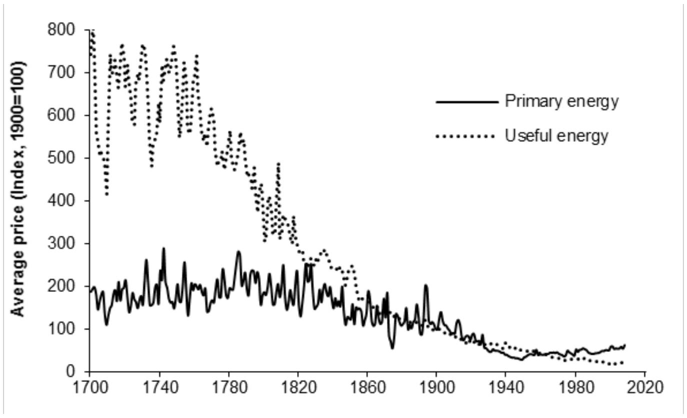

Figure 1: Average prices of primary and useful energies in Great Britain, 1700–2008. Source: Fouquet (2011).

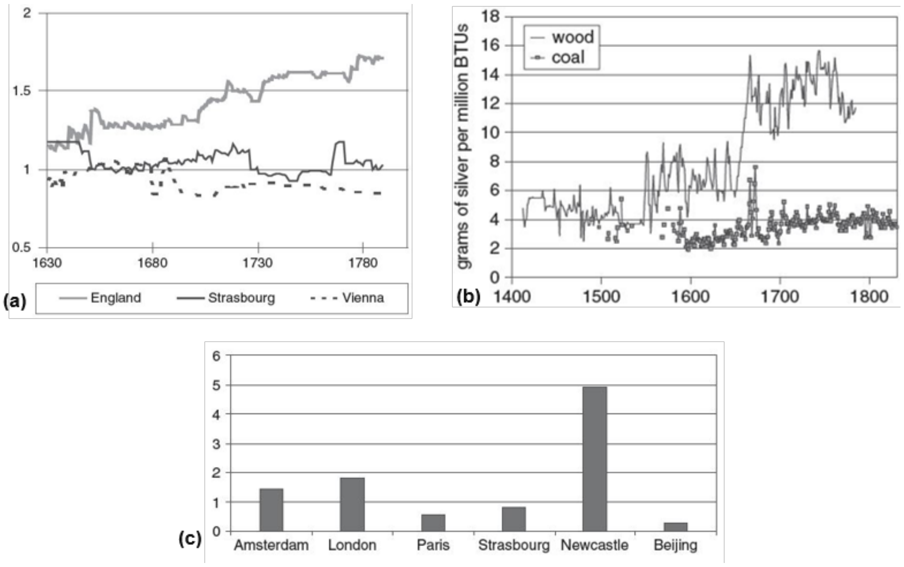

Figure 2: (a) Wage relative to price of capital in three European cities, 1630-1790; (b) Real prices of wood and coal in London, 1400-1825; (c) Wage relative to price of energy in six cities, early 1700s. Source: (Allen, 2009, p.139, 87, and 140 respectively).

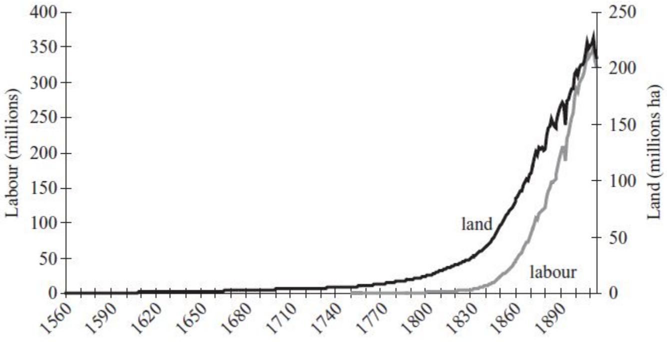

Figure 3: Land (millions of hectares, right vertical axis) and workers (millions, left vertical axis) saved by the use of coal in England and Wales, 1560–1913. Source: Malanima (2016).

sixteenth century onward, the Atlantic Trade allowed the extraction of natural resources (sugar, tea, tobacco, coffee, fur, and more specifically guano, wood, and cotton) from the New World with the extensive use of slaves, and hence flooded Western European markets with new exotic products. Institutions surely played an important role to expand Western European markets and lead to an Industrious Revolution (i.e., households-size handicraft manufacturing) in many of its constitutive states (de Vries, 1994).

As a consequence, for two Western European proto-industrial nations, Britain and the Netherlands, wages broadly increased from the sixteenth to the eighteenth centuries compared to other European Nations and development cores in other parts of the world. This so-called *Little Divergence* within Europe implied that incentives for labor-saving technologies were more important in Britain and the Netherlands compared to other European nations, while inexistent in China, Japan or India where labor remained relatively cheap. Simultaneously, because proto-industry relied heavily on wood fuel, critical levels of wood scarcity, visible both in quantity shortages and price increases, were recurrent in most of Western Europe, and especially in Britain (Pomeranz, 2000, pp.220-223). Allen (2009) comprehensively argues that the relative prices of production factors, and the existence of coal deposits close to urban centers, have been crucial to direct and foster sustained technological change. In other words, for Allen (2009), the British Industrial Revolution originated in the willingness of its people to apply knowledge brought by science (as already stressed in Section 2.1) to tap their favorable coal endowment thanks to financial incentives represented in high prices of labor and wood compared to the relatively low prices of capital and coal (Figure 2).

Similarly, comparing the role of energy in Europe and other parts of the world over the last five centuries, Kander et al. (2013, p.366) conclude that it is hard to imagine anything like modern economic growth occurring without this adoption of fossil fuels, fist of all coal. they further emphasize that they “view the transition to fossil fuels both as a necessary condition, and an enabling factor *leading* to modern growth” (italic emphasis present in original). Kander et al. (2013, pp.367-368) then assert that coal has been crucial for the British Industrial Revolution not solely as source of heat, but mostly for its high complementary with the steam engine and iron industries, delivering unprecedented amounts of mechanical power that structurally reshaped industrialized societies. Indeed, they argue that “ Steam engines saved labor, and initiated a capital-deepening growth path. [...] This capital-deepening growth was almost wholly reliant on fossil fuels and eventually, although by no means instantly, led not just to increased incomes, but set in motion a dynamic that has continued to raise incomes.”

To quantify the importance of coal as a source of both heat and mechanical power, Malanima (2016) followed the seminal contribution of Wrigley (1962) in order to estimate land- and labor-saving estimates due to coal use in England & Wales on the period 1560–1913.[8] The results presented in Figure 3 exhibit two distinct historical phases. During the first one, that lasted from the end of the sixteenth century until about 1830, the use of coal was mainly land-saving, and it is only during the second phase (from 1830 to 1900) that coal was really both land and labor-saving. Covering both phases from 1800 to 1900, the land-related (resp. labor-related) social savings grew from 1 to 14 times the extent of the entire country, that is 15 million hectares (resp. from 1 million to almost 300 million workers when the English population was 32 million and the labor force 13–14 million in 1900).

