Economic value of Information: Physically vs Economically optimized Water Quality Monitoring Network

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Abstract

Water quality monitoring networks are playing an important role in acquiring information concerning water quality. This has led to reflections regarding the type and the quantity of information provided by the monitoring networks and thus on the Economic Value of Information (EVOI).
In this study, we will focus on determining the optimal location of monitoring stations in a river in order to minimize detection time for accidental pollution. We construct a theoretical model to compare the EVOI of a physically optimized monitoring network, with only hydrological concerns, and an economically optimized monitoring network, with three scenarios of vulnerability along the river.
Our results show that the benefit of adding monitoring stations is decreasing with the number of stations. Then according to the cost of the monitoring station, a finite number of stations is recommended. Moreover, we show that the advantage of optimizing the EVOI compared to physical optimization is relative to the context, namely the number of stations, and the vulnerability scenarios. Then, according to the additional cost of economic optimization, the physical optimization could be recommended.

Keywords Water Resource Management; Quality Monitoring Network; Economic Value of Information; Optimal Location of Stations; Theoretical Modelling; Numerical Simulation.

1 Introduction

The water quality monitoring network is based on the acquisition of information concerning the physical, chemical and biological characteristics of a water body over time and space [17]. The primary purpose of a water quality-monitoring network is to provide a system that would provide sufficient and timely information to enable the decision maker (DM) to make informed decisions regarding the health risks linked to the population's exposure to this resource [20]. According to Harmancioglu et al. [10], the water quality monitoring network can be defined as an activity that collects and processes data on water quality.
The concept of obtaining information plays an important role in our society today. This concept has been used in a wide variety of scientific disciplines [13]. The tem *information* is defined as the knowledge communicated concerning a particular fact or circumstance. To take the best decisions, the DM has to know the states of nature. The monitoring networks provide information that can be considered similar to a random variable defined by messages and probabilities of occurrence regarding the states of nature.
Decades ago, developed countries became aware of the water contamination problems, caused by an increase in the population, and established strict legislative requirements regarding the wastewater disposal in rivers [1]. Yet, the reflection regarding the optimization of the network issues is relatively recent. In 2012, the European Commission (EC), in the first cycle report of the Water Framework Directive (WFD) [8], still underlined that the water quality monitoring was not satisfactory [9].
The implementation of the monitoring network faces two main issues: spatial and temporal. The spatial issues incorporate the numbers and locations of the monitoring stations, whereas temporal issues focus on the sampling frequencies. Several studies were published regarding the optimal network structure. The aim of this literature is to optimize the network, and namely to minimize imprecision concerning quality estimation. For Liu et al. [12] and Alvarez-Vázquez et al. [1] respectively, the optimized frequency and the optimized location allow the minimization of the average deviation for water pollution, in order to get a representative water quality. In Telci et al. [18], the location of the monitoring stations is optimal if it allows to minimize the detection time for accidental pollution. Do et al. [7] searched to obtain information regarding representability for water quality by using Sharp's procedure [19]. The aim is to locate monitoring stations in homogeneous water bodies. For Park et al. [13], the monitoring network has several objectives: the representativeness of river basins, compliance with water quality standards, supervision of water use, surveillance of pollution sources and examination of water

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quality changes/estimation of pollution loads. They use the genetic algorithm method with a multi-objective function in order to find the optimal location of monitoring stations.
Other studies have focused on assessing the Economic Value of Information (EVOI) of the water quality monitoring network. The value of information places a quantitative measure on the amount of information involved in any communication [11]. It has been used in several scientific areas such as health [12, 21], petroleum studies [2, 5, 18] and fisheries management [16]. EVOI evaluates the benefit of collecting additional information to reduce or eliminate uncertainty in a specific decision making context [21]. Studies about EVOI concerning water quality analyze how additional information (reduction of the imprecision on quality estimation) regarding water quality allows the DM to take the best decision (to implement the most appropriated actions according to the state of nature). For Bouma et al. [4], additional information is the use of satellite observations for water quality management in the North Sea. The objective of the study was to find the best policy to manage eutrophication. For Bouma et al. [3], the objectives was to choose between a uniform or spatialized policy to protect the Great Barrier Reef. Destandau and Diop [6] have a more theoretical approach. They identify different parameters that have an impact on the EVOI: prior probabilities on the state of nature, costs linked to a poor decision and accuracy of additional information. They study the impact of these parameters on the EVOI.
Hence, two types of literature for water quality monitoring networks exist. The first one considers physical optimization of network issues to reduce the imprecision regarding quality estimation. The second one analyzes how the modification of this imprecision influences the economic value of information. We aim in this article to combine the two types of literature by presenting a theoretical work for optimizing the location of the monitoring station. We will present two ways to optimize the location of the monitoring stations. The first one by taking into account physical (hydrologic) considerations and the second economic ones. The EVOI of both methods are compared. As for our knowledge, this is the first time that such a work, combining the two types of literature and two types of optimization, has been done.
The remaining part of this paper is organized as follows: section 2 presents the hypotheses and the location of the monitoring stations that are physically optimized. In section 3, we present the EVOI for a physically optimized monitoring network according to three scenarios of vulnerability of the river. In section 4, the location of the stations aims at maximizing the EVOI, what we call the economic optimization. Section 5 compares the EVOI of networks physically and economically optimized. Section 6 concludes.

2 Physical optimization of the network

2.1 Hypotheses

We suppose a river, represented by a segment $[0,1]$. More specifically, we denote the location $l = 0$ as the most upstream: the source, and the location $l = 1$ as the most downstream: the outlet of the river.
The aim is to determine the optimal location of n monitoring stations. In accordance with Telci et al. [20], the objective is to minimize the detection time for accidental pollution. Unlike Telci et al. [20], we suppose a homogeneous stream in a way that the detection time for accidental pollution corresponds to the distance between the location where accidental pollution is emitted, l_x, and the location of the monitoring station where the pollution is detected, l_y (Figure 1).

