Sustainable Products, Market Structure, and Welfare*

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Abstract

This paper examines the welfare consequences of private provision of sustainable goods in output markets with product differentiation. In our setting, consumers can be prone to engage in sustainable consumption, product sustainability imposes costs on firms (e.g., from investments to reduce polluting emissions) but yields welfare externalities, and firms decide both whether to invest in product sustainability and whether to enter the industry of the sustainable product. We find that the interplay of sustainable consumption and firm entry impacts on private incentives that determine the degree of product sustainability. From that interplay, the equilibrium degree of product sustainability can be insufficient or excessive relative to the socially optimal configuration because firm entry rises aggregate output and increases consumer participation in the market, but it also reduces private incentives to provide consumers with sustainable products.

Keywords: Product sustainability, polluting emissions, firm entry, monopolistic competition.

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1 Introduction

In recent decades, product sustainability has been a topic of increasing interest in the economics literature as well as in politics. One of the main reasons is that sustainable products contribute to resource efficiency and they reduce pollution and waste. Based on that, governments have adopted policies such as taxes and subsidies, public campaigns, education, and performance standards in order to increase sustainable consumption and also to encourage firms to invest in product sustainability.[1]

A basic argument behind those policies is that product sustainability leads to externalities that induce governments to promote sustainable goods. For example, in the case of polluting emissions which sustainable goods tend to reduce, one such policies consists of taxes. According to classical environmental economics, we know that Pigouvian taxes can be socially useful in the face of welfare effects from emissions not entirely internalized by agents. To the extent that output implies polluting emissions, a tax per unit of output can work as if the firms' marginal production costs had increased and then a lower level of output, and thus of emissions, is induced. In the context of product sustainability, however, the issue also includes the firms' investment in sustainable production and the consumers' willingness to pay for sustainable products. Additionally, many sustainable goods are produced under product differentiation, which can contribute to imperfect competition in the product market (Schinkel and Spiegel, 2017) and can affect the firms' incentives to enter the industry.

In this paper, we examine the welfare consequences from private provision of sustainable goods in output markets under imperfect competition. In our setting, consumers are willing to pay an extra premium for sustainable goods, but they do not fully internalize the welfare effects from product sustainability. Additionally, firms are able to invest in product sustainability at a cost that increases with the degree of that sustainability (e.g., firms must incur higher costs to reduce polluting emissions further). With those ingredients, we consider imperfect competition in the output market under spatial

[1] See OECD (2008). A conventional definition of sustainable consumption from the 1994 Oslo Symposium on Sustainable Consumption (OECD, 1999) is "the use of services and related products which respond to basic needs and bring a better quality of life while minimizing the use of natural resources and toxic materials as well as emissions of waste and pollutants over the life cycle of the service or product so as not to jeopardize the needs of future generations."

product differentiation (e.g., Hotelling, 1929, and Salop, 1979). Specifically, we consider the spokes model by Chen and Riordan (2007), which extends Hotelling (1929) to an arbitrary number of firms.[2] Our results suggest that market structure and hence firm entry can affect the equilibrium level of product sustainability, and that this level may be insufficient or excessive relative to the socially optimal level. The reason is that entry may have two contradictory effects on social welfare. On the one hand, aggregate output rises and increases the participation of consumers in the market. On the other, the average degree of product sustainability that is provided to consumers declines (e.g., polluting emissions increase). Therefore, the assumption in most of the policies adopted by governments that sustainability should be always promoted must not be taken for granted without further study, particularly when both the investment in product sustainability and firm entry are costly.[3]

A typical feature of sustainable goods is that consumers are prone to engage in sustainable consumption. Casual observation suggests, in fact, that consumers' willingness to pay an extra premium for sustainable products has increased considerably over recent decades. This may point to subjective sustainable consumerism or to objective attributes of goods (e.g., environmentally cleaner goods). Complementarily, microeconomic theory suggests a kind of altruism as a plausible rationale for sustainable consumption, according to which consumers partially internalize the overall welfare impact of sustainable products (e.g., see Andreoni, 1990, Bergstrom, 1995, Popp, 2001, Bagnoli and Watts, 2003, Schinkel and Spiegel, 2017). We rely on this literature in considering that consumers are willing to pay an extra premium for sustainable goods. In our analysis, mediated through firm entry, this plays a key role in determining the extent to which private incentives lead to an excessively high or insufficiently low degree of product sustainability relative

[2] Economic applications of the spokes model include Caminal and Claici (2007), Caminal (2010), Caminal and Granero (2012), Germano and Meier (2013), Granero (2013), and Reggiani (2014).

[3] Recent related work has examined, among other aspects, taxes on the average environmental quality consumed (Cremer and Thisse, 1999), the role of eco-labels (Crampes and Ibanez, 1996, and Kuhn, 1999), the presence of green consumerism (Eriksson, 2004), and the impact of price competition and product differentiation (Conrad, 2005, Rodríguez-Ibeas, 2007, and Espínola-Arredondo and Zhao, 2012). A common assumption in this literature is the absence of firm entry. We depart from that assumption in considering firm entry as endogenous. Our analysis shows that private incentives to enter an industry can affect product sustainability beyond the traditional framework.

to the socially optimal degree when sustainability imposes costs on firms.

When consumers are not motivated to engage in sustainable consumption, the externality from product sustainability becomes relatively large and the equilibrium outcome with endogenous firm entry resembles that without firm entry in that private incentives yield an insufficient degree of product sustainability (e.g., an excessive level of polluting emissions) relative to the socially optimal configuration. Under such circumstances, some effective policies resemble traditional instruments such as Pigouvian taxes that induce a lower output level as in the case of externalities from polluting emissions. In contrast, when consumers are motivated to engage in sustainable consumption, and then the externality from product sustainability does take place but is no longer large, the equilibrium outcome with endogenous firm entry can be in sharp contrast with that from a situation without firm entry. Specifically, in the presence of firm entry and costly investment in sustainability, here private incentives can place the degree of product sustainability above the socially optimal degree (e.g., the equilibrium level of polluting emissions can fall below the socially optimal level). In that case, several effective policies resemble subsidies rather than taxes in the presence of externalities from polluting emissions.

The rest of the paper is as follows. Section 2 introduces the model. Then, Section 3 deals with the welfare benchmark and explores the socially optimal degree of product sustainability. Subsequently, Section 4 examines the role of firm entry under monopolistic competition, and the resulting equilibrium degree of product sustainability. On the grounds of that, Section 5 compares the equilibrium configuration with the socially optimal one in order to obtain a better understanding of the welfare consequences from the private provision of sustainable goods. Finally, Section 5 gathers our main conclusions. The proofs of all the results are in the Appendix.

2 Model

We consider a product market with N potential varieties which are spatially differentiated as in the spokes model by Chen and Riordan (2007). The spokes model can be seen as a generalization of the Hotelling model of spatial product differentiation. In particular, the conventional Hotelling model is such that a line, usually of unit length, represents the product market and each extreme end of the line represents the location of one variety of the product. For $N \geq 2$ a more general situation can be examined with the spokes model, where there are N spokes of length $1/2$, which start from the same central point. Spokes are then indexed by $i = 1, ..., N$, and the producer of variety i is located at the extreme end of spoke i. In those circumstances, the Hotelling model arises as a particular case of the spokes model for $N = 2$.

