Effectiveness of decarbonisation policies in an electricity system with variable renewables*

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Can subsidies to renewable energy efficiently internalise CO_2 costs in electricity production? Under current policy design it only matters that the replaced energy is dirty, but not how dirty it is. We use a modified peak-load pricing model, including variable renewable generators and the external costs of carbon, to examine whether a unit subsidy to variable renewables successfully restores first best equilibrium. In our model, electricity is generated using any combination of three technology types: two dispatchable, thermal, and CO_2 emitting technologies, differing in their emission intensity, and a non-dispatchable renewable technology. Using this model, we show that available wind capacity is never idle, and derive equations determining optimal installed capacities for all technologies. We then demonstrate how a subsidy that does not discriminate between dirty energies fails to restore first best: it either replaces an insufficient amount of dirty energy, or does not replace the most carbon intensive energy source.

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1 INTRODUCTION

> "Its progress of late, however, has been less than stellar: Despite its aggressive deployment of wind turbines and solar panels, the carbon intensity of California's economy — measured by the CO_2 emissions per unit of economic product — declined by only 26.6 percent between 2000 and 2014. That put it in 28th place. In New York, which came in seventh, carbon intensity declined 35.4 percent."
>
> — New York Times, 17 Jan. 2017[1]

> "European lawmakers have backed measures that would substantially raise the European Union's clean-energy ambitions. By 2030, more than one-third of energy consumed in the EU should be from renewable sources such as wind and solar power, the European Parliament says — up from the existing target of just over one-quarter."
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> — Nature News, 17 January 2018[2]

Electrical energy is a fundamental energy form in many economies. It is also technically complex to manage. It has to be generated essentially simultaneously to the time of use. It is generated using a spectrum of technologies, each with its own cost and operating constraints. It can only be transported by very specialised installations, whose capacity is inflexible in the short term. It is used by consumers in processes whose timing is inflexible. It is complicated and expensive to store, whether as electrical energy or through conversion to other energy forms.

The electrical energy sector is also one of the largest sources of carbon emissions; given the potential costs of climate change, there is a strong interest in reducing its carbon emission intensity. From a carbon emissions perspective, there are two categories of electrical energy generator. The first category contains all carbon emitting, i.e. thermal generators, which are fuelled by fossil fuels. The output of these generators is generally controllable by its owners or operators, within certain technical constraints — how fast output can be changed, the maximum installed capacity and so on. The second category contains all carbon-free generators, which are generally "fuelled" by renewable energy sources such as the wind or sun. Their output often varies with changes in environmental conditions which are beyond the control of their owners or operators; they might be able to produce less than the potential offered by wind or sun, but they cannot produce more. Even so, their zero-emissions means that the goal of many a policy maker is to increase the contribution of renewable generators to electricity production. This paper analyses the effectiveness of one of the policies used to do so: subsidies to variable renewables — for simplicity we refer to them as just renewables.

Variable renewable energy has expanded greatly, in part thanks to policy support, either in the form of carbon pricing or in the form of direct subsidies.

[1] Porter, Eduardo. "On Climate Change, Even States in Forefront Are Falling Short." *New York Times* 17 Jan. 2017. Online. 30 Jan. 2018.

[2] Schiermeier, Quirin. "European Union moves to strengthen renewable-energy goals." *Nature News* 17 Jan. 2018. Online. 30 Jan. 2018.

There can be more than one policy rationale behind the provision of subsidies to renewables; one of the most important is that every unit of electrical energy generated from renewables is a unit of electrical energy generated without CO_2 emissions. Viewed through such a lens, a subsidy is an implicit tax on carbon: instead of the tax been levied at source on the generator emitting the CO_2, the subsidy — a negative tax — to renewables makes the CO_2 emitting generator less price competitive against the clean technology.

As the carbon emissions are an external cost, they are the target of policy efforts by governments. These policies either adress the source of the externality and attempt to put a price on it, by taxing it or creating a tradeable emissions permit system. Carbon taxes are often politically unpalatable, so instead focus has switched to developing technologies which produce electrical energy with no carbon emissions: variable renewables.

In one sense, electrical energy is one of the most homogenous commodities around. Electrical energy produces the same energy services, whenever and wherever it is used; contrast this to other energy commodities such as oil or coal, whose quality and energy content vary greatly. In another sense however, and as a result of the constraints in generation, transportation, storage and use, electricity is fairly heterogenous. Electricity generated by renewables cannot simply replace electricity generated from thermal generators on a one-to-one basis.

Unfortunately, and as we will demonstrate in this paper, this is inefficient in a system in which the clean electrical energy cannot discriminate between the type of CO_2 -emitting electrical energy it replaces. We want to understand how subsidies to variable renewables succeed — or fail — to restore the first best equilibrium in a system in which these are present alongside a variety of thermal technologies. Even though policy support for renewable energy is gradually being phased out as it becomes more cost-competitive, commitments being made now for expanding capacities are often scheduled to last far into the future. This justifies asking whether directing funding to variable renewable energy generators continues to be the most effective method to reduce emissions from the electricity industry.

We ask how effective a subsidy to variable renewables is as environmental policy. To examine this we consider whether a subsidy in the decentralised equilibrium replicates the social planner's solution. We introduce a subsidy in the decentralised equilibrium problem and compare the new solution to the social planner.

We analyse the operation of an electricity system using the canonical peak-load pricing model. We examine whether environmental policies are successful in restoring first best equilibrium in a power system comprising of variable and dispatchable generation technologies. Results are shown for both the social planner's optimum — where external costs are considered — and the decentralised solution — where external costs are ignored. The social planner's optimum can serve as the benchmark, against which we compare the decentralised equlibrium with and without a subsidy to variable renewable energy.

Our model has three ingredients beyond those normally found in peak-load pricing models: several technologies, variable output for at least one of these technologies, and subsidies. In a model with just two generating technologies, a subsidy to the clean technology could be found to be the perfect converse of an environmental tax on the polluting technology; we are interested in the more realistic case of how effectively a subsidy replaces an environmental tax in a system with multiple technologies. Moreover, the variability of at least one of these technologies reflects the heterogeneity of electrical energy.

Borenstein (2012) provides a qualitative review of the challenges we face in using renewables to de-carbonise the electricity sector. Our paper is most closely linked to the literature on peak-load pricing, as described by Crew et al. (1995). An accessible overview is the book by Harris (2015). This type of model has more recently been extended to consider variable renewables by Ambec & Crampes (2012) or Chao (2011); both explicitly characterise the variability of renewable generators, the former by introducing the concept of two states representing environmental conditions under which renewables can or cannot operate, the latter by representing the available capacity of an variable renewable generator by a stochastic variable.

Andor & Voss (2016) build on the approach of Chao (2011) to derive conditions under which policies supporting renewables are welfare increasing; their model examines the capacity installation and energy output of a single electricity generator that can produce external costs that are positive or negative; from the perspective of a variable renewable generator, these could be avoided CO_2 emissions or the need for more flexible conventional generators, respectively. A current working paper by Helm & Mier (2016) characterises an efficient diffusion path of renewables, again with an explicit characterisation of intermittency.