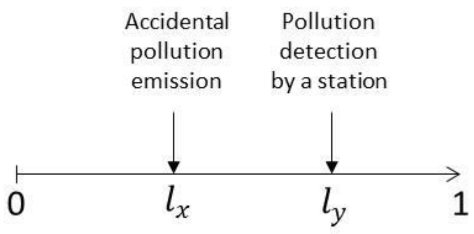

Fig. 1 River Stream

In our model, monitoring stations will always detect an upstream pollution. This article doesn't deal with the issue of measuring frequency. It can be imagined as a continuous measurement. Moreover, the quantity of pollution doesn't matter, but only whether an accidental pollution occurs or not.

2.2 Location of the monitoring stations

The objective is to find the location $\{l_{1/n} \dots l_{y/n} \dots l_{n/n}\}$ of n monitoring stations that minimizes detection time for an accidental pollution that could be emitted at any point $l_x \in [0,1]$. This is what we call the physical optimization of a monitoring network. From this perspective, we take into account hydrological considerations but not economic.
We denote T: the detection time for an accidental pollution. Optimization program can then be written as follows:

$$Min\, T = \sum_{y=1}^{n+1} \int_{l_{(y-1)/n}}^{l_{y/n}} \left(l_{y/n} - l_x\right) dl_x \quad \forall\, n \in [1; +\infty[\quad (1)$$

With $l_{0/n}$ the source of the river and $l_{(n+1)/n}$ the outlet. Then: $l_{0/n} = 0$, and $l_{(n+1)/n} = 1$.

With two monitoring stations located at $l_{(y-1)/n}$ and $l_{(y+1)/n}$, the optimal location of the y^{th} station: $l_{y/n}$, is obtained as follows:

$$Min\, T = \int_{l_{(y-1)/n}}^{l_{y/n}} (l_{y/n} - l_x) dl_x + \int_{l_{y/n}}^{l_{(y+1)/n}} (l_{(y+1)/n} - l_x) dl_x$$

$$\Leftrightarrow \; Min\, T = l_{y/n}^2 - l_{(y-1)/n}.\, l_{y/n} - l_{y/n}.\, l_{(y+1)/n} + \frac{l_{(y-1)/n}^2}{2} + \frac{l_{(y+1)/n}^2}{2}$$

$$\Leftrightarrow \; \frac{\partial T}{\partial l_{y/n}} = 2 l_{y/n} - l_{(y-1)/n} - l_{(y+1)/n} = 0$$

$$\Leftrightarrow \; l_{y/n} = \frac{l_{(y+1)/n} + l_{(y-1)/n}}{2}$$

Furthermore:

$$\frac{\partial^2 T}{\partial l_{y/n}^2} = 2 > 0$$

Hence, the optimal location of the monitoring station is the one that divides the stream into two equal parts. We use the following formula to determine the location of the monitoring stations (Figure 2):

$$l_{y/n} = \frac{y}{n+1} \quad \forall n \in [1; +\infty[\; ; \; \forall\, y \in [1, n] \quad (2)$$

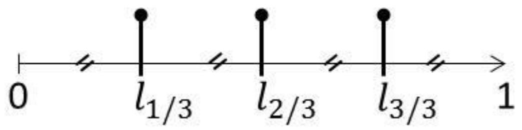

Fig. 2 Physically optimized location of three monitoring stations.

3 Economic Value of Information for a physically optimized network

3.1 Economic Value of Information

3.1.1 Methodology

To compute the EVOI, we are inspired by the Bayesian model of Destandau and Diop [6]. They construct a theoretical model by considering that two states of nature exist: one less sensitive to eutrophication $\overline{s}$, and one more sensitive, $\underline{s}$. They assign two possible actions for both states of nature, for which we each give a name, $\overline{a}$ and $\underline{a}$, corresponding to the appropriate state. Utility was based on the cost of the actions and the damage linked to eutrophication.

By designating $I(i_{\bar{s}}, i_{\underline{s}}\,; P(i_{\bar{s}}), P(i_{\underline{s}}))$ the random variable that corresponds to the additional information provided by the monitoring network composed of messages and their probabilities on the state of nature, and $U_{a/s}$ the Utility of the action a when the state of nature is s, the EVOI was written as follows:

$$EVOI = P(i_{\bar{s}}).\left[P(\bar{s}/i_{\bar{s}}).U_{\bar{a}/\bar{s}} + P(\underline{s}/i_{\underline{s}}).U_{\bar{a}/\underline{s}}\right] + P(i_{\underline{s}}).\left[P(\underline{s}/i_{\underline{s}}).U_{\underline{a}/\bar{s}} + P(\bar{s}/i_{\underline{s}}).U_{\underline{a}/\bar{s}}\right] \quad (3)$$

If we consider $'K'$ infinite states of nature and $'K'$ possible actions, EVOI (3) can then be written as follows:

$$EVOI = \sum_{k=1}^{K} P(i_{s_k}).\left[\sum_{k'=1}^{K'} P(s_{k'}/i_{s_k}).U_{a_k/s_{k'}}\right] \quad (4)$$

3.1.2 Application to our model

With n stations, we have $(n+2)$ states of nature, namely: $s_{\emptyset}$, where no accidental pollution is emitted., $s_{[l_{(y-1)/n}, l_{y/n}]}$), where accidental pollution is emitted between $l_{(y-1)/n}$ and $l_{y/n}$, and $s_{[l_{n/n}, out]}$ when accidental pollution is emitted between the last station and the outlet of the river.

$$\left\{s_{\emptyset}, s_{[0,l_{1/n}]}, s_{[l_{1/n},l_{2/n}]}, \dots, s_{[l_{(y-1)/n},l_{y/n}]}, \dots, s_{[l_{n/n},out]}\right\}$$

Then, the network can deliver $(n+2)$ messages, depending on whether no pollution is detected, pollution is detected by one of the n monitoring stations, and whether an accidental pollution is detected at the outlet of the network:

$$\left\{i_{\emptyset}, i_{l_{1/n}}, i_{l_{2/n}}, \dots, i_{l_{y/n}}, \dots, i_{out}\right\}.$$

If pollution is emitted at a point l_x and detected at a point $l_{y/n}$, the DM implement an action a in order to stop environmental damage denoted $D_{l_x, l_{y/n}}$. Without this action, environmental damage will be $D_{l_x, out}$. The action implemented by the DM has a cost C.
If the accidental pollution is detected at the outlet of the stream, it will be too late to react and it will not be necessary to implement the action. Environmental damage is then $D_{l_x, out}$.
We suppose that the network makes no mistakes. If there is no accidental pollution detected at a monitoring station or at the outlet of the river, this means that no accidental pollution was emitted. Moreover, if pollution is detected at a monitoring station or at the outlet of the river, this means that there is indeed an accidental pollution emitted just upstream.
Then, in the equation (4), $P(s_{k'}/i_k)$ is equal to 0, if $k' \neq k$, and is equal to 1 if $k' = k$.