2.1 Consumers

Consumers' utility increases with the degree of sustainability of the good that they buy. This can follow from subjective sustainable consumerism or from objective attributes, for example, of environmentally cleaner goods. Based on that, we assume that consumers partially internalize the overall welfare impact of sustainable products. For expositional purposes, we focus on polluting emissions as an indicator of the degree of product sustainability, although it will be apparent below that the analysis adapts to other motivating interpretations. A lower level of polluting emissions leads then to an increase in the degree of product sustainability. Specifically, we denote the marginal harm in total welfare and each consumer's marginal disutility from a lower level of product sustainability (a higher level of polluting emissions) by θ_w and θ, respectively. Hence, with $\theta_w > \theta$ the difference $\theta_w - \theta$ turns out to represent the marginal externality from less sustainable products.

In the product market demand is symmetric, and there is a continuum of consumers with mass $N/2$ uniformly distributed over the N spokes. Each consumer has a taste for two varieties and the pair of selected varieties differs across consumers. As is standard, consumer location represents the relative valuation of the two varieties, and each consumer has use for one unit of the good. Consumers are uniformly distributed over the $N(N-1)/2$ possible pairs. The mass of consumers with a taste for an arbitrary pair is $1/(N-1)$, and there are $N-1$ pairs that contain a particular variety, so that the mass of consumers with a taste for variety i is 1, which leads to a simplifying normalization. Consumers with a taste for varieties i and j $(i, j = 1, ..., N,$ $i \neq j)$ are uniformly distributed over the union of spokes i and j.

A consumer located on line i (her favorite brand), at a distance x from the extreme end, obtains a utility of $v - \theta e_i - tx - p_i$ if she buys one unit of variety i at the price p_i, where v is the gross utility of consuming one unit of the product, e_i is the level of polluting emissions from the production of variety i, and t is the unit transportation cost. Her second preferred brand, $j \neq i$, is chosen from the other varieties. Because all other varieties are symmetric from that consumer's viewpoint, each one becomes the chosen second preferred brand with probability $1/(N-1)$. Then, if the consumer purchases one unit of variety j at a price p_j, she obtains a utility of $v - \theta e_j - t(1-x) - p_j$, where e_j is the level of polluting emissions from the production of variety j. As is standard, markets with lower values of t can be interpreted literally as markets with lower transport costs or, in terms of product differentiation, as markets in which goods are more substitutable.

We simplify the setting by treating the number of active varieties as a continuous variable (see Caminal and Granero, 2012). To that end, we denote the fraction of active varieties by $\gamma \in [0,1]$. Then, we treat this fraction of varieties as a continuous variable by considering the limit as N goes to infinity and expressing all relevant variables relative to the total mass of consumers. Because each particular variety $i = 1, ..., N$ may or may not be supplied, if $0 < \gamma < 1$ we have that consumers can be classified into three different groups: some consumers will have access to the two varieties they have a taste for, some other consumers will only be able to purchase one of the varieties, and finally the third group of consumers will have access to neither of the preferred varieties. Given γ and N, the number of pairs of varieties for which two suppliers are active is $\gamma N(\gamma N - 1)/2$, and since the fraction of consumers with a taste for a particular pair is $2/(N(N-1))$, then the fraction of consumers with access to two varieties is $\gamma N(\gamma N-1)/(N(N-1))$. Hence, the fraction of consumers with access to two varieties is γ^2 in the limit as N goes to infinity. Similarly, the fraction of consumers with access to only one variety is $2\gamma(1-\gamma)$, and the fraction of consumers with access to neither of the two preferred varieties is $(1-\gamma)^2$.

2.2 Producers

The production of each variety involves a marginal cost c. In addition, each producer can reduce polluting emissions to reach a level e at the "green" or "sustainability" cost $\beta(e)$ per variety produced. We assume that producing more sustainable goods through lower levels of emissions is more costly, so that $\beta'(e) < 0$ and $\beta''(e) > 0$. In these circumstances, producing a variant involves a total fixed cost $F = f + \beta(e)$,[4] where $f > 0$, and the total amount of fixed costs per consumer is then $\gamma NF/(N/2) = 2\gamma F$. From the viewpoint

[4]This specification of the fixed cost resembles (due to the presence of a choice variable, e) the conventional specification of endogenous sunk costs (see Sutton, 1991, 1998; and Symeonidis, 2000).

of supplier i, the fraction of consumers that demand variety i and have the opportunity of choosing between variety i and their other selected product variety is $(\gamma N - 1)/(N - 1)$, which tends to γ as N goes to infinity. Analogously, the fraction of consumers that demand variety i and do not have access to the other selected variety is $1 - \gamma$ in the limit as N goes to infinity.

2.3 Welfare

Given the fraction of active varieties and the level of polluting emissions, in terms of total surplus it is efficient to allocate those consumers with access to their selected varieties to the closest supplier. Then, consumers with access to two variants of the product incur an average transportation cost of $\frac{t}{4}$; consumers with access to one variety only will incur a higher average transportation cost of $\frac{t}{2}$; and consumers without access to any of their selected varieties will incur no transportation cost as they get zero surplus. Hence, because the amount of fixed costs per consumer is $2\gamma F = 2\gamma\left(f + \beta(e)\right)$, total welfare can be written as

$$W = \gamma^2 \left(v - \theta_w e - \frac{t}{4} - c\right) + 2\gamma(1-\gamma)\left(v - \theta_w e - \frac{t}{2} - c\right) - 2\gamma\left(f + \beta(e)\right). \tag{1}$$

A sufficient condition to obtain interior solutions in the analysis below is $\beta''(e) > \frac{\theta_w^2}{4t}$, which we assume hereafter. Additionally, we denote by $e_0 > 0$ the bound on the level of emissions such that $\beta(e) = 0$ for all $e \geq e_0$. A maintained hypothesis that simplifies the presentation is $v > 3t + \theta_w e_0 + c$, which implies that producers want to serve as many consumers as possible for any given number of active varieties.

3 Welfare-Maximizing Benchmark

This section deals with a first-best benchmark that sets the stage for our subsequent analysis. In maximizing total welfare in expression (1), it does not matter whether γ and e are decided sequentially or simultaneously. Then,

$$\frac{\partial W}{\partial \gamma} = 2\left\{\gamma\frac{t}{4} + (1-\gamma)\left(v - \theta_w e - \frac{t}{2} - c\right) - f - \beta(e)\right\}, \tag{2}$$

$$\frac{\partial W}{\partial e} = -\gamma^2\theta_w - 2\gamma(1-\gamma)\theta_w - 2\gamma\beta'(e). \tag{3}$$

Second-order conditions for an interior solution hold under the maintained hypothesis (see Appendix), and first-order conditions lead to

$$\gamma\frac{t}{4} + (1-\gamma)\left(v - \theta_w e - \frac{t}{2} - c\right) = f + \beta(e), \tag{4}$$

$$-\beta'(e) = \frac{2-\gamma}{2}\theta_w. \tag{5}$$

Given $\gamma \in (0,1)$, equation (5) yields the optimal level of polluting emissions as increasing in γ. At the optimal solution, the marginal cost from sustainability investment per consumer, $-2\gamma\beta'(e)$, must equal the marginal increase in consumer surplus for the fraction of consumers with access to the product, $(\gamma^2 + 2\gamma(1-\gamma))\theta_w = \gamma(2-\gamma)\theta_w$. Consequently, at the optimal solution $-2\gamma\beta'(e) = \gamma(2-\gamma)\theta_w$, which leads to equation (5) for $\gamma \in (0,1)$. From that equation, ceteris paribus, a higher number of available varieties increases consumer surplus and thus allows for a higher level of emissions, which in turn lowers the green cost from reducing polluting emissions, $\beta(e)$. Since $0 \leq \gamma \leq 1$, we can define the boundary values of e that follow from (5) as $\underline{e}_w$ and $\overline{e}_w$ such that

$$-\beta'(\underline{e}_w) = \theta_w, \tag{6}$$

$$-\beta'(\overline{e}_w) = \frac{\theta_w}{2}. \tag{7}$$

With γ and e as endogenous, equations (4) and (5) give rise to the welfare-maximizing values of γ and e, denoted by the pair (γ^w, e^w):