A structured discussion on the heterogeneity of electricity can be found in Hirth et al. (2016), who ascribe three dimensions to heterogeneity: over time, between locations and across the lead time between contract and delivery. Since electricity cannot be effectively stored, it needs to be generated at the moment it is demanded; supply capacity is costly and limited in the short term. Electricity can only be transported through a specialised grid, also limited in the short term. Finally, plants can only adjust their output at a limited rate, which varies from plant to plant; this restricts the number of plants that can respond to changes in demand. This heterogeneity is what reduces the effectiveness of renewable energy subsidies as CO_2 reduction policy.

We are also motivated by the burgeoning energy economics literature whose goal is to evaluate the environmental effectiveness of renewable support policies, or of environmental policies in the energy sector generally. Much of the existing work is empirical. Using data on the hourly electricity production and emissions in the Texas ERCOT grid, Kaffine et al. (2013) quantify the emissions savings per MWh of wind power by identifying the emission intensity of the marginal plant being replaced by wind. As this varies over time and space, so will the emissions savings per MWh. Also using data over a similar time period from the ERCOT grid, Cullen (2013) calculates the change in the average emission intensity to assess whether the emissions offset by wind power justify the subsidies supporting wind power; he concludes that this is the case when the social cost of carbon is at least USD42 per ton. The paper of Gowrisankaran et al. (2016) contains both strands, quantifying the emissions savings from renewables as well as the variability costs to the system; they find that at a social cost of carbon of USD39, a share of 20% for solar power would be welfare neutral. Using data from the Spanish and German grid, Abrell et al. (2017) estimate the cost of reducing a ton of CO_2 through subsidies to wind and solar generators.

To the best of our knowledge, the existing literature does not include peak-load pricing models with multiple technologies and characterisation of variability. In addition to the description of such a model, our main contribution is to show how subsidies to renewables are a sub-optimal policy to displace carbon emitting electricity generators. This is driven by the inability of a subsidy to distinguish between which carbon-generating technology it helps displace.

We begin the analysis by describing the model setup in section 2. We then solve for the social planner's optimum in section 3. This comprises of two cases — "expensive natural gas" and "cheap natural gas" — a ramification of the assumptions we make concerning the relative cost of coal and natural gas — both of these serve as benchmarks when evaluating the effect of policies on the decentralised equibrium. Following the social planner's optimum, we solve the decentralised equilibrium in section 4. The effectiveness of subsidy as environmental policy is analysed in section 5. We close with a discussion of the model's implications, section 6.

2 Model Setup

In our model, electricity can be generated using any combination of three types of technology: two thermal — i.e. CO_2 emitting — technologies, which we represent by natural gas and coal, and a renewable technology, which we represent by wind. The two CO_2 emitting technologies differ in cost and emission intensity. Each of these three generators produces the same type of electrical energy, with the caveat that while output from the thermal generators can be determined, that of the wind generator depends on windspeed, a stochastic quantity. Given that the thermal technologies can adjust, or dispatch, their electricity production at the request of the operator, we will refer to the thermal technologies as being dispatchable, and to the renewable technology as being non-dispatchable. The use of three technologies is not accidental: the problem we want to analyse only occurs in a system with multiple carbon emitting generators of varying carbon intensity. The variation of carbon intensity is a key characteristic of our model.

In our setting, utility is derived from consuming electrical energy q, $U(q)$. We assume that utility is an increasing convex function, $MU(\cdot) > 0$, $MU'(\cdot) < 0$. Although each technology — wind w, coal c or natural gas g — differs in some characteristics, the quantities of electricity they produce $q_i > 0$, $i \in \{w, c, g\}$ are substitutes in terms of utility. Subscript i denotes the full set of technologies, while we use subscript f when specifically considering the subset of fossil based generators. There is an operating cost $b_f > 0$, $f \in \{c, g\}$, in order to produce each unit of energy; wind energy does not have any costs when producing electricity.

The quantity of energy q_i can be produced up to capacity $K_i > 0$. Each generator is installed at an increasing marginal cost, which is a function of the total capacity installed in the system, i.e. $\frac{1}{2}\beta_i K_i^2$, $\beta_i > 0$. Power plants become ever more expensive to build in capacity: as the best plants are built out, the remaining potential locations will prove ever more expensive to prepare, for instance they might not offer as good links to the natural gas/coal supply, or the power grid. In our analysis, the costs are considered over the whole lifetime of the plants, so we can consider quantity costs b_f and capacity costs β_i simultaneously, even though their original units are different.

The dependence of renewable energy upon environmental conditions means it is unavailable on occasion. This unavailability is a fundamental characteristic of the wind generators in our model, which drives the heterogeneity of electricity, a characteristic we discussed in the introduction. Hence, we introduce a stochastic variable $\alpha \in [0,1]$ that helps us distinguish between the wind capacity that is installed, K_w, and the fraction of this capacity that is available to produce electricity, αK_w. This availability factor of wind, α, is drawn from a probability distribution function $f(\alpha)$. If $\alpha = 0$, wind capacity is not available, although it is installed; if $\alpha = 1$, all installed wind capacity is available[3]. In contrast to wind, we consider the coal and natural gas capacities to be fully available, i.e. their installed and available capacities always coincide.

To solve this type of model, a standard approach in the literature is to separate the production of electricity into two stages: the decision on how much capacity to build, and the decision on how much energy to produce with this installed capacity. We name these the investment and dispatch stages respectively. The division into two stages reflects the constraints placed upon the design of a system: quantity of energy dispatched can only be produced after the requisite generation capacity has been installed. However, in our case, the dispatch stage is solved first, yielding the equilibrium quantity of electricity generated; this result is used in the investment stage to solve for the quantity of capacity required to produce this electrical energy. We refer to this process as a backward solving, stage-wise approach.

In the dispatch stage and for a given realisation of α, welfare is the utility derived from consuming electricity, $U(q(\alpha))$, less the cost of producing said quantity:

$$\mathcal{S}_q(q(\alpha)) = U\big(q(\alpha)\big) - \sum_f b_f q_f(\alpha) \quad \forall f \in \{c, g\}$$

$$\text{where} \quad q(\alpha) = \sum_i q_i(\alpha) \quad \forall i \in \{w, c, g\}$$

$$MU(\cdot) > 0 \quad MU'(\cdot) < 0$$

The quantity of energy calculated in this stage feeds into the investment stage, from which we obtain the complete results of the model.

Operating costs b_f are largely driven by the fuel costs of a technology. Since natural gas as a fuel has historically been more expensive than coal, we assume that $b_c < b_g$ throughout our analysis. This assumption is made to simplify the solution we present. The model could also be solved with natural gas being

[3]The dispatch of energy occurs for a particular value of α, while capacity is built prior to dispatch, based on a distribution of α.

cheaper than coal; in the equations that provide the solution, one would just exchange natural gas for coal. Nevertheless, in the interest of having a straightforward presentation of our results, we do not consider the case of coal being costlier than natural gas $b_c > b_g$.