These estimates strongly support Wrigley (2016, p.2-4)’s claim that “the energy required to produce, say, iron and steel on a large scale or to construct and operate a railway system implied that it was idle to expect that it could be secured from the annual *flow* of energy derived from

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[8] As noticed by Malanima (2016), usual social savings calculations based on relative costs of old and alternative technologies appear quite impossible here because it would require to compute counter-factual wood prices and labor wages in a theoretical British economy where coal would have been absent.

plant photosynthesis" (italic emphasis in original). As a corollary, "an Industrial Revolution could not be accomplished as long as mechanical energy continued to be provided principally by human and animal muscle".

Based on these evidences of the key role of energy during the Industrial Revolution, we explicitly introduce final energy carriers within our modeling setup, the production of which depends on specific primary resources and technologies. These features will allow us to (i) distinguish several form of energy (and corresponding prices), (ii) describe the technological progress of fossil energy carriers as a response to pre-modern renewable energy shortage, and (iii) emphasize the resulting key role of abundant and cheap fossil energy supply during the Industrial Revolution and the whole modern regime.

3 Structure of the model

Building on the literature review of Section 2, we present in this section a unified growth model in which demography, knowledge, and energy interact to explain the transition from limited to sustained economic growth. In our framework, time is discrete and indexed by t.

3.1 Household's preferences and demography

We consider an economy populated by N_t individuals divided into three overlapping generations: children, young adults, and old adults. Each young adult is endowed with one unit of time that can be converted one-to-one into labor force. In the first period of life (childhood), $t-1$, children earn no income and, regardless of their education, child-rearing requires a fraction of the parental unit time endowment that is logically increasing with the number of children. In addition, financial expenditure (but not parents' time attention) increase with children's formal education. In the second period of life (parenthood), t, young adults allocate their unitary time endowment between child rearing and effective labor force participation to earn a wage. The wage income is either spent through direct consumption, saved to supply the capital investment market, or spent on children's education. In the third period, $t+1$, old adults only consume their savings plus interests.

3.1.1 Household's preferences

In order to derive the main results conveniently and to get closed-form solutions, we make a number of simplifying assumptions that are usual in unified growth overlapping generation models (e.g. Galor (2005) or Strulik et al. (2013)). In each household: (i) there is one unisex parent to avoid matching issues, (ii) newborns are a continuous number denoted by b_t to avoid indivisibility issues, and (iii) the motive for child expenditure is non-operational to avoid the maximization of dynastic value functions.[9]

Adults' preferences are assumed to be represented by a utility function defined over: (i) household's immediate consumption, c_t, above a subsistence level, $\bar{c}$; (ii) future consumption (during retirement) that consists in present savings, s_t, invested on the financial market at the interest rate, r_t, defined shortly;[10] (iii) births per capita, b_t, determining family size, and (iv) the future level of human capital, h_{t+1}, that each child receives through present education. Thus, the representative household's utility function writes

$$u_t = \log(c_t - \bar{c}) + \chi \log((1+r_t)s_t) + \rho \log(h_{t+1}) + \eta \log(b_t), \tag{1}$$

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[9] This last point means that the motivation of parents to spend on their children's education is not driven by the anticipation of the increase of children's utility caused by this expenditure, but by a general desire for having "higher quality" children.

[10] We assume a saturation of the inter-temporal budget constraint regarding the old adult's consumption.

where the positive parameters χ, ρ, and η capture, relatively to current consumption, the elasticities of utility with respect to future consumption during retirement, the human capital of children, and the family size. By writing utility in such a way, we implicitly assume immediate consumption, c_t, to be strictly above the subsistence level, $\bar{c}$.

Revenues of the representative household come from two sources: labor wages and patenting revenue in the R&D sector. Regarding the latter, the potential aggregated monopoly profits of capital goods suppliers, thereafter introduced in Section 3.2.3, are driven to zero under our set of hypothesis for profit-motivated R&D.[11] Hence, household's revenue only results from labor activities, rewarded by the competitive market wage, w_t, per efficiency labor unit h_t. We can therefore defining the potential income, $z_t \equiv w_t h_t$, as the labor earning if the entire time endowment is devoted to labor force participation. As we assume that only time is required to produce children, we define τ as the fraction of the representative household's time endowment that is required to raise one child, regardless of its level of human capital. Hence, raising one child shall be seen as an opportunity cost valued τz_t. Besides, building the human capital of children, h_{t+1}, requires formal education, which is conceptualized as a financial expense, e_t, per child. In summary, one can write the budget constraint faced by an individual in parenthood as follows,

$$z_t[1-\tau b_t] = c_t + s_t + b_t e_t. \tag{2}$$

Furthermore, we assume that education expenditure e_t is converted into human capital h_{t+1} through a schooling technology that controls for schooling costs, approximated by w_t,[12]

$$h_{t+1} := A_E(Q_t)\frac{e}{w} + \bar{h}, \tag{3}$$

where $\bar{h}$ represents informal human capital acquired without formal education,[13] and $A_E(Q_t)$ is an endogenous productivity rising with the level of total applied knowledge of the economy, Q_t, introduced thereafter in Eq. (26) of Subsection 3.3. As will be shown, Q_t aggregates the history of all applied technological progress from both learning-by-doing and R&D sources. We assume that the education technology admits decreasing returns with respect to this stock of applied knowledge, so $A_E(Q_t)$ satisfies the following conditions: $A_E(0) = 0$, $\frac{\partial A_E}{\partial Q} > 0$ and $\frac{\partial^2 A_E}{\partial Q^2} < 0$. In other words, we assume that the overall level of applied knowledge, Q_t, is a good proxy for various phenomena that positively affect the efficiency of schooling, such as the rising spatial density of schools (Boucekkine et al., 2007), the evolution of social norms favoring formal education, or changes in law limiting child's labor (Doepke, 2004). Throughout this paper, we assume $A_E(Q_t) = \bar{A}_E Q_t/(1+Q_t)$, with $\bar{A}_E > 0$.

The behavior of the representative households can be formally introduced through an optimization problem introduced here for clarity.

PROBLEM (HH - HOUSEHOLD). *The representative young adult planner seeks to maximize the utility function define in Eq. (1) under five constraints: the inter-temporal budget constraint in Eq. (2); the education technology defined in Eq. (3); the constraint on consumption relatively to subsistence,* $c_t \geq \bar{c}$*; and two non-negativity constraints on the number of children and education such that* $(b_t, e_t) \geq 0$.

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[11] Relaxing these hypotheses would not change qualitatively the behavior of the representative household, because the allocation of *total* revenue would be globally unchanged.

[12] One can think of w_t as the wage of teachers, hence e_t/w_t represents efficient education expenditure.

[13] Such a basic human capital level can be thought as informal knowledge that children acquire through the time τ spends observing and imitating their parents and peers at work. This knowledge (of farming or a particular craft, for example) is useful, i.e. it creates human capital at level $\bar{h}$, but it comes for free, at no educational cost. On the contrary, e_t is a financial investment that allow the child to receive a formal education through school and cultural goods consumption in order to increase its human capital above $\bar{h}$.

Hence, taking factor prices w_t and r_t as given, the household's problems writes

$$\max_{c_t,s_t,b_t\geq 0,e_t\geq 0} u_t = \log(c_t-\bar{c}) + \chi\log((1+r_t)s_t) + \rho\log(h_{t+1}(e_t,Q_t)) + \eta\log(b_t)$$
$$\textit{s.t.}\;\; z_t[1-\tau b_t] = c_t + s_t + b_t e_t,$$

thus setting per capita levels of consumption, c_t, savings, s_t, fertility, b_t, and investment in education, e_t. Combined with the endogenous demography dynamics described in Eq. (8), the representative households ultimately sets the supply of human capital, H_t.