Proposition 1 *Define* $\underline{f}_w \equiv \frac{t}{4} - \beta(\overline{e}_w)$ *and* $\overline{f}_w \equiv v - \theta_w \underline{e}_w - \frac{t}{2} - c - \beta(\underline{e}_w)$. *Then,*

(i) $\gamma^w = 0$ *for* $f \geq \overline{f}_w$*;*

(ii) (γ^w, e^w) *is given by the solution to (4) and (5) for* $f \in [\underline{f}_w, \overline{f}_w]$*;*

(iii) $(\gamma^w, e^w) = (1, \overline{e}_w)$ *for* $f \leq \underline{f}_w$.

Corollary 1 *Both* γ^w *and* e^w *are weakly decreasing in* f*, and strictly decreasing in* f *for* $f \in (\underline{f}_w, \overline{f}_w)$.

The proofs of the results are in the Appendix. Proposition 1 shows that if the setup cost of each variety is high ($f \geq \overline{f}_w$), then not to introduce any product variety is optimal, i.e., $\gamma^w = 0$. In this case, the good is not produced and it makes no sense to examine e. For intermediate values of the setup cost ($\underline{f}_w \leq f \leq \overline{f}_w$), the solution to equations (4) and (5) yields (γ^w, e^w) such that $0 \leq \gamma^w \leq 1$ and $\underline{e}_w \leq e^w \leq \overline{e}_w$. This is the main case, and we deal with it below in detail. Finally, if the setup cost is low ($f \leq \underline{f}_w$), all varieties are produced, so that $\gamma^w = 1$, and then consumer surplus increases to the extent that $\overline{e}_w$ becomes optimal.

Proposition 1 can be illustrated by the example of a quadratic green cost $\beta(e) = \frac{1}{2}(e_0 - e)^2$, $e_0 \geq \theta_w$. Then $\underline{e}_w = e_0 - \theta_w$, $\overline{e}_w = e_0 - \frac{\theta_w}{2}$, and both $e^w(f)$ and $\gamma^w(f)$ are decreasing and convex in f for $f \in (\underline{f}_w, \overline{f}_w)$. Figure 1 displays the graph of $e^w(f)$ (that of $\gamma^w(f)$ follows analogously). The pattern that emerges from that figure has the following explanation. On the one hand, when the fixed setup cost f increases, the optimal number of product varieties falls (at an interior solution), which has a negative impact on consumer surplus. On the other hand, when f increases, the optimal level of emissions falls as well, which has a positive impact on consumer surplus but also leads to an increase in the green cost from reducing emissions. Then, in the face of a higher cost f, reducing the level of emissions compensates for the decrease in the number of varieties up to the point at which the marginal green cost from reducing emissions adjusts to make marginal total surplus equal to zero.

4 Monopolistic Competition

Now, we examine a situation characterized by monopolistic competition, where each firm produces one variety. If firm i decides to enter the market then it pays a fixed cost $f + \beta(e_i)$ with a level of polluting emissions e_i, and it sets a price p_i to maximize profits. Firms maximize profits and enter the market only if net profits are positive. We focus on symmetric free-entry equilibria.

Let us first calculate the symmetric equilibrium price and level of emissions, for a given γ. In market segments where consumers have access to two varieties, consumers will choose supplier as in the Hotelling model, and then a consumer will be indifferent between buying from firm i or from another firm that chooses a price p and a level of emissions e when

$$v - \theta e_i - tx - p_i = v - \theta e - t(1-x) - p, \tag{8}$$

from where the distance x yields the fraction of consumers that choose firm i:[5]

$$x = \frac{1}{2} + \frac{\theta(e - e_i) + p - p_i}{2t}. \tag{9}$$

In those market segments where firm i's product is the consumers' only choice, total demand is 1 whenever consumers obtain a positive surplus, that is, $v - \theta e_i - t - p_i \geq 0$. Under the maintained hypothesis ($v > 3t + \theta_w e_0 + c$) firms never find it optimal to set p_i and e_i such that their price is above $v - \theta e_i - t$, which means that they have incentives to serve as many consumers as possible (see Appendix, proof of Proposition 2). Therefore, firm i decides on p_i and e_i in order to maximize:

$$\pi_i = \left[\gamma\left(\frac{1}{2} + \frac{\theta(e - e_i) + p - p_i}{2t}\right) + 1 - \gamma\right](p_i - c) - \beta(e_i) - f, \tag{10}$$

subject to $p_i + \theta e_i \leq v - t$. If this constraint is not binding,

[5] Provided $p_i \in [\theta(e - e_i) + p - t, \theta(e - e_i) + p + t]$, so that $0 \leq x \leq 1$.

$$\frac{\partial \pi_i}{\partial p_i} = \gamma \left(\frac{1}{2} + \frac{\theta(e - e_i) + p - 2p_i + c}{2t} \right) + 1 - \gamma, \tag{11}$$

$$\frac{\partial \pi_i}{\partial e_i} = -\frac{\gamma\theta}{2t}(p_i - c) - \beta'(e_i), \tag{12}$$

from where the optimal price and level of emissions are determined by

$$p_i = \frac{2 - \gamma}{2\gamma} t + \frac{\theta(e - e_i) + p + c}{2}, \tag{13}$$

$$-\beta'(e_i) = \frac{\gamma\theta}{2t}(p_i - c), \tag{14}$$

provided that $\beta''(e_i) > \frac{\theta^2}{4t}\gamma$ (see Appendix for details on second-order conditions). Then, the symmetric equilibrium price, $p_i = p = p^*$, and level of emissions, $e_i = e = e^*$, are given by

$$p^* = c + \frac{2 - \gamma}{\gamma} t, \tag{15}$$

$$-\beta'(e^*) = \frac{2 - \gamma}{2}\theta. \tag{16}$$

From these equations, an increase in γ implies that competition intensifies and then the price falls and the level of emissions increases. Before dealing with firm entry, a baseline situation for future comparison is that in which all potential varieties are active, i.e., $\gamma = 1$. If $\gamma = 1$ then $p^* = c + t$ as in the standard Hotelling model, and $-\beta'(e^*) = \frac{\theta}{2}$. In the limiting case $\theta = \theta_w$ (no externality) this clearly yields the equilibrium level of emissions as in the first best for $\gamma = 1$, that is, $e^* = e^w$, although the equilibrium price remains above marginal cost. For $\theta < \theta_w$ (i.e., in the presence of an externality), it follows that when $\gamma = 1$ the level of emissions is higher in equilibrium relative to the first best, $e^* > e^w$. Our analysis below shows that this simple conclusion does not necessarily extends to situations where, mediated through firm entry, not all potential varieties are actually supplied, i.e., $\gamma < 1$. In particular, we will see that entry will tend to have two contradictory effects on social welfare. On the one hand, aggregate output will rise and this will increase the participation of consumers in the market. On the other hand, the average degree of product sustainability that is provided to consumers will decline (average polluting emissions will increase).