The setup of the model presented so far has no explicit characterisation of the carbon external cost, $e_f > 0$; in our setup it can be added to each technology's operating cost. Therefore, a social planner's cost is the sum of the operating cost and the carbon externality of each technology, $b_f + e_f$. The emission cost of coal is always higher than the emission cost of natural gas, $e_c > e_g$. This is a physical reality that always holds: natural gas of a particular energy content contains approximately half the carbon as coal of the same energy content.

Moving to the investment stage, the total expected welfare is considered. It is gained by consuming the quantity of electricity determined in the dispatch stage, from the capacity that is available to generate it, less the cost of installing said capacity. The subscript K denotes that this is the capacity stage.

$$\mathcal{S}_K = \mathbb{E}\big[\mathcal{S}_q\big(q(\alpha)\big)\big] - \sum_i \frac{1}{2}\beta_i K_i^2 \quad \forall i \in \{w, c, g\}$$

With the investment and capacity stages in mind, we can move on to our analysis. We begin with the social planner's problem, followed by the decentralised equilibrium. The social planner's solution consists of two cases, which serve as benchmarks against which we can assess how the decentralised equilibrium does not replicate first best. Finally, we will examine what occurs when a subsidy to wind is used in the decentralised equilibrium as a policy measure.

3 Social planner's solution

The social planner's problem depends on the sum of operating and external costs, $b_f + e_f$; this makes solving this problem a little nuanced. As we have made no assumption on the level of these parameters, our assumption on the operating costs of fossil fuels, $b_g > b_c$ and the fact that natural gas is less polluting than coal, mean the social planner faces one of two cases. Either the case of "Expensive natural gas", where the social planner's cost of natural gas is higher than that of coal: $b_c + e_c < b_g + e_g$; or the case of "Cheap natural gas", where the social planner's cost of natural gas is lower than that of coal: $b_c + e_c > b_g + e_g$[4]. We will next consider these two cases separately; they serve as a benchmark to assess how effectively a subsidy restores first best in the decentralised equilibrium, in section 5.

The first step of the solution is the dispatch stage, whose setup is common to both cases. We use the results as inputs in the investment stage, according to the backward solving, stage-wise solution approach we discussed in the setup of the model. As presented in the previous section, given capacities K_i and a measure of availability for wind α, the social planner's problem is the utility gained from a quantity of electricity, less the cost of producing said electricity. The production

[4] We will not consider the case where social costs for coal and natural gas are identical, $b_c + e_c = b_g + e_g$, because then the two technologies collapse in one.

of electricity is constrained by the available capacity of each technology, which in the case of wind is limited by the factor α. The problem reads:

$$\max_{q_i(\alpha)>0} \left\{ \mathcal{S}_q := U\big(q(\alpha)\big) - \sum_f (b_f + e_f) q_f(\alpha) \right\} \quad \forall f \in \{c, g\}$$

such that

$$\begin{aligned} & q(\alpha) = \sum_i q_i(\alpha) \quad \forall i \in \{c, g, w\} \\ & K_f \geq q_f(\alpha) \quad \forall f \in \{c, g\} \\ & \alpha K_w \geq q_w(\alpha) \end{aligned}$$

Electricity quantities from different technologies are perfect substitutes in terms of utility, so they can be summed. The Lagrangian is:

$$\begin{aligned} \mathcal{L}_q = U\big(\sum_i q_i(\alpha)\big) - \sum_f (b_f + e_f) q_f(\alpha) + \\ + \sum_f \lambda_f(\alpha)[K_f - q_f(\alpha)] + \lambda_w(\alpha)\big[\alpha K_w - q_w(\alpha)\big] \end{aligned} \tag{3.1}$$

We obtain the first order conditions from the Lagrangian's partial derivatives,

$$\begin{aligned} \frac{\partial \mathcal{L}_q}{\partial q_c(\alpha)} = 0 \;\Rightarrow\; \mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - (b_c + e_c) - \lambda_c^*(\alpha) = 0 \\ \frac{\partial \mathcal{L}_q}{\partial q_g(\alpha)} = 0 \;\Rightarrow\; \mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - (b_g + e_g) - \lambda_g^*(\alpha) = 0 \\ \frac{\partial \mathcal{L}_q}{\partial q_w(\alpha)} = 0 \;\Rightarrow\; \mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - \lambda_w^*(\alpha) = 0 \end{aligned}$$

which we combine and re-arrange to obtain the relations that control the quantity of electricity produced by each technology:

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - \lambda_w^*(\alpha) = 0 \tag{3.2}$$

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - (b_c + e_c) - \lambda_c^*(\alpha) = 0 \tag{3.3}$$

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - (b_g + e_g) - \lambda_g^*(\alpha) = 0 \tag{3.4}$$

These first order conditions tell us a few things about the amount of electricity generated by each technology, results illustrated in Figure 1 for the expensive natural gas case or Figure 2 for the cheap natural gas case. Note that in the figure, marginal utility is drawn as a linear function of quantity. This is done for simplicity and the same results hold for any non-negative, decreasing marginal utility. To continue with the solution, we need to apply our assumption on the relative operating costs of natural gas and coal. We begin with the Expensive natural gas case.

Expensive natural gas

In order to have an interior solution to our problem when $b_c + e_c < b_g + e_g$, we assume that the marginal utility from the fully used technologies — wind and coal — is greater than the social cost of operating coal, i.e. $MU(K_w + K_c) > b_g + e_g$. This condition ensures that all three technologies will be employed in the solution. Note that capacities are exogenous at this stage of the problem. The Lagrange multiplier for wind is always positive in (3.4), meaning that we will have at least some electrical energy generated from wind. In this section, coal is always fully used because it is cheaper than natural gas.

Natural gas is the most expensive technology available for use; we have assumed that $b_c + e_c < b_g + e_g$. The amount used is driven by the available capacity of wind αK_w. When α is above the threshold α_1, natural gas will only be partially used, as there is a large amount of wind energy being generated. When α falls below this threshold, natural gas capacity is fully used. The threshold α_1 is determined from (3.4), the relation controlling what is the marginal technology in the current case:

$$MU(\alpha K_w + K_c + K_g) \geq b_g + e_g$$

The threshold then is:

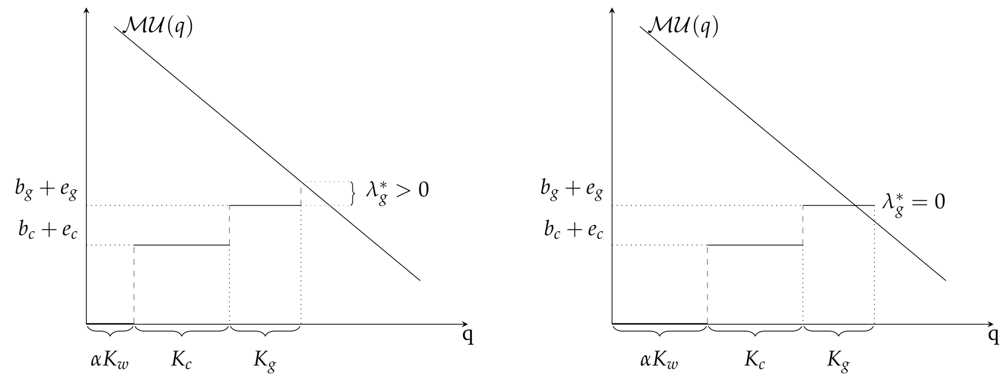
$$\alpha_1 \equiv \frac{\mathcal{MU}^{-1}(b_g + e_g) - K_c - K_g}{K_w}, \qquad \alpha_1 \in (0, 1)$$

By definition, the threshold α_1 is the highest value of α for which all three technologies fully use their installed capacity, i.e. $\mathcal{MU}(\alpha_1 K_w + K_c + K_g) = b_g + e_g$. The intuition behind this threshold can also be seen in Figure 1. If the marginal technology, in this case natural gas, becomes more expensive, that is if $b_g + e_g$ increases, then the threshold decreases. As a result, natural gas will be partially used for more values of α. When α falls below this threshold, there is insufficient available wind capacity, increasing the shadow price of all technologies. The quantity of electricity consumed is then the sum of available capacities, $\alpha K_w + K_c + K_g$. Above this threshold, there is excess capacity of the marginal technology, in this case natural gas. Now, the quantity of electricity consumed is the same, irrespective of the value of α, and equal to $\mathcal{MU}^{-1}(b_g + e_g)$.

We summarise our observations on the use of the capacities of wind, coal and natural gas in three Lemmata. The order in which technologies are used depends on their operating cost: this is zero for wind, so its available capacity is always completely used, as determined by Lemma 3.1. By assumption of this case, coal is cheaper than natural gas, so it will be the next technology to be used; it is fully used, as described in Lemma 3.2. Finally, natural gas is employed.

Lemma 3.1. *The quantity of wind dispatched will always equal the available capacity.*

Proof. Since $\mathcal{MU}\left(\sum_i q_i^*(\alpha)\right) > 0$, the Lagrange multiplier will always be positive in (3.2), $\lambda_w^* > 0$. This implies that the capacity constraint for wind always holds: $q_w^*(\alpha) = \alpha K_w$. □



(a) For $\alpha < \alpha_1$

(b) For $\alpha > \alpha_1$

Figure 1: Social planner dispatch solution for "Expensive natural gas"

Lemma 3.2. *Coal is always fully used, $q_c^* = K_c$.*

Proof. As we assume that the sum of coal's operating and carbon external costs are lower than those of natural gas, $b_c + e_c < b_g + e_g$. □

Lemma 3.3. *Natural gas is either fully or partially used, depending on the value of the availability factor α. For values below threshold α_1, all technologies are fully used. For values above it, only wind and coal are fully used; natural gas is partially used. This threshold is defined as*

$$\alpha_1(K_w, K_c, K_g) \equiv \frac{\mathcal{MU}^{-1}(b_g + e_g) - K_c - K_g}{K_w}, \qquad \alpha_1 \in (0, 1)$$

Proof. Building again on the assumption that coal is cheaper than natural gas, $b_c + e_c < b_g + e_g$, we can conclude that $q_g^*(\alpha) \leq K_g$. Depending on the wind availability, natural gas can either be fully used, i.e. $q_g^* = K_g$, or it can serve the remaining demand , $q_g^*(\alpha) = \mathcal{MU}^{-1}(b_g + e_g) - \alpha K_w - K_c < K_g$. The amount of natural gas capacity we use depends on the realization of α relative to a threshold we call α_1; below it there is insufficient wind and all three technologies must be fully used, while above it, natural gas is only partially used. The threshold is defined as the highest value of α for which all technologies are fully used. □

We now take the output quantities from the dispatch stage and use them to solve the investment stage problem, from which we obtain the optimal capacity of each technology. The expected welfare from the dispatch stage depends on the availability of wind capacity, αK_w. As discussed in the model setup, the dependence of wind generation on a stochastic phenomenon means that the full installed capacity K_w is only available when wind conditions are ideal; at other times there might be insufficient wind to operate installed capacity fully. To represent this, we use the stochastic variable α, drawn from a probability distribution function $f(\alpha)$ that assigns a probability to each realisation of α. We denote that this is the investment stage utility by S_K. Hence, this stage's problem is maximising the optimal expected welfare from the dispatch stage minus the costs of investment for the capacities:

$$\max_{K_i > 0}\Big\{\mathcal{S}_K := \int_0^1 \mathcal{S}_q^*(K_w, K_c, K_g, \alpha) f(\alpha)\mathrm{d}\alpha - \sum_i \frac{1}{2}\beta_i K_i^2\Big\} \qquad \forall i \in \{c, g, w\}$$

As previously mentioned, we will not deal with the cases which have fewer than three generation technologies. In these cases the problem of a subsidy to renewables for environmental reasons is not interesting: if wind replaces a single fossil fuel emitting technology, it is possible to correctly value a subsidy to wind with regard to the external cost of carbon that the single fossil fuel generator emits. The Lagrangian of this problem becomes a little more complex as a result of the threshold wind availability α_1. The integral of the expected welfare is split into two parts, representing the expected welfare from the quantity of energy below this threshold, and the expected welfare above this threshold. These two are not the same, due to the different utilisation of natural gas capacity and the different quantities of electricity produced from natural gas, according to Lemma 3.3.

$$\begin{aligned}\mathcal{L}_K = & \int_0^{\alpha_1(K_w, K_c, K_g)} \Big[U(\alpha K_w + K_c + K_g) - \sum_f (b_f + e_f)K_f\Big] f(\alpha)\mathrm{d}\alpha + \\ & + \int_{\alpha_1(K_w, K_c, K_g)}^1 \Big[U\big(\mathcal{MU}^{-1}(b_g + e_g)\big) - (b_c + e_c)K_c - \\ & \qquad -(b_g + e_g)\big(\mathcal{MU}^{-1}(b_g + e_g) - \alpha K_w - K_c\big)\Big] f(\alpha)\mathrm{d}\alpha \\ & - \sum_i \frac{1}{2}\beta_i K_i^2 \end{aligned} \tag{3.5}$$