The first order conditions from this problem yield the following optimal decisions regarding per capita consumption, savings and child quantity,

$$c_t = \frac{z_t-\bar{c}}{1+\chi+\eta} + \bar{c}, \qquad s_t = \chi(c_t-\bar{c}), \qquad b_t = \frac{\eta(z_t-\bar{c})}{(1+\chi+\eta)(e_t+\tau z_t)}. \tag{4}$$

One can note that those relations hold at the limit $c_t \to \bar{c}$, that is $z_t \to \bar{c}$.[14] Furthermore, it is clear from Eq. (4) that increasing education reduces fertility, which is a direct illustration of a quality-quantity trade-off.

Turning now to child quality, there exist an endogenous knowledge threshold, $\tilde{Q}_t$, such that by monotonicity of $A_E(\cdot)$,

$$e_t = \begin{cases} 0 & \text{if} \quad Q_t < \tilde{Q}_t, \\ \frac{(\rho\tau A_E(Q_t)h_t - \eta\bar{h})w_t}{A_E(Q_t)(\eta-\rho)} & \text{if} \quad Q_t \geq \tilde{Q}_t. \end{cases} \tag{5}$$

This threshold is defined as the solution of $e_t(\tilde{Q}_t) = 0$, that is $\tilde{Q}_t = \left(\bar{A}_E\rho\tau/(\eta\bar{h})h_t - 1\right)^{-1}$ according to the functional form we gave to $A_E(\cdot)$. For a given h_t, whenever $Q_t > \tilde{Q}_t$, the schooling technology is sufficiently efficient to be worth investing by the representative household. Moreover, one can easily see that $\frac{\partial \tilde{Q}_t}{\partial h_t} < 0$, which means that the per capital level of human capital tends to decrease the profitability threshold $\tilde{Q}_t$ of education. This fact can be interpreted as the positive effect of average per capita knowledge, and concomitantly of teachers' training, on the overall inclination of society for schooling. Besides, one can easily check from Eq. (5) that $\frac{\partial e_t}{\partial h_t} > 0$ and $\frac{\partial e_t}{\partial Q_t} > 0$, and from Eq. (3) that $\frac{\partial h_t}{\partial e_t} > 0$. Hence, given that applied knowledge, Q_t, is cumulative in our model, as soon as $Q_t > \tilde{Q}_t$, a virtuous circle is established between education, human capital, and general knowledge, such that these three variables all tend to grow together. This last comment finally allows us to give a simple interpretation of the transition from limited to sustained modern growth. Initially, education expenditure, e_t, is null while human capital is stuck to its minimal level, $\bar{h}$. In such a state, the knowledge threshold $\tilde{Q}_t$ is fully exogenous and only depends on the parameters of the model, and we have $\tilde{Q}_{h_t=\bar{h}} = \left(\bar{A}_E\rho\tau/\eta - 1\right)^{-1}$. Due to learning-by-doing technological progress introduced in Section 3.3, Q_t gradually increases but education expenditure (resp. human capital) remains null (resp. equal to $\bar{h}$) up to the point where $\tilde{Q}_t$ is crossed. From then, the representative households begins to invest into education, the demographic transition sets in, and both per capita education and human capital begin to rise.

3.1.2 Mortality and aggregate population dynamics

Following Jones (2001), the number of death per capita, $d_t(\tilde{c}_t)$, is assumed to be a function of $\tilde{c}_t$ representing the average level of per capita consumption c_t relative to the subsistence level $\bar{c}$, that

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[14] Recall that whenever $z_t < \bar{c}$, the utility function of Eq. (1) is not defined and we assume that the potential income, z_t, is fully dedicated to present consumption. This case is excluded from both our theoretical and empirical analysis.

is $\tilde{c}_t \equiv (c_t - \bar{c})/\bar{c}$. This variable is viewed as a proxy measure of hygiene levels and of the sensitivity of the population to diseases and natural disasters. With $\underline{d} \geq 0$ representing the lower bound of the mortality rate when consumption is infinitely large, we have

$$d_t = \frac{1}{\theta_1 \tilde{c}_t^{\theta_2} + \theta_3 \tilde{c}_t} + \underline{d}, \tag{6}$$

where θ_1, θ_2, and θ_3 are positive parameters.

Finally, one can easily computes the population growth rate, $g_{N,t} \equiv b_t - d_t$, and accordingly the evolution of total population, N, is given by

$$N_{t+1} = (1 + g_{N,t})N_t, \tag{7}$$

Taking the child-rearing time into account, the size of the workforce, L_t, is given by

$$L_t = (1 - \tau b_t)N_t, \tag{8}$$

whereas the aggregate human capital supply, H_t, corresponds to

$$H_t = h_t L_t. \tag{9}$$

3.2 Aggregate production

We turn now to the description of the production side of the economy. We consider a final good sector combining intermediate capital goods (i.e., machines), human capital, and aggregate final energy. Following Acemoglu et al. (2012) and Golosov et al. (2014), the aggregate final energy input of the composite good sector comes from the imperfect substitutability of k final energy inputs. The latter are supplied by k energy sectors extracting and refining different primary energies, coming either from renewable or exhaustible resources. The extraction of those primary energy inputs and their refining into final carriers require machines, human capital, and incur a sector-specific extraction cost.

All sectors are assumed to be perfectly competitive at the exception of intermediate capital goods. Indeed, technological progress occurring in this sector follow a Schumpeterian innovation process where quality improvements are specific to each machine line (Aghion and Howitt, 1992). Those machines are provided by monopolists owning a patent on their variety, endogenously supplanted by successful innovators in a process of creative destruction. Moreover, as described in Subsection 3.3, this profit-motivated R&D innovation interacts with *General Purpose Technologies* (GPTs) , that also shapes the level of a learning-by-doing knowledge affecting production.

3.2.1 Final good production

The final composite good, Y_t, is supplied under perfect competition through a constant returns-to-scale Cobb-Douglas production function combining, (i) a continuum of machines of measure one, $\{x_{i,t}\}_{i\in[0,1]}$, with specific endogenous quality, $\{q_{i,t}\}_{i\in[0,1]}$, (ii) human capital allocated to the final sector, $H_{Y,t}$, and (iii) aggregate final energy E_t. With α, β, and γ representing the respective output elasticities of intermediate machines, human capital, and final energy, we have

$$Y_t = A_{Y,t}\left[\int_0^1 (q_{i,t}x_{i,t})^{\alpha} di\right] H_{Y,t}^{\beta} E_t^{\gamma}, \tag{10}$$

where $A_{Y,t}$ is the technological level achieved through learning-by-doing in the final good sector, and $\alpha + \beta + \gamma = 1$. Building on Acemoglu et al. (2012) and Golosov et al. (2014), we consider that the aggregate final energy, E_t, comes from the combination of imperfectly substitutable final energy inputs, coming from k primary resources taken in a set $\mathcal{K}$ which are either: (i) renewable energy resources corresponding to biomass and wind/water/solar flows; or (ii) nonrenewable energy resources corresponding to fossil fuels, such as coal/gas/oil. Considering an elasticity of substitution σ between all final energy carriers, we have

$$E_t = \left[\sum_{k\in\mathcal{K}} \vartheta_k E_{k,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{11}$$

where $\sum_{k\in\mathcal{K}} \vartheta_k = 1$, with ϑ_k measuring the relative economic usefulness of the final energy carrier E_k. The final good sector can then be formally described through an usual optimization problem.