For future reference, we define $\underline{e}$ and $\overline{e}$ analogously to the thresholds $\underline{e}_w$ and $\overline{e}_w$ in (6)-(7):

$$-\beta'(\underline{e}) = \theta, \tag{6'}$$

$$-\beta'(\overline{e}) = \frac{\theta}{2}. \tag{7'}$$

The equilibrium is given by (15)-(16) provided $p^* + \theta e^* < v - t$, which is equivalent to $\gamma > \frac{2t}{v - \theta e^* - c}$. This condition can be written as $\gamma > \widehat{\gamma}$, where $\widehat{\gamma}$ is a threshold that depends on the parameters of the model. For example, with a quadratic green cost $\beta(e_i) = \frac{1}{2}(e_0 - e_i)^2$, $e_0 \geq \theta$, we have $p^* = c + \frac{2-\gamma}{\gamma} t$ and $e^* = e_0 - \frac{2-\gamma}{2}\theta$ provided $\gamma > \widehat{\gamma}$, where

$$\widehat{\gamma} = \frac{1}{\theta^2}\left(v - \theta(e_0 - \theta) - c - \sqrt{(v - \theta(e_0 - \theta) - c)^2 - 4\theta^2 t}\right). \tag{17}$$

Furthermore, if $p^* + \theta e^* < v - 2t$, an individual firm i may find it optimal to deviate from $p^* + \theta e^*$ and set $p_i + \theta e_i = v - t$. Such a deviation is not profitable provided:

$$\frac{v - c}{t} \leq \Psi(\gamma) + \frac{\theta}{t} e^*(\gamma) \equiv 1 + \frac{(2 - \gamma)^2}{2\gamma(1 - \gamma)} + \frac{\theta}{t} e^*(\gamma). \tag{18}$$

If this condition does not hold, a symmetric equilibrium does not exist.[6]

Hence, provided $\gamma \in [\widehat{\gamma}, 1]$ the fraction of active varieties in equilibrium, γ^*, will be given by the zero profit condition:

$$\pi^*(\gamma^*) = \frac{(2-\gamma^*)^2}{2\gamma^*}t - \beta(e^*(\gamma^*)) - f = 0. \tag{19}$$

Equivalently, if $f \in [\underline{f}, \widehat{f}]$ then (provided condition (18) holds) $\gamma^*(f)$ is given by the solution to (19), and if $f \leq \underline{f}$ then $\gamma^* = 1$, where $\underline{f} \equiv \frac{t}{2} - \beta(\overline{e})$, and $\widehat{f}$ is defined as the value of f such that $\pi^*(\widehat{\gamma}) = 0$ in equation (19).

If instead $\gamma \leq \widehat{\gamma}$, which occurs whenever $f \geq \widehat{f}$, each firm faces little competition and finds it optimal to set $p^* + \theta e^* = v - t$, and serve all consumers with no other choice. Then, each firm's profits are

$$\pi_i = \frac{2-\gamma}{2}(v - \theta e_i - t - c) - \beta(e_i) - f, \tag{20}$$

so that

$$p^* = v - \theta e^* - t, \tag{21}$$

$$-\beta'(e^*) = \frac{2-\gamma}{2}\theta, \tag{22}$$

where (22) is as (16) above. In this case, the zero profit condition is:

$$\pi^*(\gamma^*) = \frac{2-\gamma^*}{2}(v - \theta e^*(\gamma^*) - t - c) - \beta(e^*(\gamma^*)) - f = 0. \tag{23}$$

[6] The function $\Psi(\gamma)$ reaches a minimum at $\gamma = \frac{2}{3}$, where $\Psi\left(\frac{2}{3}\right) = 5$, and e^* increases with γ. For the example of a quadratic green cost $\beta(e_i) = \frac{1}{2}(e_0 - e_i)^2$, $e_0 \geq \theta$, condition (18) reads as $\frac{v-c}{t} \leq \Psi(\gamma) + \frac{\theta^2}{2t}\gamma + \frac{\theta}{t}(e_0 - \theta)$.

Consequently, if $f \in [\widehat{f}, \overline{f}]$ then $\gamma^*(f)$ is given by the solution to (23), and if $f \geq \overline{f}$ then $\gamma^* = 0$, where $\overline{f} \equiv v - \theta \underline{e} - t - c - \beta(\underline{e})$.

This discussion is summarized as follows:

Proposition 2 *Define* $\underline{f} \equiv \frac{t}{2} - \beta(\overline{e})$ *and* $\overline{f} \equiv v - \theta\underline{e} - t - c - \beta(\underline{e})$. *Then,*

(i) $\gamma^* = 0$ *for* $f \geq \overline{f}$*;*

(ii) (γ^*, p^*, e^*) *is given by the solution to (21), (22) and (23) for* $f \in [\widehat{f}, \overline{f}]$*;*

(iii) (γ^*, p^*, e^*) *is given by the solution to (15), (16) and (19) for* $f \in [\underline{f}, \widehat{f}]$*;*

(iv) $(\gamma^*, p^*, e^*) = (1, c + t, \overline{e})$ *for* $f \leq \underline{f}$*.*

Corollary 2 *(i)* p^* *is weakly increasing in* f*, and strictly increasing in* f *for* $f \in (\underline{f}, \overline{f})$*; (ii) both* γ^* *and* e^* *are weakly decreasing in* f*, and strictly decreasing in* f *for* $f \in (\underline{f}, \overline{f})$*.*

We draw $e^*(f)$ in Figure 2 (the graph of $\gamma^*(f)$ follows analogously). Intuitively, the impact of f on equilibrium emissions is mediated through the impact of firm entry on price, and of price on emissions. An increase in the entry cost f reduces net profit and thus firm entry. Because the equilibrium price decreases with the number of active firms, an increase in f turns out to increase the free-entry equilibrium price. Therefore, because the incentives to reduce emissions increase with the price-cost margin, we have that an increase in f reduces the number of active firms, a lower number of firms increases the equilibrium price, and a higher price reduces in turn the free-entry equilibrium level of emissions. Hence, γ^* and e^* decrease with f. At this point, recall that the welfare-maximizing counterpart values γ^w and e^w decrease with f as well.

5 Equilibrium vs. Socially Optimal Levels of Product Sustainability

This section compares the levels of product sustainability that follow from the decentralized equilibrium and from the socially optimal configuration. In particular, we find three different situations depending on the consumers' willingness to pay for sustainable products: a small, an intermediate, and a large willingness to pay for sustainability. In our setting, the marginal externality on social welfare from polluting emissions is captured by the difference between the marginal harm in total welfare and each consumer's marginal disutility from a higher level of emissions (i.e., relative willingness to pay for sustainability), as given by $\theta_w - \theta$. Ceteris paribus, the greater this difference, the larger the externality from emissions. Then, we obtain the following result:

Proposition 3 *There exist thresholds* $\underline{\alpha}$ *and* $\overline{\alpha}$, *with* $0 < \underline{\alpha} < \overline{\alpha}$, *such that:*

(i) With an exogenous number firms, the equilibrium level of polluting emissions is above the first-best level for all $\theta_w - \theta > 0$.