We derive the first order conditions by applying the Leibniz integral rule to the Lagrangian and assuming a continuous uniform distribution for the availability factor of wind $0 \leq \alpha \leq 1$, that is $f(\alpha) = 1$. Superscript ex denotes that this is the expensive natural gas case:

$$\begin{aligned}&\frac{\partial \mathcal{L}_K}{\partial K_w} = 0 \Rightarrow \\ &\int_0^{\alpha_1(K_w^{ex}, K_c^{ex}, K_g^{ex})} \mathcal{MU}(\alpha K_w^{ex} + K_c^{ex} + K_g^{ex})\alpha \mathrm{d}\alpha - \\ &-(b_g + e_g)\left(\frac{\mathcal{MU}^{-1}(b_g + e_g) - K_c^{ex} - K_g^{ex}}{K_w^{ex}}\right)^2 - \beta_w K_w^{ex} + \frac{b_g + e_g}{2} = 0\end{aligned} \tag{3.6}$$

$$\frac{\partial \mathcal{L}_K}{\partial K_c} = 0 \Rightarrow \tag{3.7}$$

$$\int_0^{\alpha_1(K_w^{ex},K_c^{ex},K_g^{ex})} \mathcal{MU}(\alpha K_w^{ex} + K_c^{ex} + K_g^{ex})\mathrm{d}\alpha -$$

$$-(b_g + e_g)\frac{\mathcal{MU}^{-1}(b_g + e_g) - K_c^{ex} - K_g^{ex}}{K_w^{ex}} - (b_c + e_c) - \beta_c K_c^{ex} + b_g + e_g = 0$$

$$\frac{\partial \mathcal{L}_K}{\partial K_g} = 0 \Rightarrow \tag{3.8}$$

$$\int_0^{\alpha_1(K_w^{ex},K_c^{ex},K_g^{ex})} \mathcal{MU}(\alpha K_w^{ex} + K_c^{ex} + K_g^{ex})\mathrm{d}\alpha -$$

$$-(b_g + e_g)\frac{\mathcal{MU}^{-1}(b_g + e_g) - K_c^{ex} - K_g^{ex}}{K_w^{ex}} - \beta_g K_g^{ex} = 0$$

Combining and re-arranging equations (3.7) and (3.8), we obtain a condition for the equilibrium, and a Lemma summarising some of the system's characteristics.

Condition 1. *The total costs for the two fossil fuel technologies must be equal in the equilibrium:*

$$b_g + e_g + \beta_g K_g^{ex} = b_c + e_c + \beta_c K_c^{ex}$$

Lemma 3.4 shows that the optimal capacity of the thermal technologies is determined by the operating, carbon externality, and investment costs of these technologies, as well as their relative levels. However, optimal thermal capacities do not depend on the investment cost of wind.

Lemma 3.4. *The equilibrium capacities of coal and natural gas are independent of the equilibrium capacity of wind.*

Proof. By observation of equations (3.7) and (3.8). □

Unfortunately the form of the first order conditions for the capacity stage — (3.6), (3.7), (3.8) — does not lend itself to easy interpretation of the solution to the social planner's problem. To make the problem tractable, we make a simplifying assumption about the properties of α: instead of allowing it to take any value in an interval, it can only take the limiting values of this interval, 0 or 1, i.e. α is now a discrete stochastic variable. There is an equal probability that each value occurs $Pr(\alpha \leq \alpha_1) = Pr(\alpha > \alpha_1) = \frac{1}{2}$, and that gives us the probability mass function. We justify this assumption by arguing that the key difference between values is whether they lie above or below the threshold α_1, identified in Lemma 3.3. Above it, every value has the same effect on our problem: natural gas is only partially used, while below it, natural gas is fully used. Collapsing all values above and beyond the threshold to 0 or 1 does not change our result, but does make it more tractable. The Lagrangian for capacity (3.5) can be re-written as,

$$
\begin{aligned}
\mathcal{L}_K = \frac{1}{2}\Big[& U(K_c + K_g) - \sum_f (b_f + e_f) K_f \Big] + \\
& + \frac{1}{2}\Big[U\big(\mathcal{MU}^{-1}(b_g + e_g)\big) - (b_c + e_c)K_c - \\
& \qquad - (b_g + e_g)\big(\mathcal{MU}^{-1}(b_g + e_g) - K_w - K_c\big)\Big] - \\
& - \sum_i \frac{1}{2}\beta_i K_i^2
\end{aligned} \tag{3.9}
$$

and the first order conditions (3.6), (3.7) and (3.8), re-computed as:

$$
\frac{1}{2}(b_g + e_g) = \beta_w K_w^{ex} \tag{3.10}
$$

$$
\frac{1}{2}\mathcal{MU}(K_c^{ex} + K_g^{ex}) - (b_c + e_c) + \frac{1}{2}(b_g + e_g) = \beta_c K_c^{ex} \tag{3.11}
$$

$$
\frac{1}{2}\mathcal{MU}(K_c^{ex} + K_g^{ex}) - \frac{1}{2}(b_g + e_g) = \beta_g K_g^{ex} \tag{3.12}
$$

It can be easily verified that Condition 1 still holds by combining (3.11) and (3.12),

$$
K_c^{ex} = \frac{\beta_g K_g^{ex} + b_g + e_g - b_c - e_c}{\beta_c} \tag{3.13}
$$

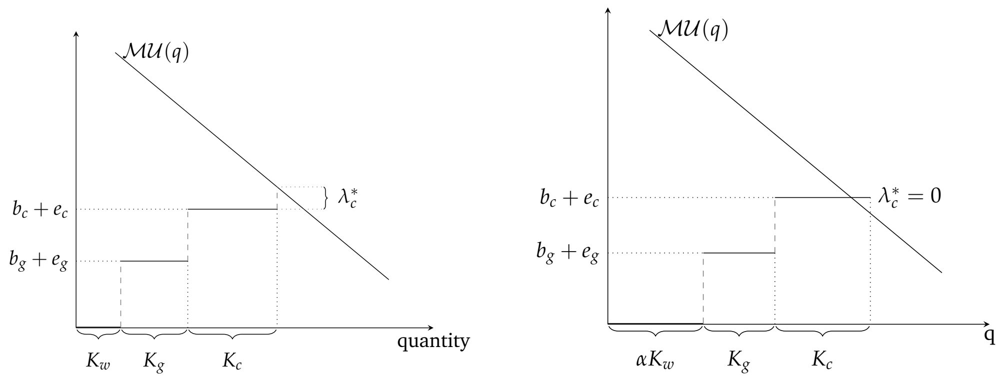
and the open form solution for K_g^{ex} is:

$$
\mathcal{MU}\left(\frac{\beta_g K_g^{ex} + b_g + e_g - b_c - e_c + \beta_c K_g^{ex}}{\beta_c}\right) - (b_g + e_g) - 2\beta_g K_g^{ex} = 0 \tag{3.14}
$$

The simplified first order condition for wind (3.10), and expressions (3.13) & (3.14), allow us to draw some initial conclusions. The installed capacity of wind only depends on two variables: its own capacity cost, β_w, and the cost of the marginal technology, in this case of natural gas, $b_g + e_g$. More specifically, there is a positive relation between K_w^{ex} and $b_g + e_g$, while there is a negative one between K_w^{ex} and β_w. As the marginal technology becomes more expensive, there is a higher investment in wind. On the other hand, when wind technology becomes more expensive, there is less wind capacity installed. The comparative statics for the two thermal technologies can be found in the appendix. Investment in these technologies decreases in their own costs but decreases in the cost of the other thermal technology. It is worth noting here that the cost of the variable technology does not affect the installed capacities of the thermal technologies, as already noted in Lemma 3.4.