Problem (FG - Final good producer). *The final good sector is perfectly competitive and makes use of the production technology given by Eq. (10). Its price is chosen as the numeraire. The representative firm takes prices of machines, human capital, and final energy forms (r_t, w_t, and $\{p_{E_k,t}\}_{k\in\mathcal{K}}$ respectively) as given, as well as the current level of technology, $A_{Y,t}$, and $\{q_{i,t}\}_{i\in[0,1]}$ partially supplied by capital good producers at price $\{p_{i,t}^x\}_{i\in[0,1]}$, to solve*

$$\max_{\{x_{i,t}\}_{i\in[0,1]},H_{Y,t},\{E_{k,t}\}} A_{Y,t}\left[\int_0^1 (q_{i,t}x_{i,t})^{\alpha} di\right] H_{Y,t}^{\beta} E_t^{\gamma} - p_{i,t}^x \int_0^1 p_{i,t}x_{i,t}di - w_t H_{Y,t} - \sum_{k\in\mathcal{K}} p_{E_k,t}E_{k,t},$$

where E_t is defined by Eq. (11), thus setting demand schedules in machines, $\{x_{i,t}(\hat{v})\}_{i\in[0,1]}$, human capital, $H_{Y,y}(\hat{v})$, and useful energy, $\{E_{k,t}(\hat{v})\}_{k\in K}$, where "$\hat{v}$" stands for "$r_t$,$w_t$,$\{p_{E_k,t}\}_{k\in K}$,$\{p_{i,t}^x\}_{i\in[0,1]}$".

The first order conditions from this problem yield the following optimal demand schedules regarding each production factor

$$p_{i,t}^x = \alpha \frac{A_{Y,t} q_{i,t}^{\alpha} H_{Y,t}^{\beta} E_t^{\gamma}}{x_{i,t}^{1-\alpha}}, \tag{12}$$

$$w_t = \beta \frac{Y_t}{H_{Y,t}}, \tag{13}$$

$$p_{E_k,t} = \gamma \frac{Y_t}{E_t} \vartheta_k \left[\frac{E_t}{E_{k,t}}\right]^{\frac{1}{\sigma}}. \tag{14}$$

In Section 3.2.4, we show that solving the capital goods provider's problem yields the price $p_t^x = (r_t + \delta)/\alpha$ which is common to all machine lines i.[15] One can then compute the aggregate raw physical capital demand of the final good sector, $K_{Y,t} = \int_0^1 x_{i,t}di$, which can be inserted into the production technology given by Eq. (10) to yield the aggregate production function

$$Y_t = A_{Y,t} Q_{Y,t}^{1-\alpha} K_{Y,t}^{\alpha} H_{Y,t}^{\beta} E_t^{\gamma}, \tag{15}$$

where $Q_{Y,t} = \int_0^1 q_{i,t}^{\frac{\alpha}{1-\alpha}} di$ stands for a quality index that is formally equivalent to the final sector technological level achieved through R&D.

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[15] As mentioned shortly, technological improvements are assumed to be sufficiently large such that the monopoly price can fully be charged. The alternative assumption that only a limit price can be set, as in O'Rourke et al. (2013), would not change the qualitative results of our model.

3.2.2 Primary energy extraction and refining into final forms

For each energy carrier, $k \in \mathcal{K}$, the provision of final energy, $\{E_{k,t}\}_{k\in\mathcal{K}}$, results from the combination of (i) a continuum of machines of measure one, $\{x_{k,j,t}\}_{j\in[0,1]}$ with specific endogenous quality, $\{q_{k,j,t}\}_{j\in[0,1]}$, (ii) human capital, $H_{k,t}$, and (iii) a primary energy input flow, $F_{k,t}$. Those elements are combined according to a Cobb-Douglas production function with constant returns-to-scale,

$$E_{k,t} = A_{k,t}\left[\int_0^1 (q_{k,j,t}x_{k,j,t})^{\alpha_k} dj\right] H_{k,t}^{\beta_k} F_{k,t}^{\gamma_k}, \tag{16}$$

where $\alpha_k + \beta_k + \gamma_k = 1$, and $A_{k,t}$ is the technological level achieved through learning-by-doing in sector k. The provision of the final energy inputs, $\{E_{k,t}\}_{k\in K}$, is perfectly competitive. Each final energy input is ultimately sold, at a price $p_{E_k,t}$, to the final good sector. In addition to the cost of sector-specific machines, $p^x_{k,j,t}$, and of human capital, w_t, the energy provider must also endure a convex cost, $\Psi(\cdot)$, associated with the extraction of primary energy.[16] This cost is a function of the remaining level of primary energy resource, $\mathcal{R}_{k,t}$, that is still not captured in the case of renewable energy, and still underground in the case of nonrenewable energy. In the case of a renewable resource, we have

$$\mathcal{R}_{k,t} = \mathcal{R}_{k,0} - F_{k,t-1}, \tag{17}$$

while in the case of a nonrenewable resource, we have

$$\mathcal{R}_{k,t} = \mathcal{R}_{k,0} - \sum_{a<t} F_{k,a}. \tag{18}$$

The set $\{\mathcal{R}_{k,0}\}_{k\in\mathcal{K}}$ is determined by the natural environment and corresponds to the levels of (renewable or nonrenewable) primary energy virgin resources.[17] Following Court et al. (2018), we suppose that as each stock of resource $\mathcal{R}_{k,t}$ gradually decreases towards zero, it becomes increasingly difficult to extract primary energy. On the other hand, technological improvements can lower the extraction cost of primary energy. We use the stock of applied knowledge, Q_t (introduced thereafter in Eq. (26) of Subsection 3.3), as a proxy for the technological advancements that decrease extraction costs, so as to extend the amount of economically profitable *reserves* out of physically bounded *resources*). We thus assume that all kinds of applied innovations contribute to decrease extractions costs, featuring a technological externality of applied technical progress. [18] We then define $\Psi(\mathcal{R}_{k,t}, Q_t)$ as follows

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[16] This extraction cost might also be seen as the price charged by a perfectly competitive primary energy extracting firm. Thus, our framework is neutral regarding an integrated or segmented energy sector.

[17] Nonrenewable and renewable primary energies are both physically bounded by the finite character of planet Earth. This point is straightforward for nonrenewable energies that come from finite stocks. The untaped level of a nonrenewable primary resource, $\mathcal{R}_{k,0}$, formally corresponds to the Ultimately Recoverable Resource (URR). According to British Petroleum (2015), the "URR is an estimate of the total amount of a given resource that will ever be recovered and produced. It is a subjective estimate in the face of only partial information. Whilst some consider URR to be fixed by geology and the laws of physics, in practice estimates of URR continue to be increased as knowledge grows, technology advances and economics change. The ultimately recoverable resource is typically broken down into three main categories: cumulative production, discovered reserves and undiscovered resource." Renewable energies are also bounded by the ultimate size of their annual flows (as an illustration, one might consider that, for a given year, the maximum solar energy ultimately recoverable cannot exceed the natural sun radiation), which is called the Technical Potential (TP) and corresponds to $\mathcal{R}_{k,0}$ in our framework. For the (IIASA, 2012, chapter 7, p.434), the renewable Technical Potential is "the degree of use that is possible within thermodynamic, geographical, or technological limitations without a full consideration of economic feasibility."