(ii) With an endogenous number of firms, and cutoffs f_I^a, f_I^b, f_{II}^a *and* f_{II}^b, *where* $f_{II}^a < f_{II}^b < f_I^a < f_I^b$,
(ii.1) if $\theta_w - \theta \geq \overline{\alpha}$ *(small willingness to pay for sustainability) then the equilibrium level of polluting emissions is above the first-best level;*
(ii.2) if $\underline{\alpha} \leq \theta_w - \theta < \overline{\alpha}$ *(intermediate willingness to pay for sustainability) then the equilibrium level of polluting emissions is above the first-best level for* $f < f_I^a$ *and for* $f > f_I^b$, *and it is below the first-best level for* $f_I^a < f < f_I^b$;
(ii.3) if $0 < \theta_w - \theta < \underline{\alpha}$ *(large willingness to pay for sustainability) then the equilibrium level of polluting emissions is above the first-best level for* $f < f_{II}^a$, *for* $f_{II}^b < f < f_I^a$ *and for* $f > f_I^b$, *and it is below the first-best level for* $f_{II}^a < f < f_{II}^b$ *and for* $f_I^a < f < f_I^b$.

The explanation of this result is as follows. Part (i) deals with a situation where the number of firms is given. Then, without firm entry, the presence of an externality from polluting emissions leads to an excessive level of emissions relative to the socially optimal level. This outcome is immediate and can be seen as a baseline situation with which to compare situations with firm entry.

Part (ii) with an endogenous number of firms is the main part in the result. Then, if consumers are not willing to pay for product sustainability, so that the externality from polluting emissions is large, as in part (ii.1), firms have no incentives to invest in sustainable products in order to avoid the cost $\beta(e)$, which reduces $F = f + \beta(e)$ and implies excessive firm entry. Then, the outcome becomes analogous to that from the baseline situation without firm entry, and the equilibrium level of emissions is above the welfare-maximizing level. Figure 3 illustrates the case in part (ii.1) of the result, where the regular line represents free-entry equilibrium emissions, $e^*(f)$, and the bold line socially optimal emissions, $e^w(f)$. Since there is excessive firm entry with an excessive equilibrium level of emissions, this situation yields an insufficient level of product sustainability. Consequently, it is socially optimal to reduce the equilibrium level of emissions. In that context, several instruments can be used to achieve a fall in equilibrium emissions. For example, a conventional tax per unit of output can contribute to that aim. Because this tax works as if the firms' marginal production cost had increased, it impacts on their price-cost margins and thus on the incentives to invest in product sustainability. Then, under free entry a fall in effective price-cost margins due to the tax will reduce the number of active firms and the equilibrium level of emissions, which ends up placing the equilibrium configuration closer to the socially optimal one. Graphically, in Figure 3, the tax is able to place the line that represents free-entry equilibrium emissions, $e^*(f)$, closer to the line that represents socially optimal emissions, $e^w(f)$. An alternative way to achieve a similar outcome follows from a tougher product-market competition, which reduces prospective profits and hence the incentives to enter the market. In our setting, the parameter that captures transportation costs or, more generally, spatial product differentiation, t, is a measure of the toughness of price competition as defined by Sutton (1991). Therefore, regulatory or other changes that reduce t can also contribute to place the equilibrium configuration closer to the socially optimal one.

When consumers are relatively willing to pay for product sustainability, so that the externality from polluting emissions is intermediate, as in part (ii.2), the optimal regulation on the product market in order to affect the decentralized configuration is no longer unconditional. Figure 4 illustrates this case. As in part (ii.1), if f is relatively low then there is excessive firm entry ($\gamma^* > \gamma^w$) and the equilibrium level of emissions is above the socially optimal level ($e^* > e^w$). However, if f is high then there is insufficient entry ($\gamma^* < \gamma^w$) and the equilibrium level of emissions can fall below the socially optimal level ($e^* < e^w$). The latter situation (Region I in Figure 4) contrasts with the case of small willingness to pay for sustainability (i.e., large externality) in part (ii.1) and with the baseline case in part (i). Specifically, with a small number of firms, each of the active firms will have an incentive to invest in product sustainability because, first, there is little competition with a small number of active producers in the market and, second, now consumers are relatively willing to pay for sustainability. Those two aspects make firms to reduce polluting emissions even if that is costly. As in traditional spatial models of localized competition (e.g., Salop, 1979), if the entry cost is higher than a certain threshold then all firms are local monopolists, and the equilibrium number of firms is insufficient because they cannot appropriate all the surplus they create by entering the market if they cannot price discriminate among heterogeneous consumers. Since an insufficient number of firms tends to reduce the level of emissions below the socially optimal level, here a subsidy per unit of output (which increases the firms' incentives to enter the industry) can contribute to place the equilibrium configuration closer to the socially optimal one. Alternatively, policies such as advice or more generally help to obtain eco-labels, which can reduce the cost f, could contribute to that aim (e.g., by approaching f_I^a from above in Region I of Figure 4). In contrast, if the entry cost is below the threshold then the entire market is served, and the firms' decisions are driven by business stealing. Private incentives to invest in product sustainability are then altered, and both firm entry and the equilibrium level of polluting emissions become excessive. The reason is that a sizable fraction of an entrant firm's customers is stolen from existing firms, given that many firms are active when the entry cost is low, and profits made out of stolen customers are higher than the reduction in transportation costs by these reallocated consumers. Then, a tax per unit of output (which reduces the firms' incentives to enter the industry) can contribute to place the equilibrium configuration closer to the socially optimal one.

Figure 5 draws the case of large willingness to pay for sustainability (i.e., small externality) as in part (ii.3). In this case, for extreme values of f ($f \leq f_{II}^a$ and $f \geq f_I^a$), we have situations similar to those arising from intermediate willingness to pay for sustainability. If f is sufficiently low to support equilibrium values of γ close to one, then business stealing induces excessive firm entry, and the resulting equilibrium level of emissions ends up above the socially optimal level. If f is sufficiently high to support equilibrium values of γ close to zero, firms are close to being local monopolies and firm entry is insufficient, so that the equilibrium level of emissions falls below the socially optimal level for a relevant range of values of f ($f_I^a < f < f_I^b$). For the remaining situations in Figure 5 ($f_{II}^a \leq f \leq f_I^a$) it is not obvious to determine the net effect of a change in f on the difference between equilibrium and first-best emission levels. As f decreases from levels that support equilibrium values of γ close to zero, then firm entry increases and business stealing intensifies. At a certain point, business stealing becomes the dominant effect and private incentives to enter become socially excessive, which leads to polluting emissions above the welfare-maximizing level of emissions. If f falls so much that firm entry intensifies to the point that γ increases above $\widehat{\gamma}$, price competition intensifies considerably and private incentives to enter moderate thereafter. Since this limits the impact of business stealing for intermediate values of f, the equilibrium level of emissions can fall below the first-best level. This situation rests on the fact that consumers are very prone to pay for product sustainability. The reason is that, in those circumstances (Region II, i.e., $f_{II}^{a} \leq f \leq f_{II}^{b}$), firms face a high degree of product-market competition (many firms have entered the market), which they can "relax" through product sustainability (this becomes profitable due to the consumers' willingness to pay for sustainability). Consequently, in the case of Figure 5 we can have excessive or insufficient equilibrium levels of emissions. For values of f in regions I and II of Figure 5 the equilibrium level of emissions is insufficient (in contrast with the baseline case without firm entry), whereas for the rest of values of f in Figure 5 the equilibrium level of emissions is excessive (as in that baseline case). Then, a subsidy or a tax per unit of output can contribute, accordingly, to place the equilibrium configuration closer to the socially optimal one along the lines argued above.