Cheap natural gas

We now consider the case when the social planner's cost of natural gas is lower than that of coal, $b_g + e_g < b_c + e_c$. The switch from natural gas being the most



(a) For $\alpha < \alpha_2$

(b) For $\alpha > \alpha_2$

Figure 2: Social planner dispatch solution for "Cheap natural gas"

expensive to the cheapest changes the merit order of the two thermal technologies. Although the operating cost of coal is lower, when its level is close enough to that of natural gas, combining it with its carbon external cost could mean that the social planner considers it more expensive than natural gas. As a result, natural gas is the technology which is always fully used, while coal serves the remaining demand; the two thermal technologies have exchanged roles.

The social planner's problem at this point was already described at the beginning of this section, therefore the first order conditions (3.2), (3.3), (3.4) give the solution to the dispatch stage. Note that the condition to ensure internal solutions now reads $\mathcal{MU}(K_w + K_c) > b_c + e_c$. While Lemma 3.1, which describes the amount of wind generated, still holds as is, Lemmata 3.2 and 3.3 describing coal and natural gas generation respectively, essentially switch positions: natural gas is now fully used, so behaves as coal did in the case of expensive natural gas, while coal is the marginal technology, whose use depends on the value of α. The value of the threshold can be found similarly to the expensive natural gas case to be:

$$\alpha_2 \equiv \frac{\mathcal{MU}^{-1}(b_c + e_c) - K_c - K_g}{K_w}, \qquad \alpha_2 \in (0, 1)$$

Once more, a more expensive marginal technology results in a lower threshold α_2 which in turn leads to coal being partially used for more realizations of α.

We restate the Lemmata to reflect the new merit order of the thermal technologies. Contrast Figure 2 with Figure 1; they are identical in form, and differ only in the order with which the three technologies are dispatched. Wind is again the first to be used, generating the available capacity αK_w, now followed by the cheap(er) natural gas and the coal, which is now the marginal technology. Regarding the total quantity of electricity consumed, it is equal to $\alpha K_w + K_c + K_g$ when $\alpha \leq \alpha_2$, and equal to $\mathcal{MU}^{-1}(b_c + e_c)$ when $\alpha > \alpha_2$.

Lemma 3.5 (Restating Lemma 3.2 for cheap natural gas)**.** *Assuming natural gas is cheaper than coal, $b_g + e_g < b_c + e_c$, we can conclude that natural gas is always fully used, $q_g^* = K_g$.*

Lemma 3.6 (Restating Lemma 3.3 for cheap natural gas)**.** *When $b_c + e_c > b_g + e_g$, we can conclude that $q_c^*(\alpha) \leq K_c$. Depending on the wind availability, coal* *can either be fully used, i.e.* $q_c^* = K_c$*, or it can serve the remaining demand,* $q_c^* = \mathcal{MU}^{-1}(b_c + e_c) - K_w - K_g < K_c$.

After applying our simplifying assumptions on α, the Lagrangian for the capacity stage can now be written as:

$$\begin{aligned}\mathcal{L}_K = &\frac{1}{2}\left[U(K_c + K_g) - \sum_f (b_f + e_f)K_f\right] + \\ &+ \frac{1}{2}\Big[U\Big(\mathcal{MU}^{-1}(b_c + e_c)\Big) - (b_g + e_g)K_g - \\ &\quad -(b_c + e_c)\Big(\mathcal{MU}^{-1}(b_c + e_c) - K_w - K_g\Big)\Big] - \\ &- \sum_i \frac{1}{2}\beta_i K_i^2 \end{aligned} \tag{3.15}$$

Condition 1, referring to the requirement that operating cost of the two fossil technologies are equal, still holds, hence we can modify the first order conditions (3.10), (3.13), (3.14) to derive the capacities for the cheap natural gas case, denoted by the superscript *ch*:

$$K_w^{ch} = \frac{b_c + e_c}{2\beta_w} \tag{3.16}$$

$$\mathcal{MU}\left(\frac{\beta_c K_c^{ch} + b_c + e_c - b_g - e_g + \beta_g K_c^{ch}}{\beta_g}\right) - (b_c - e_c) - 2\beta_c K_c^{ch} = 0 \tag{3.17}$$

$$K_g^{ch} = \frac{\beta_c K_c^{ch} + b_c + e_c - b_g - e_g}{\beta_g} \tag{3.18}$$

Once again, we see in (3.16) that the installed wind capacity depends on its own investment cost and on the operating and carbon external cost of the marginal technology, this time coal, $b_c + e_c$. Similar to the previous case, K_w^{ch} decreases in β_w, while it increases in $b_c + e_c$. The thermal capacities, K_g^{ch} and K_c^{ch}, follow the same comparative statics as in the "Expensive natural gas" case, increasing when the other thermal technology becomes more expensive and decreasing in their own investment costs.

The key difference between the two cases of the social planner, namely "Expensive natural gas" and "Cheap natural gas", is the order of the fossil fuels. In both cases wind, being the cheapest generator to operate, is used to the extent permitted by α. The result that thermal capacities do not depend on the investment cost of wind also holds in both cases. However, while natural gas is the marginal technology in the "Expensive natural gas" case, this role is taken on by coal in the "Cheap natural gas" case, due to the difference in operating cost being relatively smaller than the difference in the cost of the carbon externality. This change in the merit order creates some interesting implications, when the policy measure will be introduced.

We next analyse which generation technologies are used when the external cost of carbon is not considered, and apply a policy to the decentralised equilibrium to correct this. We will then be able to compare the decentralised equilibrium with a policy to the two cases of the social planner's solution.

4 DECENTRALISED EQUILIBRIUM PROBLEM

As discussed in the setup of the model, we assume that the operating cost of coal is lower than that of natural gas, $b_c < b_g$. This assumption is justified by the fact that the biggest part of the operating costs, fuel costs, have been lower for coal than for natural gas. In the decentralised case, operating costs are the only costs per unit of energy produced that agents consider. As in the social planner's case, we solve the model in two stages, starting with the dispatch stage, then feeding the resulting quantity of electrical energy into the investment stage, which gives us the capacity of generators to be installed.

In this section, the dispatch stage is identical to the "Expensive natural gas" case if we set the external costs of carbon to zero, $e_c = e_g = 0$. Equations (3.2), (3.3), (3.4) now become

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - \lambda_w^*(\alpha) = 0 \tag{4.1}$$

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - b_c - \lambda_c^*(\alpha) = 0 \tag{4.2}$$

$$\mathcal{MU}\big(\sum_i q_i^*(\alpha)\big) - b_g - \lambda_g^*(\alpha) = 0 \tag{4.3}$$

while the threshold for α is given by:

$$\alpha_3 \equiv \frac{\mathcal{MU}^{-1}(b_g) - K_c - K_g}{K_w}, \qquad \alpha_3 \in (0, 1)$$

Following the same approach as before, when $\alpha \le \alpha_3$, the quantity of electricity consumed is equal to $\alpha K_w + K_c + K_g$, whereas when $\alpha > \alpha_3$, the quantity is $\mathcal{MU}^{-1}(b_g) - K_w - K_c$.