[18] For instance, steam engines were first used to pump out water from flooded coal mines. Despite their poor efficiencies, such a use of steam engines decreased extraction costs because older methods using mechanical power from horses were more inefficient and expansive. Moreover, the depth of accessible coal seams increased as a result of this technological breakthrough. Similarly, one can think about the appearance of water wheels (resp. fracking) as a new technology that implicitly decreased the capture and conversion costs of previously unused water flows (respec. shale oil deposits) so as to make them economically profitable.

$$\Psi(\mathcal{R}_{k,t}, Q_t) = \bar{\Psi}_k \mathcal{R}_{k,t}^{\psi_{1,k}} Q_t^{\psi_{2,k}}. \tag{19}$$

with $\bar{\Psi}_k > 0$, $\psi_{1,k} < 0$ and $\psi_{2,k} < 0$ some sector-specific parameters respectively measuring the scale and the convexity of the extraction costs. The technology of the energy provider being set, we can formally describe its behavior through an usual optimization problem.

PROBLEM (EP – FINAL ENERGY PROVIDER). *The energy input provider $k \in \mathcal{K}$ is perfectly competitive and uses the technologies defined in Eq. (16) and (19). The representative firm takes as given the price of energy inputs, $p_{E_k,t}$, the current level of technology, $A_{k,t}$, $\{q_{k,j,t}\}_{j\in[0,1]}$, and Q_t, as well as the current level of resource, $\mathcal{R}_{k,t}$ to solve*

$$\max_{\{x_{k,j,t}\}_{j\in[0,1]}, H_{k,t}, F_{k,t}} \quad p_{E_k,t} A_{k,t} \left[\int_0^1 (q_{k,j,t} x_{k,j,t})^{\alpha_k} dj\right] H_{k,t}^{\beta_k} F_{k,t}^{\gamma_k} - p^x_{k,j,t} \int_0^1 x_{k,j,t} dj - w_t H_{k,t} - \Psi(\mathcal{R}_{k,t}, Q_t) F_{k,t}$$

thus, in each sector $k \in \mathcal{K}$, setting demand schedules in machines, $x_{k,j,t}(\hat{v})$, human capital, $H_{k,t}(\hat{v})$ and primary energy, $F_{k,t}(\hat{v})$, where "$\hat{v}$" stands for "r_t, w_t, $\{p^x_{k,j,t}\}_{j\in[0,1]}$, $\Psi(\mathcal{R}_{k,t}, Q_t)$".

The first order condition corresponding to this program yield the optimal demand schedules for each factor as follows

$$\frac{p^x_{k,j,t}}{p_{E_k,t}} = \alpha_k \frac{A_{k,t} q_{k,j,t}^{\alpha_k} H_{k,t}^{\beta_k} F_{k,t}^{\gamma_k}}{x_{k,j,t}^{1-\alpha_k}}, \tag{20}$$

$$\frac{w_t}{p_{E_k,t}} = \beta_k \frac{E_{k,t}}{H_{k,t}}, \tag{21}$$

$$\frac{\Psi(\mathcal{R}_{k,t}, Q_t)}{p_{E_k,t}} = \gamma_k \frac{E_{k,t}}{F_{k,t}}. \tag{22}$$

Similarly to the final good sector, each machine line, $x_{k,j}$ is identified to an intermediate capital good supplied under monopolistic competition. Taking the optimal demand schedule defined by Eq. (20) as given, each innovator-producer owning a patent on the current highest generation of the machine line sets the price of the latter to maximize her profit. We retrieve the optimal price of the machines of the final good sector, equal for all machine line to $p^x_{k,t} = (r_t + \delta)/\alpha_k$.[19] As proposed *supra*, one can compute the aggregate raw physical capital demand for each energy input sector, $K_{k,t} = \int_0^1 x_{k,j,t} dj$, to yield the aggregate production function of each energy carrier

$$E_{k,t} = A_{k,t} Q_{k,t}^{1-\alpha_k} K_{k,t}^{\alpha_k} H_{k,t}^{\beta_k} F_{k,t}^{\gamma_k}, \tag{23}$$

where $Q_{k,t} = \int_0^1 q_{k,j,t}^{\frac{\alpha_k}{1-\alpha_k}} dj$ stands for a quality index in sector k that is formally equivalent to the technological level of sector k achieved through R&D.

3.2.3 Capital goods production

Machines used in final good and energy sectors are seen as intermediate capital goods produced under monopolistic competition from the stock of available raw capital. Machine lines in a specific

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[19] As mentioned *supra*, quality improvements are assumed to be sufficiently high for the full monopoly price to be charged.

sector, hereafter indexed by $u \in \{i; k, j\}$ can be treated in the same way assuming the relevant demand schedule. As a slight abuse of notation, the index u will also refer to the corresponding sector, $u \in \{Y, k\}$, whenever there is no ambiguity. Each machine producer is endowed with a one-period patent on her machine line. Either it is a successful innovator replacing a former incumbent in this specific line, or it results from a random allocation of property rights of incumbent technologies, precluding any congestion, at a purchasing price equal to $\pi_{u,t}$. This assumption, along with the free-entry condition into R&D introduced in Eq. (35), ensures that aggregate monopoly profits (ultimately redistributed to households) net of patent acquisition costs are null.[20]

For each machine line u, the production technology is linear and transforms one unit of the raw capital stock, K_t—which is rented from households at the interest rate r_t *plus* the depreciation rate of capital δ—to one unit of specialized machine $x_{u,t}$. Hence, the corresponding operating profit writes $\pi_{u,t}(x_{u,t}) = (p^x_{u,t} - r_t - \delta)x_{u,t}$, where $x_{u,t}$ stands for the demand in machine line u (given by Eqs (12) and (20)).[21] This behavior can be formalized through an usual optimization problem.

PROBLEM (CG – CAPITAL GOOD PRODUCERS). *Each capital good sector $u \in \{Y, k\}$ sustains a monopolistic competition setup where former successful innovators replace previous incumbents, acting as monopolists due to the patent they hold. In each sector-specific production line, u, the monopolist takes his specific demand schedule, $x_{u,t}$, as given by Eqs (12) and (20) as well as the price of capital, r_t, to solve*

$$\max_{p_{u,t}} \pi_{u,t} = (p_{u,t} - r_t - \delta)x_{u,t}(p_{u,t})$$

thus setting the price of intermediate capital goods, $p_{u,t}(r_t)$.

The solution immediately yields the price of intermediate capital goods, $p^x_t = (r_t + \delta)/\alpha$, which is common to all machine lines and every sector. To derive this result, we make a customary assumption of the patent-race literature. Hence, we suppose that the monopoly price can fully be charged without possibilities for the monopolist holding a patent on a previous version of the technology vintage to reap out the technology market. By doing so, we assume technological improvements to be sufficiently large such that a drastic innovation regime occurs (see Aghion and Howitt, 1992, for an analysis of the non-drastic case that yields similar comparative static results when the production function is of Cobb-Douglas type). An alternative view shall be to assume that patent holders on a specific machine line engage in a competing setting *a la Bertrand*, such that the most recent innovator (holding the highest quality) implements a limit price, ensuring him to remain below the marginal cost of the next quality vintage holder, such that he remains alone on the market. The key issue here is that each patent holder realizes a positive profit, yielding incentives to enter profit-motivated R&D activities.