The Utility is estimated compared to the status-quo, namely, for us, no action. Then, the Utility is the difference between environmental damage $D_{l_x, out} - D_{l_x, l_{y/n}} = D_{l_{y/n}, out}$ to which we subtract the cost C of the action that stop the environmental damage. If no accidental pollution is detected or if accidental pollution is detected at the outlet of the river, Utility is null.
Considering n monitoring stations, equation (4) is rewritten as follows:

$$EVOI_n = \sum_{y=1}^{n} P(i_{l_{y/n}}).\left[D_{l_{y/n}, out} - C\right] \quad (5)$$

3.2 Economic value of information: three scenarios

We denote P the probability of existence for accidental pollution. The pollution could be emitted at any point in the stream with equal probability.
With a physically optimized network, monitoring stations are located as defined by the equality (2) of section (2.2). EVOI from (5) becomes:

$$EVOI_n = \sum_{y=1}^{n} \frac{P}{n+1}.\left[D_{\frac{y}{n+1}, out} - C\right] \quad (6)$$

In order to determine $D_{\frac{y}{n+1},out}$, we suppose three scenarios. In the first scenario (section 3.2.1), we suppose a uniform vulnerability. Environmental damage will depend only on the distance between the emission of pollution and its detection, whatever the upstream or downstream positioning in the river.
In the second scenario (section 3.2.2), we suppose a decreasing vulnerability. Pollution will generate more damage at the upstream of the river.
In the third and final scenario (section 3.2.3), we suppose an increasing vulnerability. Pollution will generate more damage at the downstream of the river.

3.2.1 EVOI with a uniform vulnerability scenario

Figure 3 illustrates the first hypothesis, namely the uniform vulnerability. Marginal damage is calculated by using the function $f^0(l) = \delta$.

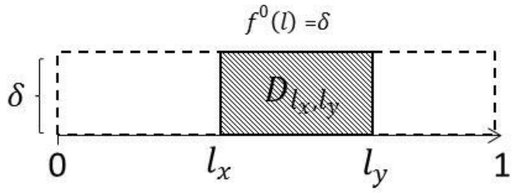

Fig. 3 Uniform vulnerability

By integrating the hypothesis of uniform vulnerability into equation (6), we can rewrite the EVOI as follows:

$$EVOI_n^0 = \sum_{y=1}^{n} \frac{P}{n+1} . \left[\left(1 - \frac{y}{n+1}\right) . \delta - C \right]$$

$$\Leftrightarrow \quad EVOI_n^0 = P\left(\frac{n}{n+1}\right)\left(\frac{\delta}{2} - C\right) \quad (7)$$

3.2.2 EVOI with a decreasing vulnerability scenario

Figure 4 illustrates the second hypothesis regarding the decreasing vulnerability. Marginal damage is given by the following function: $f^{-}(l) = -2\delta l + 2\delta$.

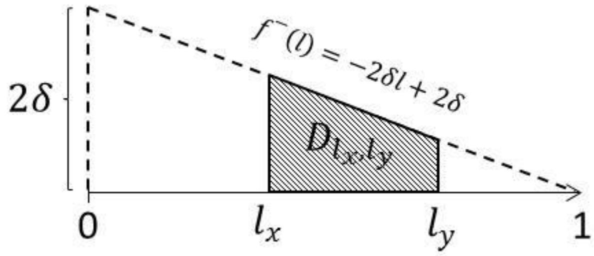

Fig. 4 Decreasing vulnerability

By integrating the hypothesis of decreasing vulnerability into equation (6), we can rewrite the EVOI as follows:

$$EVOI_n^{-} = \sum_{y=1}^{n} \frac{P}{n+1} . \left[\int_{\frac{y}{n+1}}^{1} -2\delta l + 2\delta \ dl - C \right]$$

$$\Leftrightarrow EVOI_n^{-} = P\left(\frac{n}{n+1}\right)\left[\delta\left(\frac{2n+1}{6n+6}\right) - C\right] \quad (8)$$

3.2.3 EVOI with an increasing vulnerability scenario

Figure 5 illustrates the third hypothesis for the increasing vulnerability. Marginal damage is given by the following function: $f^{+}(l) = 2\delta l$.

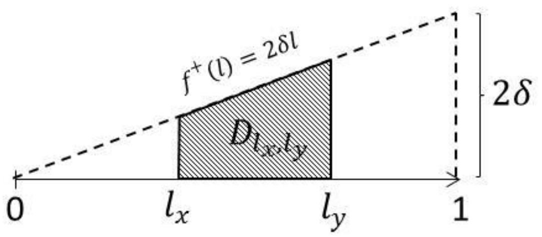

Fig. 5 Increasing vulnerability

By integrating the hypothesis of increasing vulnerability into equation (6), we can rewrite the EVOI as follows:

$$EVOI_n^{+} = \sum_{y=1}^{n} \frac{P}{n+1} \cdot \left[\int_{\frac{y}{n+1}}^{1} 2\delta l \, dl - C\right]$$

$$\Leftrightarrow \; EVOI_n^{+} = P\left(\frac{n}{n+1}\right)\left[\delta\left(\frac{4n+5}{6n+6}\right) - C\right] \quad (9)$$

3.3 Discussion

In the three scenarios, we note that the EVOI can take a negative value if the cost of implementing the action is excessive, respectively when the cost is higher than $\frac{\delta}{2}$, $\delta\left(\frac{2n+1}{6n+6}\right)$, and $\delta\left(\frac{4n+5}{6n+6}\right)$. In this case, it is not preferable to take action to stop damage.