The investment stage Lagrangian of this section is:

$$\begin{aligned}\mathcal{L}_K = &\frac{1}{2}[U(K_c + K_g) - \sum_f b_f K_f] + \\ &+ \frac{1}{2}\Big[U\big(\mathcal{MU}^{-1}(b_g)\big) - b_c K_c - b_g\big(\mathcal{MU}^{-1}(b_g) - K_w - K_c\big)\Big] - \\ &- \sum_i \frac{1}{2}\beta_i K_i^2 \end{aligned} \tag{4.4}$$

As we have already discussed, the decentralised equilibrium is similar to the social planner's expensive natural gas case, if the external costs e_f were ignored. Consequently, the system of equations defining the capacities now is:

$$K_w^* = \frac{b_g}{2\beta_w} \tag{4.5}$$

$$K_c^* = \frac{\beta_g K_g^* + b_g - b_c}{\beta_c} \tag{4.6}$$

$$\mathcal{MU}\left(\frac{\beta_g K_g^* + b_g - b_c + \beta_c K_g^*}{\beta_c}\right) - b_g - 2\beta_g K_g^* = 0 \tag{4.7}$$

Comparing the results of the decentralised equilibrium, with those of the social planner, we can see that the solutions of the decentralised case do not coincide with the social planner's solutions. The costs considered in the decentralised equilibrium are now different than in the social planner's case, and that is why the installed capacities are not optimal. We see this by comparing the system of equations from the decentralised equilibrium, (4.5), (4.6) & (4.7), with the two sets of equations controlling capacity for the social planner; either for the Expensive natural gas case, (3.10), (3.13) & (3.14), or the Cheap natural gas case, (3.16), (3.17) & (3.18).

Note that there is an important difference between the cases of expensive and cheap natural gas. When the social planner and the decentralised equilibrium both consider natural gas to be more expensive than coal, then their difference is only a matter of sub-optimal investments in capacity. On the other hand, when the social planner considers natural gas to be cheaper than coal, the difference between the two solutions is that the market fails not only to invest optimally in the capacities of each technology, but also to dispatch the thermal technologies optimally. Indeed, the social planner solution demands natural gas being fully used and coal serving the remaining demand, while in the decentralised equilibrium the more polluting thermal energy, i.e. coal, is fully used, and natural gas serves the remaining demand .

5 Effect of a subsidy for wind energy

Variable renewables are supported through the use of some sort of direct subsidy scheme. Under such a scheme, a policy maker offers a subsidy σ for each unit of electrical energy that these generators produce. Although awarded via the quantity of energy generated, this is a subsidy that offsets the cost of installing wind generator capacity. We modify our model to reflect this thinking: the final cost of each unit of energy generated by wind is reduced by σ. Since wind technology's operating costs are equal to zero when the subsidy is not in place, they become negative when a positive subsidy is implemented. This is not the same as implementing a Pigouvian tax to deal with the external cost of carbon. Once the wind subsidy has been added, the first order conditions of the decentralised equilibrium (4.1), (4.2), (4.3) become

$$\mathcal{MU}\Big(\sum_i q_i^*(\alpha)\Big) + \sigma - \lambda_w^*(\alpha) = 0 \tag{5.1}$$

$$\mathcal{MU}\Big(\sum_i q_i^*(\alpha)\Big) - b_c - \lambda_c^*(\alpha) = 0 \tag{5.2}$$

$$\mathcal{MU}\Big(\sum_i q_i^*(\alpha)\Big) - b_g - \lambda_g^*(\alpha) = 0 \tag{5.3}$$

The threshold and the total quantities consumed remain identical to the previous section, that is when a subsidy to the variable renewable is not in place. Hence, the threshold is

$$\alpha_3 = \frac{\mathcal{MU}^{-1}(b_g) - K_c - K_g}{K_w}, \qquad \alpha_3 \in (0,1)$$

and the quantity is either equal to $\alpha K_w + K_c + K_g$, or $\mathcal{MU}^{-1}(b_g) - K_w - K_c$, depending on the realization of α.

Turning to the Langrangian for the investment stage, it now reads:

$$
\begin{aligned}
\mathcal{L}_K = \frac{1}{2}[U(K_c + K_g) - \sum_f b_f K_f] + \\
&+ \frac{1}{2}\Big[U\big(\mathcal{MU}^{-1}(b_g)\big) - b_c K_c - b_g\big(\mathcal{MU}^{-1}(b_g) - K_w - K_c\big) + \sigma K_w\Big] - \\
&- \sum_i \frac{1}{2}\beta_i K_i^2
\end{aligned} \tag{5.4}
$$

for which the solutions are:

$$\tilde{K}_w = \frac{b_g + \sigma}{2\beta_w} \tag{5.5}$$

$$\tilde{K}_c = \frac{\beta_g \tilde{K}_g + b_g - b_c}{\beta_c} \tag{5.6}$$

$$\mathcal{MU}\left(\frac{\beta_g \tilde{K}_g + b_g - b_c + \beta_c \tilde{K}_g}{\beta_c}\right) - b_g - 2\beta_g \tilde{K}_g = 0 \tag{5.7}$$

The tilde denotes the solution parameters of the decentralised equilibrium with a subsidy. We rewrite equation (5.5):

$$\tilde{K}_w = \frac{b_g}{2\frac{b_g}{b_g+\sigma}\beta_w}$$

comparing it with Equation 4.5, which controls wind capacity for the decentralised equilibrium, we can easily see that the investment costs of wind capacity are now equivalent to $\dfrac{b_g}{b_g + \sigma}\beta_w < \beta_w$, when $\sigma > 0$. Note that the policy measure σ only appears in the solution for wind; when a positive subsidy is applied, $\sigma > 0$, the subsidised capacity of wind $\tilde{K}_w$ is higher than the unsubsidised K_w^*. In order to specify the subsidy that would replicate the first best wind capacity, we need to know whether the marginal technology for the social planner is natural gas or coal; this depends on whether natural gas is the most expensive technology or not. If natural gas is the marginal technology, $b_g + e_g > b_c + e_c$, we know that the social planner's solution for the wind capacity is given by (3.10):

$$K_w^{ex} = \frac{b_g + e_g}{2\beta_w}$$

from which it is quite clear that the subsidy should be $\sigma = e_g$. On the contrary, if coal is the marginal technology for the social planner, $b_g + e_g < b_c + e_c$, then we know that the social planner's solution for the wind capacity is given by (3.16):

$$K_w^{ch} = \frac{b_c + e_c}{2\beta_w}$$

from which we can determine the optimal subsidy to be $\sigma = e_c - (b_g - b_c)$. Recall that throughout this analysis we have assumed that the operating cost of natural gas alone — without the external cost of carbon — is higher than that of coal, $b_g > b_c$. The two social planner's cases arise because it would be too constraining to make an assumption about the level of these costs, so cannot clearly conclude whether the fully internalised costs of natural gas, $b_g + e_g$ are larger or smaller than those of coal, $b_c + e_c$. Hence if we are in the cheap natural gas case of the social planner, using a subsidy to restore first best would entail internalising the cost of carbon, less the additional operating cost of natural gas over coal.