3.3 Knowledge and endogenous technological change

Following Strulik et al. (2013), we have considered two kinds of technological progress: non-profit motivated learning-by-doing occurring in the final good sector, and profit-motivated R&D affecting both final and energy-producing sectors. As suggested by Schaefer et al. (2014), all these technological advancement should be interrelated by the evolution of a *General Purpose Technology* (GPT) denominated by G. Lipsey et al. (2005, p.98) define a GPT as "a single generic technology,

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[20] This assumptions prevent any issue of inter-temporal patent allocation and pricing, without precluding the set of incentives central to profit-motived R&D (Acemoglu, 2002; Aghion and Howitt, 1992, 1998).

[21] To derive this result we assume that, as a clearing condition on the capital market, the rental rate of capital shall equalize its rate of net return.

recognizable as such over its whole lifetime, that initially has much scope for improvement and eventually comes to be widely used, to have many uses, and to have many spillover effects”.[22]

3.3.1 Endogenous dynamics of GPTs

Following Schaefer et al. (2014), we assume that successive vintages of GPTs, $G_{v,t}$, are developed endogenously as a result of non-profit motivated activities. In the following setting, we do not suppose that a GPT directly affects the level of (learning-by-doing or R&D) technological progress, but rather that it increases the *efficiency* of those processes. In a sense, GPTs gather all kinds of technological externalities fostering technological change. We assume that several vintages of GPTs, indexed by v, succeed over time. While it is still active, the *level* of a given vintage of GPT might also evolve over time, featuring learning-effects. The magnitude of the latter phenomenon, along with the evolution of the expected duration before the arrival of a new GPT, have a crucial impact on the degree of complementarity between past and current knowledge. Together, those features allow to comply with two stylized facts regarding the historical arrival of GPTs: (i) the initially slow evolution of the efficiency of new GPTs, and (ii) the decreasing time interval between successive GPTs. We set apart from Schaefer et al. (2014) in two ways. First, following the endogenous growth literature centered on human capital (Jones, 1995; Strulik et al., 2013), we consider researchers (i.e., human capital allocated to R&D) rather than machines (i.e., lab-equipment purchased though financial expenditures) to be the key driver of innovation processes. Moreover, we consider that all kinds of technological progress are involved in the evolution of GPTs.

To start with, let’s assume that new GPT vintages follow a Poisson process with endogenous mean, μ_t, define as

$$\mu_t = \mu_0 G_{v,t-1}, \tag{24}$$

with $\mu_0 \in [0,1]$. As one can see, the level of the active GPT, $G_{v,t-1}$ ease the arrival of the new vintage.[23] Once discovered, a new GPT is initialized with a level of

$$G_{v+1,t} = \bar{G}\frac{\tilde{q}_t}{H_{t-1}} \tag{25}$$

where $\bar{G}$ is a positive scale parameter, H_{t-1} is the aggregate stock of human capital in previous period, and $\tilde{q}_t$ is an index of available applied knowledge. The normalization by the total stock of human capital, H_{t-1}, allows to account for the economic efficiency of researchers dedicated to R&D and workers involved in learning-by-doing. To further characterize the index of available applied knowledge, $\tilde{q}_t$, let us define two stocks of knowledge, $\tilde{Q}_{v+1,t}$ and $\tilde{Q}_{v,t}$. The former represents the improvement history of the current (and potentially newly introduced) GPT, whereas the latter tracks the improvement history of all previous vintages of GPTs. Introducing the aggregate quality index of the economy

$$Q_t = \sum_{X\in(A,Q),u\in(Y,\mathcal{K})} X_{u,t}, \tag{26}$$

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[22] Lipsey et al. (2005, p.97) further stress that GPTs are typically use-radical but not technology-radical, meaning that GPTs do not stand out from other technologies because of a revolutionary technological basis, but rather because of outstanding applications and adaptations to other technologies and sectors of the economy. GPTs are typically not born in their final form, so they often start off as something we would never call a GPT and then develop into something that transforms an entire economy. The considerable scope of improvement of GPTs is explored as their range and variety of use increase, which in the meantime generate knowledge and practical spillovers on other technologies and organizational processes.

[23] In other words, the time interval between successive GPT vintages, T, is given by the cumulative distribution: $\mathbb{P}(T \leq t)$ where T follows the Poisson process of mean μ, hence the average waiting time corresponds to $\mathbb{E}(T) = 1/\mu$.

that measures the extent of all technological developments through learning-by-doing and R&D, one can write the following identity

$$\tilde{Q}_{v+1,t} + \tilde{Q}_{v,t} = Q_t. \tag{27}$$

It is worth noting that the total quality index of the economy, Q_t, should be interpreted as the overall stock of *applied* knowledge —already introduced in the efficiency of the schooling technology, Eq. (3), and the extraction technology of primary energy inputs, Eq. (19)— that aggregates according to Eq. (27). Following Mokyr (1990, 2011), *applied* knowledge, taking the form of learning-by-doing and R&D technologies, shall be distinguished from *useful* knowledge contained in human capital and in GPT's waves that evolve concomitantly with the development of applied knowledges. The index of applied knowledge then just capture the quantity of applied knowledge that can be used to strengthen the current GPT vintage. Depending of the complementarity between past and current applied knowledge, quantified by the parameter $\zeta \in [0,1]$,[24] one can then define this index as

$$\tilde{q}_t = \left[\frac{\tilde{Q}_{v+1,t}}{H_{t-1}}\right]^{\zeta} \left[\frac{\tilde{Q}_{v,t}}{H_{t-1}}\right]^{1-\zeta}. \tag{28}$$

As long as it remains active, the quality increments of GPT vintage evolve over time through a logistic path that is a function of the quality index $\tilde{q}_t$,

$$G_{v,t} = G_{v,t-1} + \frac{\bar{G}}{1 + \left[\bar{G}/\underline{G} - 1\right] \exp(-\xi \bar{G}[\tilde{q}_t - \bar{q}])}, \tag{29}$$

with ξ, $\bar{q}$, $\underline{G}$ some constants. Hence, technological progress, through learning-by-doing or R&D, increases the index of applied knowledge, $\tilde{q}_t$, which raises the current GPT's level. Thus, the efficiency of each kind of technological knowledge is improved (as specified thereafter), strengthening again the level of the current GPT according to Eq. (27). Moreover, as GPTs are improved, the waiting time interval before the arrival of a new GPT decreases according to Eq. (24).