6 Concluding Remarks

In this paper, we have explored the welfare consequences of private provision of sustainable goods with product differentiation. In our setting, consumers can be prone to engage in sustainable consumption. Additionally, product sustainability is costly to firms (e.g., through investments to reach lower levels of polluting emissions) but yields welfare externalities, and firms decide whether to invest in product sustainability and whether to enter the industry of the sustainable product.

Our analysis reveals that the interplay of sustainable consumption and firm entry impacts on private incentives that determine the degree of product sustainability in equilibrium. From that interplay, the private provision of product sustainability can be insufficient or excessive relative to the socially optimal configuration. In our setting, entry may have two contradictory effects on social welfare. On the one hand, aggregate output rises and increases the participation of consumers in the market. On the other, the average degree of product sustainability that is provided to consumers declines (e.g., polluting emissions increase). For extreme situations with many active firms or only a few of active firms, one of those two contradictory effects dominates and conclusions in terms of welfare tend to be unconditional. In contrast, for the intermediate cases of firm entry the conclusions are less obvious without further study of the consumers' willingness to pay for sustainable products, particularly for intermediate and large willingness to pay for sustainability (i.e., when externalities are not very large). These situations point to the relevance of consumer valuation of product sustainability regarding the private provision of sustainable goods relative to the socially optimal provision.

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8 Appendix

Proof of Proposition 1 Making use of (2)-(3),

$$\frac{\partial^2 W}{\partial \gamma^2} = -2\left(v - \theta_w e - \frac{3}{4}t - c\right) < 0, \quad \frac{\partial^2 W}{\partial e^2} = -2\gamma\beta''(e) < 0,$$

$$\frac{\partial^2 W}{\partial \gamma^2}\frac{\partial^2 W}{\partial e^2} - \left(\frac{\partial^2 W}{\partial \gamma \partial e}\right)^2 = 4\gamma\beta''(e)\left\{v - \theta_w e - \frac{3}{4}t - c - \frac{\gamma\theta_w^2}{4\beta''(e)}\right\},$$

where

$$\frac{\partial^2 W}{\partial \gamma \partial e} = \frac{\partial^2 W}{\partial e \partial \gamma} = -2\left((1-\gamma)\theta_w + \beta'(e)\right).$$

Since $v - \theta_w e - (3t/4) - c > 5t/4$, then for all $\gamma \in (0,1)$ we can write

$$\frac{\partial^2 W}{\partial \gamma^2}\frac{\partial^2 W}{\partial e^2} - \left(\frac{\partial^2 W}{\partial \gamma \partial e}\right)^2 > \gamma\beta''(e)\left(5t - \frac{\theta_w^2}{\beta''(e)}\right) > 0.$$

Thus, an interior solution (γ^w, e^w) follows from (4)-(5) for $f \in (\underline{f}_w, \overline{f}_w)$. If $f \geq \overline{f}_w$ then $\gamma^w = 0$ from (2), whereas if $f \leq \underline{f}_w$ then $\gamma^w = 1$ from (2) and $-\beta'(e^w) = \frac{\theta_w}{2}$ from (3), that is, $e^w = \overline{e}_w$. Hence, the result follows.

Proof of Corollary 1 Under Proposition 1, (γ^w, e^w) follows from (4)-(5) for $f \in (\underline{f}_w, \overline{f}_w)$. Then, we can use (4) to define

$$\Omega(\gamma^w) \equiv \frac{t}{4}\gamma^w + (1-\gamma^w)\left(v - \theta_w e^w(\gamma^w) - \frac{t}{2} - c\right) - \beta(e^w(\gamma^w)) - f = 0,$$

where $e^w(\gamma^w)$ is determined by equation (5) such that

$$-\beta'(e^w) = \frac{2-\gamma^w}{2}\theta_w.$$

Hence,

$$\frac{d\gamma^w}{df} = -\frac{\partial\Omega/\partial f}{\partial\Omega/\partial\gamma^w} = -\frac{1}{v - \theta_w e^w - \frac{3}{4}t - c - \frac{\theta_w^2\gamma^w}{2\beta''(e^w)}},$$

which is negative if and only if $v - \theta_w e^w - (3t/4) - c > \theta_w^2\gamma^w/(2\beta''(e^w))$. From $\gamma^w \leq 1$ and the second-order conditions (SOC) for an interior solution, a sufficient condition for $d\gamma^w/df < 0$ is $v - \theta_w\overline{e}_w - (3t/4) - c > 2t$, i.e., $v > \theta_w\overline{e}_w + (11t/4) + c$, which holds under the maintained hypothesis ($v > \theta_w e_0 + 3t + c$). From Proposition 1, this completes the proof.

Proof of Proposition 2 From equations (11)-(12),

$$\frac{\partial^2\pi_i}{\partial p_i^2} = -\frac{\gamma}{t} < 0, \quad \frac{\partial^2\pi_i}{\partial e_i^2} = -\beta''(e_i) < 0,$$

$$\frac{\partial^2\pi_i}{\partial p_i^2}\frac{\partial^2\pi_i}{\partial e_i^2} - \left(\frac{\partial^2\pi_i}{\partial p_i\partial e_i}\right)^2 = \frac{\gamma}{t}\left\{\beta''(e_i) - \frac{\gamma}{t}\left(\frac{\theta}{2}\right)^2\right\},$$

where

$$\frac{\partial^2\pi_i}{\partial p_i\partial e_i} = \frac{\partial^2\pi_i}{\partial e_i\partial p_i} = -\frac{\gamma\theta}{2t}.$$

Hence, $(\partial^2\pi_i/\partial p_i^2)(\partial^2\pi_i/\partial e_i^2) - (\partial^2\pi_i/\partial p_i\partial e_i)^2 > 0$ iff $\beta''(e_i) > \gamma\theta^2/(4t)$, which is implied by $\beta''(e_i) > \theta_w^2/(4t)$ given that $\gamma \leq 1$ and $\theta \leq \theta_w$. Then, expressions (13) and (14) yield the profit-maximizing solution (p_i, e_i), and thus at a symmetric equilibrium $(p_i, e_i) = (p^*, e^*)$ as given by equations (15) and (16) for $f \in [\underline{f}, \widehat{f}]$, i.e., $\gamma \geq \widehat{\gamma}$, provided condition (18) holds. In those circumstances, free entry yields γ^* as the solution to the zero profit condition

(19). For any $\gamma \geq \widehat{\gamma}$ that condition is equivalent to

$$g(\gamma) \equiv \gamma\pi^*(\gamma) = \frac{(2-\gamma)^2}{2}t - \gamma\beta(e^*(\gamma)) - \gamma f = 0,$$

where equation (16) determines $e^*(\gamma)$ such that $-\beta'(e^{MC}) = \theta(2-\gamma)/2$. We have that $g(\gamma)$ is a continuous function, $g'(\gamma) < 0$ when $\pi^*(\gamma) \geq 0$, $g(\widehat{\gamma}) > 0$ for $f < \widehat{f}$, and $g(1) \leq 0$. Hence, there exists one solution to $g(\gamma) = 0$ given by $\gamma^* \in (\widehat{\gamma}, 1)$. If $f = \widehat{f}$ then $\gamma^* = \widehat{\gamma}$ as given by

$$\frac{(v - \theta e^*(\widehat{\gamma}) - t - c)^2}{v - \theta e^*(\widehat{\gamma}) - c} - \beta(e^*(\widehat{\gamma})) = \widehat{f},$$

with $e^*(\widehat{\gamma})$ such that $-\beta'(e^*) = \theta(2-\widehat{\gamma})/2$; and if $f \leq \underline{f}$ then $\gamma^* = 1$.