The installed capacities of the thermal technologies are not altered by the presence of the subsidy. This is problematic for the subsidy-paying policy maker: although we can specify the optimal subsidy in order to replicate the social planner's solution for wind capacity, we cannot find a subsidy that would do the same for the thermal capacities. When the wind is not blowing, i.e. $\alpha = 0$, the system is exactly identical to the one without a subsidy. However, we have already seen in the decentralised equilibrium case that the remaining demand covered by natural gas is given by $q_g^* = \mathcal{MU}^{-1}(b_g) - K_w - K_c$. When a subsidy is in place, the installed capacity of wind is higher, while the installed capacity of coal and natural gas are unaffected. As a consequence, when the wind is blowing $\alpha = 1$, and a subsidy is in place, a larger part of the energy consumed is covered by wind production and the remaining demand that is covered by natural gas is reduced. Indeed, the energy mix at the dispatch stage is altered, having a smaller part of the quantity coming from natural gas. However, that is an issue for the efficacy of the policy measure, especially if the social planner considers natural gas to be the cheaper of the two thermal technologies Therefore, given that thermal capacities are not being replicated, this policy measure cannot succeed in restoring the first best.

6 CONCLUSION

In this paper, we have sought to understand how subsidies to variable renewable generators are effective as environmental policy. We do this by testing to what degree the subsidy to each unit of variable renewable energy replicates the first best with regards to the carbon external costs of fossil fueled electricity.

To do so, we have describe a peak load pricing type model which includes a characterisation of the variable of output from renewables and the external cost of carbon, which varies across fossil based generators. Our model includes a set of three technologies which is typical in a large scale electricity grid: wind, coal and natural gas. More importantly, we characterise the variability in the output of renewable generators, a characteristic we argue makes electricity heterogenous in production. We use this model to analyse the effectiveness of subsidies, as environmental policy, to variable renewables.

We solve the model for the social planner's problem and the decentralised equilibrium. Our simplifying assumptions — made to obtain a tractable result — give us two cases in the social planner's solution. Both are used as benchmarks, against which we evaluate the effectiveness of a subsidy applied in the decentralised equilibrium. We show that the subsidy either fails to displace the most polluting thermal technology, or if it does displace the most polluting, cannot displace the optimal amount. Whether one situation or the other occurs depends solely on conditions that are exogenous to the subsidy.

Our analysis makes no statement on whether subsidies are an effective tool to increase the quantity of renewables per se. We argue that subsidies alone are not an effective measure, if the ultimate goal of increasing the amount of renewable energy generated in an electricity system is decarbonisation. They either do not completely exploit the abatement opportunities that exist — by displacing natural gas instead of the more polluting coal — or they cannot displace the optimal amount of natural gas. This finding should be a consideration for a policy designer or system regulator trying to decrease the carbon intensity of an electricity system.

APPENDIX

Comparative statics

As we have noticed in the setup of the model, what differentiates the two thermal technologies is whether they are always fully used or if when the wind is blowing they serve the residual demand. The decisive factor for which role the thermal technologies have in the energy system is the relation between the costs. Therefore, in the appendix, we will refer to the cheap thermal technology as $th1$ and to the expensive one as $th2$, $b_{th2} > b_{th1}$. If we want to see the comparative statics when gas (coal) is expensive, then we have to replace $th2$ with g (c) and $th1$ with c (g). Without loss of generality, the system of equations defining the capacities is:

$$K_w = \frac{b_{th2}}{2\beta_w}$$

$$K_{th1} = \frac{\beta_{th2}K_{th2} + b_{th2} - b_{th1}}{\beta_{th1}} \tag{1}$$

$$\mathcal{MU}\left(\frac{\beta_{th2}K_{th2} + b_{th2} - b_{th1} + \beta_{th1}K_{th2}}{\beta_{th1}}\right) - b_{th2} - 2\beta_{th2}K_{th2} = 0$$

The comparative statics for the variable technology are quite straightforward:

$$\frac{dK_w}{d\beta_w} = -\frac{b_{th2}}{2\beta_w^2} < 0$$

$$\frac{dK_w}{d\beta_{th1}} = 0$$

$$\frac{dK_w}{d\beta_{th2}} = 0$$

$$\frac{dK_w}{db_{th1}} = 0$$

$$\frac{dK_w}{db_{th2}} = \frac{1}{2\beta_w} > 0$$

After using the implicit function theorem, the comparative statics for the thermal technologies are:

$$\frac{dK_{th2}}{d\beta_w} = 0$$

$$\frac{dK_{th2}}{d\beta_{th1}} = \frac{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)(\beta_{th2}K_{th2}+b_{th2}-b_{th1})}{\beta_{th1}^2[\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}]} > 0$$

$$\frac{dK_{th2}}{d\beta_{th2}} = -\frac{\left[\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right) - 2\right]K_{th2}}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}} < 0$$

$$\frac{dK_{th2}}{db_{th1}} = \frac{\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}} > 0$$

$$\frac{dK_{th2}}{db_{th2}} = \frac{1-\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}} < 0$$

Having computed these effects, we can now use them for the comparative statics of equation (1)

$$\frac{dK_{th1}}{d\beta_w} = 0$$

$$\frac{dK_{th1}}{d\beta_{th1}} = \frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}}{\beta_{th1}^2}\left[-1+\frac{\beta_{th2}}{\beta_{th1}}\frac{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}}\right] < 0$$

$$\frac{dK_{th1}}{d\beta_{th2}} = \frac{K_{th2}}{\beta_{th1}}\left[1-\frac{\beta_{th2}[\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)-2]}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}}\right] > 0$$

$$\frac{dK_{th1}}{db_{th1}} = \frac{1}{\beta_{th1}}\left[\beta_{th2}\frac{\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}} - 1\right] < 0$$

$$\frac{dK_{th1}}{db_{th2}} = \frac{1}{\beta_{th1}}\left[\beta_{th2}\frac{1-\frac{1}{\beta_{th1}}\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)}{\mathcal{MU}'\left(\frac{\beta_{th2}K_{th2}+b_{th2}-b_{th1}+\beta_{th1}K_{th2}}{\beta_{th1}}\right)\frac{\beta_{th2}+\beta_{th1}}{\beta_{th1}} - 2\beta_{th2}} + 1\right] > 0$$

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