3.3.2 Technological progress through learning-by-doing

We model the technological level achieved through learning-by-doing in (final good and energy) sectors, $A_{u,t}$ with $u \in \{Y, \mathcal{K}\}$, as a function of (i) the current human capital stock allocated to the specific production sector, $H_{u,t}$, and (ii) the current GPT's level, $G_{v,t}$, capturing the conventional technological externality (i.e., the so-called standing-on-giants-shoulders effect).[25] With $\Omega > 0$ representing the efficiency with which useful knowledge contained in human capital and GPT-related know-how are converted into applied learning-by-doing knowledge for production, we have

$$A_{u,t+1} - A_{u,t} = \Omega H_{u,t}^{\omega_1} G_{v,t}^{\omega_2}, \quad u \in \{Y, \mathcal{K}\}. \tag{30}$$

We suppose that there are deceasing returns in both human capital and technological externalities, i.e., $\omega_1 \in]0,1[$ and $\omega_2 \in]0,1[$, meaning that in the long run there is no more technological progress through leaning-by-doing. This assumption calls for another source of technological progress, namely profit-motivated R&D presented below, to sustain growth in the long-run. We finally define the growth rate of the technological level (i.e., the technological change) obtained through learning-by-doing as $g_{A_u,t} \equiv \frac{A_{u,t} - A_{u,t-1}}{A_{u,t-1}} = \Omega H_{u,t-1}^{\omega_1} G_{v,t-1}^{\omega_2} A_{u,t-1}^{-1}$, with $u \in \{Y, \mathcal{K}\}$.

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[24] Past (respectively current) knowledge is useless whenever $\zeta = 1$ ($\zeta = 0$).

[25] Given that our formulation of $G_{v,t}$ depends on technological levels $A_{u,t}$ achieved through learning-by-doing, Eq. (30) is strictly in line with the formulation of Jones (1995).

3.3.3 R&D-based technological progress

As conventionally assumed in the endogenous growth literature (Acemoglu, 2002; Aghion and Howitt, 1992, 1998), R&D is profit-motivated. In each sector of innovation associated with a given productive sector (final good or final energy carrier), intermediate capital goods (i.e., machines) are produced by monopolists that are former successful innovators. We assume that each machine line follows a specific quality ladder, such that one can write the quality of machine line in a specific sector as $q_{u,t} = q_u^{\kappa_{u,t}}, \forall u \in \{i; k, j\}$, where $\kappa_{u,t}$ is the number of successful innovations up to time t and $q_u > 1$ stands for the sector-specific rung of each quality ladder. At the beginning of each period, successful innovations bring the corresponding sector-specific machines to the higher rung of the specific quality ladder, meaning that the corresponding $\kappa_{u,t}$ becomes $\kappa_{u,t} + 1$. Otherwise, the quality of machines remains constant. In what follows, we express R&D processes as a function of the level in the technological ladder of the corresponding machine line, κ, and show that this level ultimately disappear at the aggregated level. This is why, as a slight abuse of notation, we immediately refer to κ without distinguishing the corresponding sectoral machine line and temporal index (i.e. κ stands for $\kappa_{u,t}$). We follow Schaefer et al. (2014) and assume that the probability of success of a potential innovator in a specific sector u writes

$$\lambda_{u,\kappa,t} = \Phi_{u,t,\kappa} H_{R,u,t,\kappa} G_{v,t}, \tag{31}$$

where $\Phi_{u,t,\kappa}$ captures the increasing difficulty to perform R&D with the complexity κ of the production line of sector u, $H_{R,u,\kappa,t}$ stands for the amount of human capital dedicated to research in the machine line κ of sector u, and $G_{v,t}$ is the level of the current GPT vintage. It is worth mentioning that such a modeling choice of the probability of successfulness does not preclude *per se* an upper bound for productivity gains in each sector—in others words this probability is unlikely to reach its zero lower bound. As suggested by Ayres and Warr (2009, p.52-53), thermodynamics constraints of real processes necessarily imply an impossibility for infinite technological progress. Given that the present paper focuses on economic take-off and the energy transition associated with the Industrial Revolution, we consider that the introduction of such thermodynamics limits is beyond its scope. A functional form $\Phi_{u,t}(\cdot)$ capturing decreasing returns, as argued by Kortum (1993) and Stokey (1995), is proposed by Schiess and Wehrli (2008) as follows

$$\Phi_{u,\kappa,t} = \frac{1 - \lambda_{u,\kappa,t}}{\phi_u} q_u^{-\frac{\alpha_u}{1-\alpha_u}[\kappa+1]}, \tag{32}$$

where ϕ_u is a parameter capturing the cost of innovation in sector u. One can thus write the probability of an innovation success as

$$\lambda_{u,\kappa,t} = \frac{H_{R,u,\kappa,t} G_{v,t} q_u^{-\frac{\alpha_u}{1-\alpha_u}[\kappa+1]}}{\phi_u + H_{R,u,\kappa,t} G_{v,t} q_u^{-\frac{\alpha_u}{1-\alpha_u}[\kappa+1]}}. \tag{33}$$

As already mentioned, we follow Acemoglu et al. (2012) and Strulik et al. (2013) and assume that patents hold for one period (or one household's generation), afterwards monopoly rights are randomly reallocated among the pool of innovators. Thus, innovation decisions (and ultimately the free-entry condition defined shortly) are driven by the expected one-period monopoly profit that can be written as a function of the current level of the sector-specific machine line,

$$\pi_{u,\kappa,t} = \bar{\pi}_{u,t} \left[\frac{q_u^{\kappa}}{p_t^{x}}\right]^{\frac{\alpha_u}{1-\alpha_u}}, \tag{34}$$

where $\bar{\pi}_{u,t} = (1-\alpha)\left[\alpha A_{Y,t} H_{Y,t}^{\beta} E_t^{\gamma}\right]^{\frac{1}{1-\alpha}}$ if $u = i$ and $\bar{\pi}_{u,t} = (1-\alpha_k)\left[p_{E_k,t} \alpha_k A_{k,t} H_{k,t}^{\beta_k} F_{k,t}^{\gamma_k}\right]^{\frac{1}{1-\alpha_k}}$ otherwise. R&D processes and monopoly profits being defined, we can then describe the behavior of innovator through a free-entry condition.

Problem (R&D – Sectoral innovators). *Each R&D sector $u \in \{Y, \mathcal{K}\}$ is viewed as a pool of innovators, willing to enter the capital good production market through a successful innovation. Each potential monopolist takes as given the current level of general purpose technology, $G(v,t)$, in the targeted production line, κ, as well as the price of human capital, w_t, and maximize her expected profit, $\lambda_t(\kappa)\pi_{u,t}(\kappa+1)$, such that at the equilibrium the following free entry condition in R&D (i.e., that expected profit on innovation equals its cost) holds*

$$\lambda_{u,\kappa,t}\pi_{u,t}(\kappa+1) = w_t H_{R,u,\kappa,t} \tag{35}$$

This condition sets the level of the probability of successful R&D, which ultimately determine the optimal demand for human capital in each research sector, $H_{R,u,\kappa,t}(w_t, \pi_{u,t})$.