Now, consider $f > \widehat{f}$, i.e., $\gamma < \widehat{\gamma}$. Here, we need to check that the only symmetric equilibrium involves $p^* = v - \theta e^* - t$ and $-\beta'(e^*) = \theta(2-\gamma)/2$. A representative firm i chooses (p_i, e_i) to maximize

$$\pi_i = \left[\gamma\left(\frac{1}{2} + \frac{\theta(e - e_i) + p - p_i}{2t}\right) + (1-\gamma)\frac{v - \theta e_i - p_i}{t}\right](p_i - c) - \beta(e_i) - f,$$

subject to $p_i + \theta e_i \geq v - t$. The first-order conditions for an interior solution can be written as

$$\gamma\left(t + \theta(e - e_i) + p - 2p_i + c\right) + 2(1-\gamma)(v - \theta e_i - 2p_i + c) = 0,$$

$$-\frac{\theta}{2t}(2-\gamma)(p_i - c) - \beta'(e_i) = 0.$$

If a symmetric equilibrium exists, then the price is given by

$$p(\gamma) = \frac{2(1-\gamma)(v - \theta e(\gamma) + c) + \gamma(t + c)}{4 - 3\gamma},$$

where $e(\gamma)$ is such that

$$-\beta'(e) = \frac{\theta}{2t}\frac{2-\gamma}{4-3\gamma}(2(1-\gamma)(v-\theta e-c)+\gamma t).$$

It turns out that $p(0)+\theta e(0) = \frac{1}{2}(v+\theta e(0)+c) < v-t$, and $p'(\gamma) < 0$. Thus, a contradiction follows. If other firms set $p+\theta e = v-t$ then according to firm i's first-order conditions its best response is such that $p_i+\theta e_i = \frac{1}{2}(v+\theta e_i+c) < v-t$. Therefore, the only symmetric equilibrium involves $p+\theta e = v-t$, from where $p^* = v-\theta e^* - t$ and $-\beta'(e^*) = \theta(2-\gamma)/2$. Then, free entry determines γ^* as the solution to the zero profit condition (23) if $f \in [\widehat{f}, \overline{f}]$, and $\gamma^* = 0$ if $f \geq \overline{f}$. This shows the result.

Proof of Corollary 2 Consider first that $f \in (\underline{f}, \widehat{f})$. Then, γ^* is given by the solution to the zero profit condition (19). That condition is equivalent to

$$g(\gamma) \equiv \gamma\pi^*(\gamma) = \frac{(2-\gamma)^2}{2}t - \gamma\beta(e^*(\gamma)) - \gamma f = 0,$$

where equation (16) determines $e^*(\gamma)$ such that $\beta'(e^*) = -\theta(2-\gamma)/2$. Hence, γ^* is given by the only solution to $g(\gamma) = 0$ such that $\gamma^* \in (\widehat{\gamma}, 1)$, and implicit differentiation gives rise to

$$\frac{d\gamma^*}{df} = -\frac{\gamma^*}{(2-\gamma^*)\left(t - \frac{\theta^2}{4\beta''(e^*)}\gamma^*\right) + \beta(e^*) + f} < 0.$$

If $f = \widehat{f}$ then $\gamma^* = \widehat{\gamma}$ such that

$$\frac{(v-\theta e^*(\widehat{\gamma})-t-c)^2}{v-\theta e^*(\widehat{\gamma})-c} - \beta(e^*(\widehat{\gamma})) = \widehat{f},$$

with $e^*(\widehat{\gamma})$ such that $-\beta'(e^*) = \frac{2-\widehat{\gamma}}{2}\theta$, and if $f \leq \underline{f}$ then $\gamma^* = 1$.

Next, consider that $f \in (\widehat{f}, \overline{f})$. Then, γ^* is given by the solution to the zero profit condition (23), and implicit differentiation yields

$$\frac{d\gamma^*}{df} = -\frac{2}{v - \theta e^* - t - c} < 0,$$

and if $f \geq \overline{f}$ then $\gamma^* = 0$. Thus, γ^* is weakly decreasing in f, and it is strictly decreasing in f for $f \in (\underline{f}, \overline{f})$. Since e^* is strictly increasing in γ for $f \in (\underline{f}, \overline{f})$, this implies that e^* is weakly decreasing in f, and it is strictly decreasing in f for $f \in (\underline{f}, \overline{f})$. Finally, because γ^* and e^* are strictly decreasing in f for $f \in (\underline{f}, \widehat{f})$, it follows that p^* is weakly increasing in f, and it is strictly increasing in f for $f \in (\underline{f}, \overline{f})$. Thus, the result is shown.

Proof of Proposition 3 With an exogenous number of firms, part (i) follows directly from equations (5), (15) and (16) under $\theta < \theta_w$. Next, with an endogenous number of firms it can be seen that there exists a positive threshold $\overline{\alpha}$ such that $\partial W(\gamma^*, e^*)/\partial e < 0$ for all $\theta_w - \theta > \overline{\alpha}$, where $\overline{\alpha}$ depends on the parameters in the model. Hence, there exists no crossing point at which $e^*(\gamma^*) = e^w(\gamma^w)$ whenever the difference $\theta_w - \theta$ is above $\overline{\alpha}$. In particular, $e^*(\gamma^*) > e^w(\gamma^w)$ for all $\theta_w - \theta > \overline{\alpha}$, so that part (ii.1) holds.

Consider now part (ii.2), so that $\theta_w - \theta < \overline{\alpha}$. Here, there exists a positive threshold $\underline{\alpha} < \overline{\alpha}$ such that with $\theta_w - \theta \in (\underline{\alpha}, \overline{\alpha})$ we have $\partial W(\gamma^*, e^*)/\partial e \gtreqless 0$ as $f \gtreqless f_I^a$ for $f \in [\underline{f}, \overline{f})$, and $\partial W(\gamma^*, e^*)/\partial e \lesseqgtr 0$ as $f \gtreqless f_I^b$ for $f \in [\overline{f}, \overline{f}_w]$. Consequently, in the region where $\widehat{f} \leq f \leq \overline{f}$ we can use equations (4) and (23) to see that as long as $\theta_w - \theta < \overline{\alpha}$ there exists one crossing point at which $e^w(\gamma^w) = e^*(\gamma^*)$, where $e^w(\gamma^w)$ follows from (5), and $e^*(\gamma^*)$ follows from (16). However, in the region where $\underline{f} \leq f \leq \widehat{f}$ no crossing point at which $e^w(\gamma^w) = e^*(\gamma^*)$ does exist whenever $\theta_w - \theta > \underline{\alpha}$. Hence, part (ii.2) follows.

Finally, consider part (iii.3), where $\theta_w - \theta < \underline{\alpha}$. First, consider the region $\widehat{f} \leq f \leq \overline{f}_w$. Making use of (4) and (23), here γ^* and γ^w cross as long as

$$\frac{2-\gamma}{2}\left(v - \theta e^*(\gamma) - t - c\right) = \gamma\frac{t}{4} + (1-\gamma)\left(v - \theta e^w(\gamma) - \frac{t}{2} - c\right),$$

with $e^*(\gamma) = e^w(\gamma)$ for $\gamma = \gamma^* = \gamma^w$ under θ arbitrarily close to θ_w. This holds whenever

$$h_I(\gamma) \equiv \gamma \left(v - \theta e^w(\gamma) - \frac{t}{2} - c \right) - t = 0.$$

It can be seen that $h_I(0) < 0$, $h_I(1) > 0$, and $h_I'(\gamma) > 0$. Hence, in this region there exists one solution $\gamma \in (0, 1)$ to $h_I(\gamma) = 0$. That is, there exists one crossing point at which $\gamma^* = \gamma^w$ when θ approaches θ_w. Denote that crossing point by $\gamma(f_I^a)$.