In the case of decreasing vulnerability or increasing vulnerability, the condition for a positive EVOI depends on the number of monitoring stations.

In the decreasing vulnerability scenario, $\left(\frac{2n+1}{6n+6}\right)$ is an increasing function of n[3], the condition $C < \frac{1}{4}\delta$ is sufficient to get a positive EVOI.

In the increasing vulnerability scenario, $\left(\frac{4n+5}{6n+6}\right)$ is a decreasing function that converge to $\left(\frac{2}{3}\right)$[4], the condition $C < \frac{2}{3}\delta$ is sufficient to get a positive EVOI.

When the positive EVOI conditions are obtained, we find, for the scenarios 1 and 2, that when the number of monitoring stations is increased, the EVOI increases with a decreasing rate. This is not the case with the third scenario (see Appendix).

These results show that a physical optimization of the network do not lead to an EVOI optimization, what is the issue of the next section.

4 Optimization of the economic value of information

4.1 Economic considerations

In the previous section, we have seen that the EVOI could take a negative value according to the value of the cost C. The reason is that the network was optimized only by taking into account physical or hydrological considerations. Therefore, for monitoring stations located at the very downstream of the river, the cost of action could be higher than the damage saved.

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[3] $\frac{\partial\left(\frac{2n+1}{6n+6}\right)}{\partial n} = \frac{1}{6(n+1)^2} > 0$

[4] $\frac{\partial\left(\frac{4n+5}{6n+6}\right)}{\partial n} = -\frac{1}{6(n+1)^2} < 0$; $\lim_{n\to+\infty}\left(\frac{4n+5}{6n+6}\right) = \frac{4n}{6n} = \frac{2}{3}$

In order to optimize EVOI, the location of the monitoring stations must take into account economic considerations, namely the cost of the action and the ecological damage. The location of the monitoring station $l_{y/n}$ should first respect the following condition:

$$\int_{l_{y/n}}^{1} f(l)dl > C \qquad \forall n \in [1;+\infty[\ ;\ \forall\, y \in [1,n] \quad (10)$$

If this condition is not met, the action to stop the damage is not economically rational.
The network structure should also take into account the vulnerability along the river. The objective of the network is no longer to minimize the detection time for an accidental pollution, but to minimize the damage generated by this pollution. A pollution that generates less damage could be detected later.
Then, equation (1) of the section (2.2) is rewritten as follows:

$$Min\ D = \sum_{y=1}^{n+1} \int_{l_{(y-1)/n}}^{l_{y/n}} D_{l_x, l_{y/n}} dl_x \qquad \forall n \in [1;+\infty[\quad (11)$$

4.2 Uniform vulnerability

4.2.1 Location of the monitoring stations

With the hypothesis: $f^0(l) = \delta$, the condition (10) becomes:

$$l^0_{y/n} < 1 - \frac{C}{\delta} \qquad \forall n \in [1;+\infty[\ ;\ \forall\, y \in [1,n] \quad (12)$$

Then, there is no station if:

$$1 - \frac{C}{\delta} < 0 \quad \Leftrightarrow \quad C < \delta \quad (13)$$

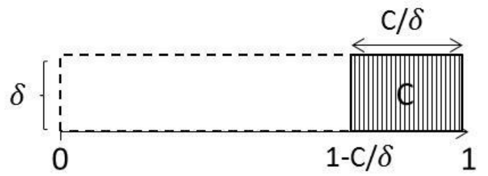

Fig. 6 Part of the river where monitoring stations could be localized or not in scenario 1

In the uniform vulnerability scenario, minimizing detection time or minimizing environmental damage is the same. The demonstration in section 2.2 remains valid to indicate that the monitoring stations divide the part of the river where stations could be located (Figure 6) into equal parts.
The location of the monitoring stations in equation (2) becomes:

$$l^{*0}_{y/n} = \frac{y}{n+1}\left(1 - \frac{C}{\delta}\right) \qquad \forall n \in [1;+\infty[\ ;\ \forall\, y \in [1,n] \quad (14)$$

4.2.2 Economic value of information

EVOI from equation (5) becomes:

$$EVOI^{*0}_n = \sum_{y=1}^{n} \frac{P}{n+1}\left(1 - \frac{C}{\delta}\right).\left[\left(1 - \frac{y}{n+1}\left(1 - \frac{C}{\delta}\right)\right).\delta - C\right]$$

$$\Leftrightarrow \quad EVOI^{*0}_n = P\left(\frac{n}{n+1}\right)\left(\delta - 2C + \frac{C^2}{\delta}\right) \quad (15)$$

The condition (13) ensures that the EVOI in equation (15) is always positive. In addition, it increases at a decreasing rate according to n. In fact:

$$\frac{\partial EVOI_n^{*0}}{\partial n} = \frac{P}{(n+1)^2}\left(\delta - 2C + \frac{C^2}{\delta}\right) > 0$$

$$\frac{\partial^2 EVOI_n^{*0}}{\partial n^2} = -P\frac{2}{(n+1)^3}\left(\delta - 2C + \frac{C^2}{\delta}\right) < 0$$

Considering the cost of the monitoring, increasing the number of monitoring stations indefinitely is not rational.