In writing this problem, we assume that funds are ultimately lent by households to potential innovators, and then repaid by profits (i.e. dividends) whenever innovation is successful. Substituting Eq. (34) into Eq. (35) gives the following expression of the free-entry condition in each research sector,

$$\lambda_{u,\kappa,t}\bar{\pi}_{u,t}\left[\frac{q_u^{\kappa+1}}{p_t^x}\right]^{\frac{\alpha_u}{1-\alpha_u}} = w_t H_{R,u,\kappa,t}. \tag{36}$$

Substituting Eq. (36) into Eq. (33) yields an explicit expression for the probability of successful R&D, that does not depend on the level in the quality ladder, κ, anymore[26]

$$\lambda_{u,\kappa,t} = 1 - \frac{\phi_u w_t [p_t^x]^{\frac{\alpha_u}{1-\alpha_u}}}{G_{v,t}\bar{\pi}_{u,t}}. \tag{37}$$

This expression is increasing in the level of the current GPT, $G(v,t)$, as well as an indicator of the value of innovation, $\bar{\pi}_{u,t}$, and decreasing in the sectoral research cost, ϕ_u, in the wage level, w_t, as well as in the cost of producing machines, p_t^x. An interesting analytical feature of this probability of success is that it is independent from the idiosyncratic level of quality of the sector-specific production line, κ. As a result, human capital allocations, $H_{R,u,\kappa,t}$, are uniformly distributed among all machines lines of a sector and do not depend on the level in the quality ladder, κ. In each sector, one can thus use Eqs. (31) and (32) to derive the aggregate amount of human capital dedicated to research as

$$H_{R,t} = \sum_{u\in\{Y,\mathcal{K}\}}\left[\frac{G_{v,t}\bar{\pi}_{u,t} - \phi_u w_t [p_t^x]^{\frac{\alpha_u}{1-\alpha_u}}}{G_{v,t} w_t [p_t^x]^{\frac{\alpha_u}{1-\alpha_u}}}\right] q_u^{\frac{\alpha_u}{1-\alpha_u}} Q_{u,t} \tag{38}$$

Besides, the law of motion of the quality index, $Q_{u,t}$, can be computed in each sector by using the law of large numbers ensuring that the probability of innovation success, $\lambda_{u,t}$, coincides with the fraction of machine-lines that will experience a success in R&D. This lead to the following quality dynamics

$$Q_{u,t+1} = \lambda_{u,t} q_u^{\frac{\alpha_u}{1-\alpha_u}} Q_{u,t} + (1-\lambda_{t,u})Q_{u,t}, \tag{39}$$

and thus the growth rate of innovation in each sector, $g_{Q_u,t}$, writes

$$g_{Q_u,t} = \left[q_u^{\frac{\alpha_u}{1-\alpha_u}} - 1\right]\lambda_{u,t}. \tag{40}$$

Finally, we can compute the aggregate technological growth rate of the economy, g_t, as a weighted average of the different technological growth rate obtained, either through learning-by-doing, $\{g_{A_{u,t}}\}_{u\in\{Y,\mathcal{K}\}}$, or through sector-specific R&D, $\{g_{Q_u,t}\}_{u\in\{Y,K\}}$. Formally, we have

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[26] This property is due to the assumption that rungs of the quality ladder are proportionally distant.

$$g_t = \sum_{X\in(A,Q),u\in(Y,\mathcal{K})} g_{X_{u,t}} \frac{X_{u,t}}{Q_t} \tag{41}$$

where Q_t has already been defined in Eq. (26).

3.4 Market-clearing and general equilibrium solution

At each time period, real flows must ensure that all markets —namely (i) final good, (ii) physical capital, (iii) human capital, and (iv) financial assets— clear, that is

$$Y_t = C_t + I_t + \sum_{k\in\{\mathcal{K}\}} \Psi_{k,t}(Q_t, \mathcal{R}_{k,t})F_{k,t}, \tag{42}$$

$$K_t = K_{Y,t} + \sum_{k\in\{\mathcal{K}\}} K_{k,t}, \tag{43}$$

$$H_t = H_{Y,t} + \sum_{k\in\{\mathcal{K}\}} H_{k,t} + H_{R,t}, \tag{44}$$

$$s_t N_t = I_t. \tag{45}$$

Those conditions ensure that the provision of real flows equal their uses. Moreover, physical and economic resource constraints shall hold in the provision of energy inputs —as already stated in Eq. (18) and (17)— and in the provision of capital goods. Hence, we have

$$\begin{aligned} R_{k,t} &= R_{k,0} - F_{k,t-1} \text{ for a renewable resource,} \\ R_{k,t} &= R_{k,0} - \sum_{a<t} F_{k,a} \text{ for an exhaustible resource,} \\ K_{Y,t} &= \int_0^1 x_{i,t}di, \text{ and } \forall k \in \{\mathcal{K}\}, K_{k,t} = \int_0^1 x_{k,j,t}dj. \end{aligned} \tag{46}$$

We know turn to the explicit definition of a decentralized dynamic general equilibrium in our theoretical framework.

DEFINITION 3.1. *An equilibrium is a sequence of level of per capita consumption,* $\{c_t\}$*, savings,* $\{s_t\}$*, fertility* $\{b_t\}$*, educational investment* $\{e_t\}$*, physical capital allocations,* $\{\{K_{u,t}\}_{u\in\{Y,\mathcal{K}\}}\}$*, human capital allocation,* $\{\{H_{u,t}\}_{u\in\{Y,\mathcal{K}\}}\}$*, final energy consumption and primary energies' extractions,* $\{E_t, \{F_{k,t}\}_{k\in\{\mathcal{K}\}}\}$*, as well as prices* $\{r_t, w_t, p_{E,t}, \{p_{E_{k},t}\}_{k\in\{\mathcal{K}\}}, \{p^x_{u,t}\}_{u\in\{Y,\mathcal{K}\}}\}$ *such that*

- *(i)* $\{c_t, s_t, b_t, e_t\}$ *solve Problem HH;*
- *(ii)* $\{\{x_{i,t}\}_{i\in[0,1]}, H_{Y,t}, \{E_{k,t}\}_{k\in\mathcal{K}}\}$ *solve Problem FG;*
- *(iii)* $\{\{x_{k,j,t}\}_{k\in\{\mathcal{K}\},j\in[0,1]}, \{H_{k,t}\}_{k\in\{\mathcal{K}\}}, \{F_{k,t}\}_{k\in\{\mathcal{K}\}}\}$ *solve Problem EP under primary energy resource constraints defined by Eq. (17) and (18);*
- *(iv)* $\{\{p^x_{u,i,t}\}_{u\in\{Y,\mathcal{K}\},i\in[0,1]}\}$ *solves Problem CG along with the free-entry condition of Problem R&D;*
- *(v)* $\{r_t\}$ *is such that the physical capital market clears, that is Eqs (43) and (45) hold;*
- *(vi)* $\{w_t\}$ *is such that the human capital market clears, that is Eq. (44) holds;*
- *(vii)* $\{\{x_{u,i,t}\}_{u\in\{Y,\mathcal{K}\},i\in[0,1]}\}$ *are such that the capital resource constraint is satisfied, that is Eq. (46) holds;*

(viii) *Population and human capital follow the endogenous dynamics described in Eq. (8) and (9);*

(ix) *GPTs are generated from a Poisson process of endogenous mean given by Eq. (24), and evolve according to Eq. (27) and (29);*

(x) *Learning-by-doing technological progresses endogenously evolve according to Eq. (30);*

(xi) *R&D-based technological progresses endogenously evolve according to Eq. (33) and (35).*

4 Analysis of the balanced growth path

The aim of this in progress section will be to analyze the balanced growth path (BGP) of the model.

5 Numerical analysis of the adjustment dynamics

The goal of this in progress section will be to calibrate the model on the historical experience of England for the period 1560–2010. In a second step, we plan to reiterate these simulations to compare the differential trajectories of Western Europe and Eastern Asia. This simulation exercise should allow an identification of the most crucial parameters explaining the differential timing of the transition from limited to sustained growth of these two world regions.

6 Conclusion

A summary of the contributions of this article will be given in concluding section, along with recommendations for future research.

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