4.3 Decreasing vulnerability

4.3.1 Location of the monitoring stations

With the hypothesis: $f^{-}(l) = -2\delta l + 2\delta$, condition (10) becomes:

$$l_{y/n}^{-} < 1 - \sqrt{\frac{C}{\delta}} \quad \forall n \,\epsilon\, [1; +\infty[\; ;\ \forall\, y \,\epsilon\, [1, n] \quad (16)$$

There is no stations if:

$$1 - \sqrt{\frac{C}{\delta}} < 0 \quad \Leftrightarrow \quad C < \delta \quad (17)$$

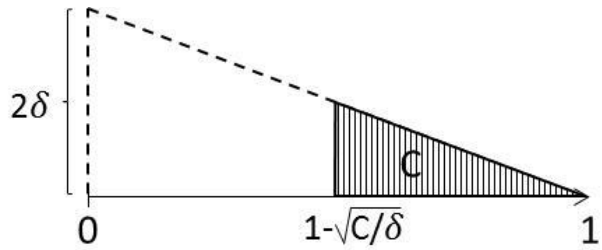

Fig. 7 Part of the river where monitoring stations could be located or not in scenario 2

To locate the monitoring stations in the part of the river where it could be (Figure 7), the optimization program is written as follows:

$$Min\ D^{-} = \sum_{y=1}^{n+1} \int_{l_{(y-1)/n}}^{l_{y/n}} \left[\int_{l_x}^{l_{y/n}} -2\delta l + 2\delta\ dl\right] dl_x$$

With two monitoring stations located at $l_{(y-1)/n}$ and $l_{(y+1)/n}$, the optimal location $l_{y/n}$ of the y^{th} monitoring station is computed as follows:

$$Min\ D^{-} = \int_{l_{(y-1)/n}}^{l_{y/n}} \left[\int_{l_x}^{l_{y/n}} -2\delta l + 2\delta\ dl\right] dl_x + \int_{l_{y/n}}^{l_{(y+1)/n}} \left[\int_{l_x}^{l_{(y+1)/n}} -2\delta l + 2\delta\ dl\right] dl_x$$

$$\frac{\partial D^{-}}{\partial l_{y/n}} = -3l_{y/n}^2 + \left(4 + 2l_{(y-1)/n}\right)l_{y/n} - 2l_{(y-1)/n} + l_{(y+1)/n}^2 - 2l_{(y+1)/n} = 0$$

$$\frac{\partial^2 D^{-}}{\partial l_{y/n}^2} = -6.\, l_{y/n} + 2l_{(y-1)/n} + 4 > 0 \quad (18)^5$$

5 This condition should be checked in the case of decreasing vulnerability, to ensure that the location of the selected stations minimizes the damage.

$$\Leftrightarrow$$

$$l_{y/n}^{*-} = \frac{1}{3}\left[2 + l_{(y-1)/n}^{*-} - \sqrt{l_{(y-1)/n}^{*-\,2} - 2l_{(y-1)/n}^{*-} + 3l_{(y+1)/n}^{*-\,2} - 6l_{(y+1)/n}^{*-} + 4}\right], \quad \forall n \in [1; +\infty[\,;\ \forall\, y \in [1, n] \quad (19)$$

4.3.2 Economic value of information

The EVOI from equation (5) becomes:

$$EVOI_n^{*-} = \sum_{y=1}^{n} P(i_{l_{y/n}^{*-}}).\left[D_{l_{y/n}^{*-},out}^{-} - C\right]$$

$$\Leftrightarrow$$

$$EVOI_n^{*-} = P\sum_{y=1}^{n} (l_{y/n}^{*-} - l_{(y-1)/n}^{*-}).\left[\delta + \delta l_{y/n}^{*-2} - 2\delta l_{y/n}^{*-} - C\right] \quad (20)$$

With the location of the monitoring stations calculated in equation (19).

4.4 Increasing vulnerability

4.4.1 Location of the monitoring stations

With the hypothesis: $f^+(l) = 2\delta l$, condition (10) becomes:

$$l_{y/n}^{+} < \sqrt{1 - \frac{C}{\delta}} \quad \forall n \in [1; +\infty[\,;\ \forall\, y \in [1, n] \quad (21)$$

There is no station if:

$$1 - \frac{C}{\delta} < 0 \quad \Leftrightarrow \quad C < \delta \quad (22)$$

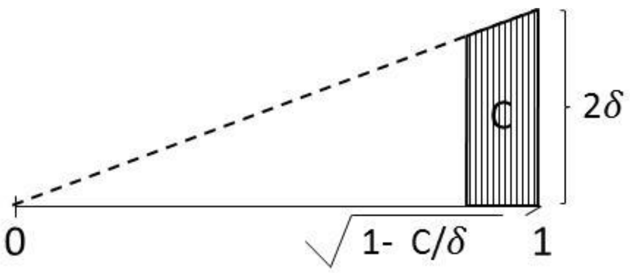

Fig. 8 Part of the river where monitoring stations could be located or not in scenario 3

To locate the monitoring stations in the part of the river where it could be (Figure 8), the optimization program is written as follows:

$$Min\ D^+ = \sum_{y=1}^{n+1} \int_{l_{(y-1)/n}}^{l_{y/n}} \left[\int_{l_x}^{l_{y/n}} 2\delta l\, dl\right] dl_x$$

With two monitoring stations located at $l_{(y-1)/n}$ and $l_{(y+1)/n}$, the optimal location $l_{y/n}$ of the y^{th} monitoring station is computed as follows:

$$Min\ D^+ = \int_{l_{(y-1)/n}}^{l_{y/n}} \left[\int_{l_x}^{l_{y/n}} 2\delta l\, dl\right] dl_x + \int_{l_{y/n}}^{l_{(y+1)/n}} \left[\int_{l_x}^{l_{(y+1)/n}} 2\delta l\, dl\right] dl_x$$

$$\frac{\partial D^+}{\partial l_{y/n}} = 3l_{y/n}^2 - 2l_{y/n}l_{(y-1)/n} - l_{(y+1)/n}^2 = 0$$

$$\frac{\partial^2 D^+}{\partial l_{y/n}^2} = 6.\, l_{y/n} - 2l_{(y-1)/n} > 0$$

$$\Leftrightarrow$$

$$l_{y/n}^{*+} = \frac{1}{3}\left[l_{(y-1)/n}^{*+} + \sqrt{l_{(y-1)/n}^{*+\,2} + 3l_{(y+1)/n}^{*+\,2}}\right] \quad \forall n \in [1; +\infty[\ ;\ \forall\, y \in [1, n] \quad (23)$$

4.4.2 Economic value of information

The EVOI from equation (5) becomes:

$$EVOI_n^{*+} = \sum_{y=1}^{n} P(i_{l_{y/n}^{*+}}).\left[D_{l_{y/n}^{*+},out}^{+} - C\right]$$

$$\Leftrightarrow$$

$$EVOI_n^{*+} = P\sum_{y=1}^{n}(l_{y/n}^{*+} - l_{(y-1)/n}^{*+}).\left[\delta - \delta l_{y/n}^{*+\,2} - C\right] \quad (24)$$

With the location of the monitoring stations calculated in equation (23).

5 Network physically vs economically optimized

Now, we compare the values of information of a physically optimized network (section 3), and of an economically optimized network (section 4).
We will only solve the ratio of both EVOIs algebraically for scenario 1, case of a uniform vulnerability. For the three scenarios, we will observe the difference between both EVOIs by a simulation, with the values of parameters, chosen arbitrarily, as follows:

$$P = 10\%\ ;\ \delta = 10\ ; C = 2 \quad (25)$$

The hypothesis (24) respects the condition of positivity of all the EVOIs.

5.1 Uniform vulnerability

According to the results from equation (14), we can compute the location of the monitoring stations economically optimized. We simulate our model by using Excel. We represent the results in table 1:

Table 1 Economically optimized locations of the monitoring stations from scenario 1

n	L1	L2	L3	L4	L5
1	0,400				
2	0,267	0,533			
3	0,200	0,400	0,600		
4	0,160	0,320	0,480	0,640	
5	0,133	0,267	0,400	0,533	0,667

From the results of (7) and (15) we have:

$$\frac{EVOI_n^{*0}}{EVOI_n^{0}} = \frac{P\left(\frac{n}{n+1}\right)\left(\delta - 2C + \frac{C^2}{\delta}\right)}{P\left(\frac{n}{n+1}\right)\left(\frac{\delta}{2} - C\right)} = \frac{\left(\delta - 2C + \frac{C^2}{\delta}\right)}{\left(\frac{\delta}{2} - C\right)} = \frac{(\delta - C)^2}{\delta\left(\frac{\delta}{2} - C\right)}$$

With $C < \frac{\delta}{2}$.

We note that the relationship between the EVOIs of economically optimized network and physically optimized network depends on the number of monitoring stations.
The ratio decreases with the damage δ and increases with the cost C. Under condition (13): we find:

$$\frac{\partial\left(\frac{EVOI_n^{*0}}{EVOI_n^0}\right)}{\partial\delta} = \frac{C^3 - \delta C^2}{\left(\frac{\delta^2}{C} - \delta C\right)} < 0$$

$$\frac{\partial\left(\frac{EVOI_n^{*0}}{EVOI_n^0}\right)}{\partial C} = \frac{C\delta^2 - \delta C^2}{\left(\frac{\delta^2}{C2} - \delta C\right)} > 0$$

From hypotheses (25), we find that economic optimization can multiply the value of information by 2.133. Figure 9 illustrates the EVOI of physically and economically optimization, with accordance to the number of monitoring stations and these hypotheses.

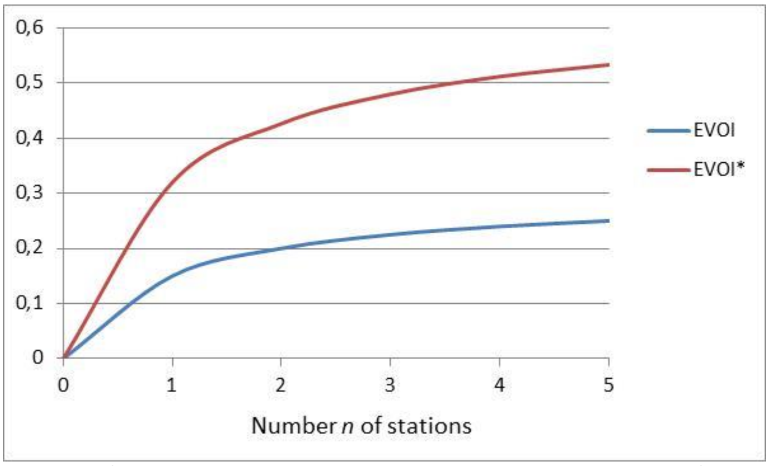

Figure 9 EVOI of physically (EVOI) and economically (EVOI*) optimized networks for scenario 1

Figure 9 illustrates that, in absolute value, the higher the number of monitoring stations is, the more their location need to be chosen rationally, taking into account economic considerations.

5.2 Decreasing vulnerability

The results of equation (19) and the hypotheses of equation (25) give us the economically optimized location of the monitoring stations. Results are shown in Table 2. Condition (18) is met.

Table 2 Economically optimized locations of the monitoring stations from scenario 2

n	L1	L2	L3	L4	L5
1	0,245				
2	0,160	0,339			
3	0,119	0,247	0,389		
4	0,095	0,195	0,303	0,420	
5	0,079	0,161	0,248	0,341	0,441

Comparing Table 1 and Table 2, we note that, with a decreasing vulnerability, the stations are located further upstream.
From (8) and (20), and from the hypotheses (25), we compare the EVOI for scenario 2. Results are represented in Table 3 and Figure 10:

Table 3 EVOI of physically (EVOI) and economically (EVOI*) optimized networks for scenario 2

n	EVOI	EVOI*	EVOI*/EVOI
0	0	0	-
1	0,025	0,091	3,6262
2	0,052	0,123	2,3780
3	0,069	0,140	2,0386
4	0,080	0,150	1,8805
5	0,088	0,157	1,7890

If we compare Figures 9 and 10, we note that the value of information is weaker when we have a decreasing vulnerability. In absolute value, the loss related to a location of non-economically optimal stations is less important. In relative value, this loss is important for few stations, but decreases with the number of stations.

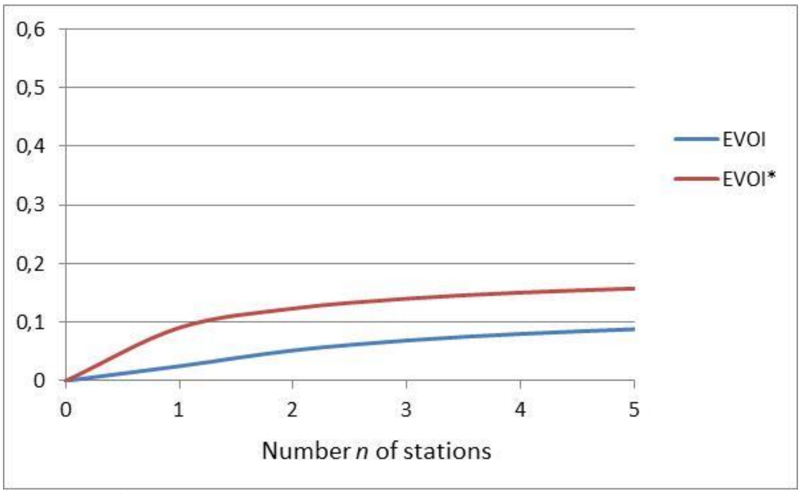

Fig. 10 EVOI of physically (EVOI) and economically (EVOI*) optimized networks for scenario 2

5.3 Increasing vulnerability

The results of equation (23) and the hypotheses of equation (25) give us the economically optimized location of the monitoring stations. Results are shown in Table 4.

Table 4 Economically optimized locations of the monitoring stations from scenario 3

n	L1	L2	L3	L4	L5
1	0,516				
2	0,380	0,658			
3	0,308	0,533	0,724		
4	0,262	0,453	0,616	0,761	
5	0,230	0,398	0,540	0,668	0,785

Comparing to the first two scenarios, the stations are located more downstream when we have an increasing vulnerability (Table 4).
From (9) and (22), and from the hypotheses (25), we compare the EVOIs for scenario 3. Results are represented in Table 5 and Figure 11:

Table 5 EVOI of physically (EVOIO) and economically (EVOI*) optimized networks for scenario 3

n	EVOI	EVOI*	EVOI*/EVOI
0	0	0	-
1	0,275	0,275	1,001
2	0,348	0,351	1,009
3	0,381	0,386	1,012
4	0,400	0,406	1,014
5	0,412	0,419	1,016

We conclude that with increasing vulnerability, failure to take economic considerations into account has little impact on the value of information. A physical optimization of the stations locations can therefore be justified, in the case where the search for an economic optimization would be more expensive.

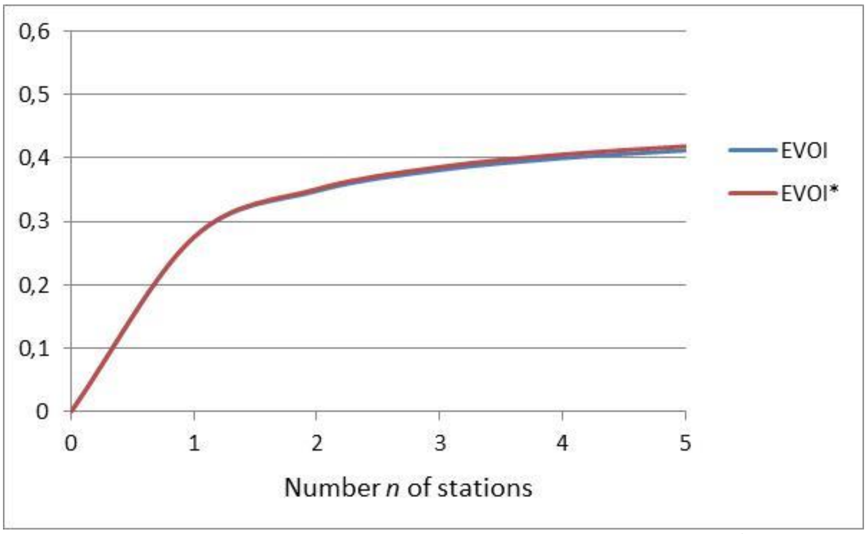

Figure 11 EVOI physically (EVOI) and economically (EVOI*) optimized networks for scenario 3

6 Conclusion

The aim of this article is to combine two types of literature. The first one optimizes the monitoring network and the second analyzes the economic value of information provided by the water quality monitoring network. In this paper, the monitoring network aims at minimizing the detection time for accidental pollution. For this purpose, the optimization of the network focuses on finding the optimal locations of monitoring stations. We construct a theoretical model to compare the EVOI of a physically optimized monitoring network, with only hydrological concern, and the EVOI of an economically optimized monitoring network, according to three scenarios of vulnerability along the river: uniform, decreasing and increasing vulnerability.

Our results show that the benefit to add monitoring stations decrease with the number of stations. Then according to the cost of the monitoring station, a finite number of stations are recommended. Moreover, we show that the advantage of optimizing the EVOI compared to physical optimization of the network is relative to the context, namely the number of stations, and the vulnerability scenarios. Then, according to the additional cost of economic optimization, the physical optimization could be recommended.

Previous works searched on the optimization of the monitoring network by spatial or temporal issues. In our paper, we only focus on the spatial issues. However, a temporal issue could be added by considering that accidental pollution could not be detected, with a probability α, by a monitoring station if the sampling frequency is too low. An increase in the frequency would lead to decrease the probability α. This new hypothesis will increase the uncertainty concerning the location of an emitted pollution detected by a station. This could be the topic of another research paper.

Appendix

* Uniform vulnerability: Positivity condition for the EVOI: $C < \frac{\delta}{2}$.
In relation to the condition of positivity, we find:

$$\frac{\partial EVOI_n^0}{\partial n} = \frac{P}{(n+1)^2}.\left(\frac{\delta}{2} - C\right) > 0$$

$$\frac{\partial^2 EVOI_n^0}{\partial n^2} = -\frac{2P}{(n+1)^3}.\left(\frac{\delta}{2} - C\right) < 0$$

* Decreasing vulnerability: Positivity condition for the EVOI: $C < \frac{1}{4}\delta$.
In relation to the condition of positivity, we find:

$$\frac{\partial EVOI_n^-}{\partial n} = \frac{P}{(n+1)^2}.\left[\delta\left(\frac{3n+1}{6n+6}\right) - C\right] > 0$$ [6]

$$\frac{\partial^2 EVOI_n^-}{\partial n^2} = -\frac{2P}{(n+1)^3}\left[\delta\left(\frac{n}{2n+2}\right) - C\right] \leq 0$$ [7]

* Increasing vulnerability: Positivity condition for the EVOI: $C < \frac{2}{3}\delta$.
In relation to the condition of positivity, we find:

$$\frac{\partial EVOI_n^+}{\partial n} = \frac{P}{(n+1)^2}.\left(\delta\left(\frac{3n+5}{6n+6}\right) - C\right)$$

$$\frac{\partial^2 EVOI_n^+}{\partial n^2} = -\frac{2P}{(n+1)^3}\left[\delta\left(\frac{n+2}{2n+2}\right) - C\right]$$

The condition $C < \frac{2}{3}\delta$ is not sufficient to get an increasing EVOI. We need $C < \delta\left(\frac{3n+5}{6n+6}\right)$ [8].
In the same way, the sign of the second derivative will depend on the value of C with respect to $\delta\left(\frac{n+2}{2n+2}\right)$ [9].

[6] For $n \geq 1$, $\left(\frac{3n+1}{6n+6}\right) > \frac{1}{4}$, thus: $C < \delta\left(\frac{3n+1}{6n+6}\right)$.

[7] For $n \geq 1$, $\left(\frac{n}{2n+2}\right) \geq \frac{1}{4}$, thus: $C \leq \delta\left(\frac{n}{2n+2}\right)$.

[8] Yet for $n \geq 1$, $\left(\frac{3n+5}{6n+6}\right) \leq \frac{2}{3}$.

[9] The condition of positivity of the EVOI is not sufficient because $\frac{n+2}{2n+2} < \frac{2}{3}$ when $n > 2$.

References

1. Alvarez-Vázquez, L.J., Martínez, A., Vázquez-Méndez, M.E., & Vilar, M.A. (2006). Optimal location of sampling points for river pollution control. Mathematics and Computers in Simulation 71, 149–160.

2. Bickel, J.E. (2008). The Relationship Between Perfect and Imperfect Information in a Two-Action Risk-Sensitive Problem. Decision Analysis 5, 116–128.

3. Bouma, J.A., Kuik, O., & Dekker, A.G. (2011). Assessing the value of Earth Observation for managing coral reefs: An example from the Great Barrier Reef. Science of The Total Environment 409, 4497–4503.

4. Bouma, J.A., van der Woerd, H.J., & Kuik, O.J. (2009). Assessing the value of information for water quality management in the North Sea. Journal of Environmental Management 90, 1280–1288.

5. Bratvold, R., Bickel, J.E., & Lohne, H.P. (2007). Value of Information in the Oil and Gas Industry: Past, Present, and Future. Society of Petroleum Engineers.

6. Destandau, F., & Diop, A.P. (2016). An Analysis of the Value of Additional Information Provided by Water Quality Measurement Network. Journal of Water Resource and Protection 08, 767–776.

7. Do, H.T., Lo, S-L., Chiueh, P-T., & Phan, Thi. LA. (2012). Design of sampling locations for mountainous river monitoring. Environmental Modelling & Software 27–28, 62–70.

8. European Commission (2000) Directive 2000/60/EC of the European Parliament and of the Council, of 23 October 2000: establishing a framework for Community action in the field of water policy.

9. European Commission (2012). Rapport de la Commission au Parlement Européen et au Conseil relatif à la mise en oeuvre de la directive-cadre sur l'eau (2000/60/CE). Plans nationaux de gestion des bassins hydrographiques..

10. Harmancioglu, N.B., Fistikoglu, O., Ozkul, S.D., Singh, V.P., & Alpaslan, M.N. (1999). Definition of Monitoring Objectives. In: Water Quality Monitoring Network Design.

11. Howard, R.A. (1966). Information Value Theory.

12. Koerkamp, B.G., Spronk, S., Stijnen, T., & Hunink, M.G.M. (2010). Value of Information Analyses of Economic Randomized Controlled Trials: The Treatment of Intermittent Claudication. Value in Health 13,242–250.

13. Lawrence, D.B. (2012). The Economic Value of Information. Springer Science & Business Media.

14. Liu, Y., Zheng, B., Wang, M., Xu, Y., & Qin, Y. (2014). Optimization of sampling frequency for routine river water quality monitoring. Science China Chemistry 57, 772–778.

15. Park, S-Y., Choi, J.H., Wang, S., Park, S.S. (2006). Design of a water quality monitoring network in a large river system using the genetic algorithm. Ecological Modelling 199,289–297.

16. Prellezo, R. (2017). Expected economic value of the information provided by fishery research surveys. Fisheries Research 190,95–102.

17. Sanders, T.G., Ward, R.C., Loftis, J.C., Steele, T.D., Adrian, D.D., & Yejevich, V. (1983) Design of Networks for Monitoring Water Quality. Water Resources Publication

18. Santos, S.M.G., Gaspar, A.T.F.S., & Schiozer, D.J. (2017). Value of information in reservoir development projects: Technical indicators to prioritize uncertainties and information sources. Journal of Petroleum Science and Engineering 157, 1179–1191.

19. Sharp, W.E. (1971). A Topologically Optimum Water-Sampling Plan for Rivers and Streams. Water Resources Research 7, 1641–1646.

20. Telci, I.T., Nam, K., Guan, J., & Aral, M.M. (2009). Optimal water quality monitoring network design for river systems. Journal of Environmental Management 90, 2987–2998. d.

21. Yokota, F., & Thompson, K.M. (2004) Value of Information Analysis in Environmental Health Risk Management Decisions: Past, Present, and Future. Risk Analysis 24, 635–650.