Next, consider the region $\underline{f}_w \leq f \leq \widehat{f}$. From (4) and (19), here γ^* and γ^w cross as long as

$$\frac{(2-\gamma)^2}{2\gamma} t = \gamma \frac{t}{4} + (1-\gamma) \left(v - \theta e^w(\gamma) - \frac{t}{2} - c \right),$$

with $e^*(\gamma) = e^w(\gamma)$ for $\gamma = \gamma^* = \gamma^w$ under θ arbitrarily close to θ_w. This holds whenever

$$h_{II}(\gamma) \equiv 2\gamma(1-\gamma) \left(v - \theta e^w(\gamma) - \frac{t}{2} - c \right) + \gamma^2 \frac{t}{2} + (2-\gamma)^2 t = 0.$$

We have that $h_{II}(0) < 0$, $h_{II}(1) < 0$, $h_{II}(\gamma) > 0$ for some $\gamma \in (0, 1)$, and there exists a threshold value $\widetilde{\gamma} \in (0, 1)$ such that $h_{II}'(\gamma) \gtreqless 0$ as $\gamma \lesseqgtr \widetilde{\gamma}$. Therefore, there are two roots to $h_{II}(\gamma) = 0$ from the Intermediate Value Theorem on the interval $(0, \widetilde{\gamma})$, and on the interval $(\widetilde{\gamma}, 1)$. Hence, here there exist two crossing points at which $\gamma^* = \gamma^w$ when θ approaches θ_w. Denote those crossing points by $\gamma(f_{II}^a)$ and $\gamma(f_{II}^b)$, where $f_{II}^a < f_{II}^b$. By continuity, this shows part (ii.3) and completes the proof.

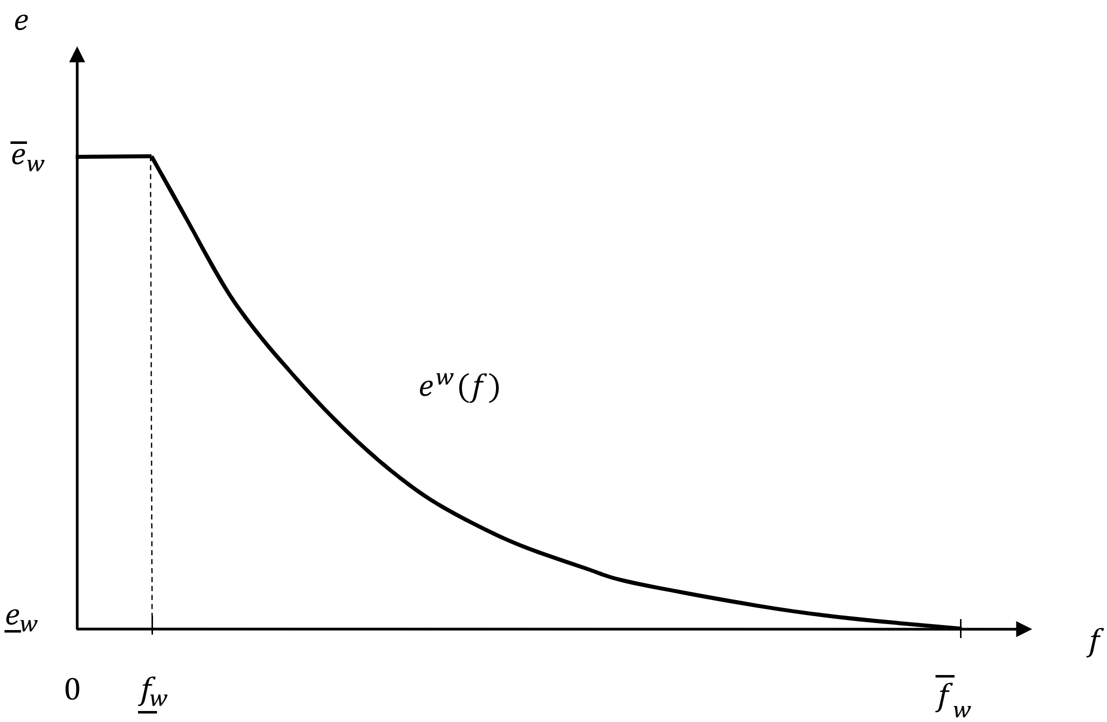

Figure 1 Socially optimal level of emissions.
$[\beta(e) = (e_0 - e)^2/2, e_0 \geq \theta_w]$

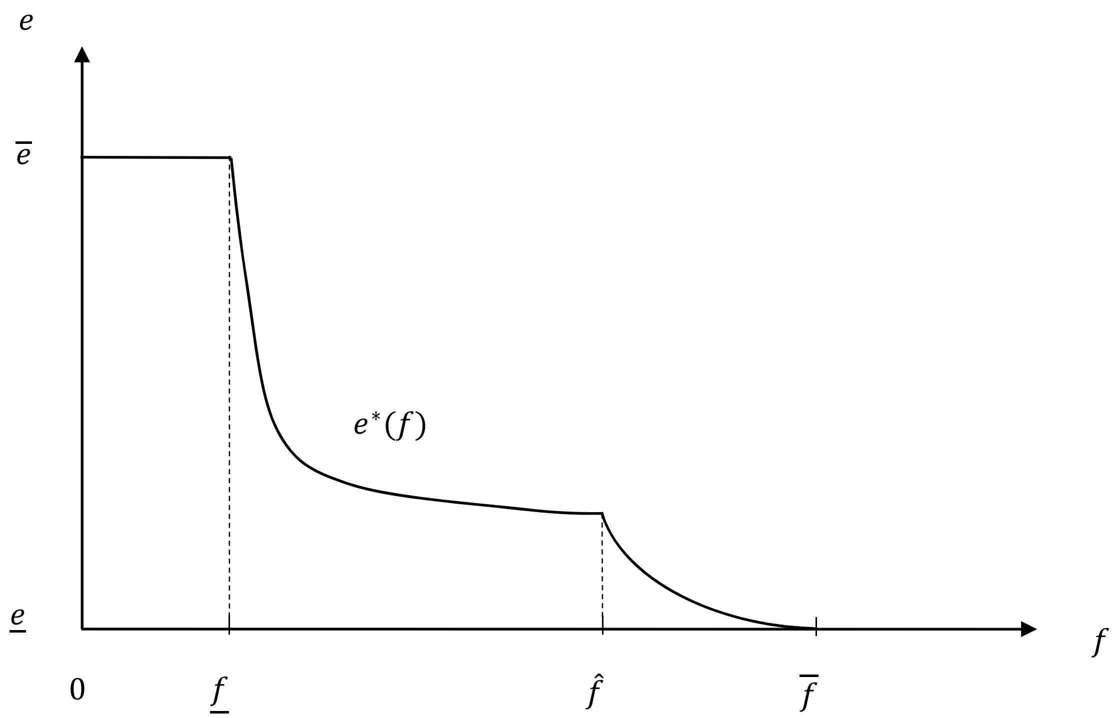

Figure 2 Free-entry equilibrium level of emissions.
$[\beta(e) = (e_0 - e)^2/2, e_0 \geq \theta]$

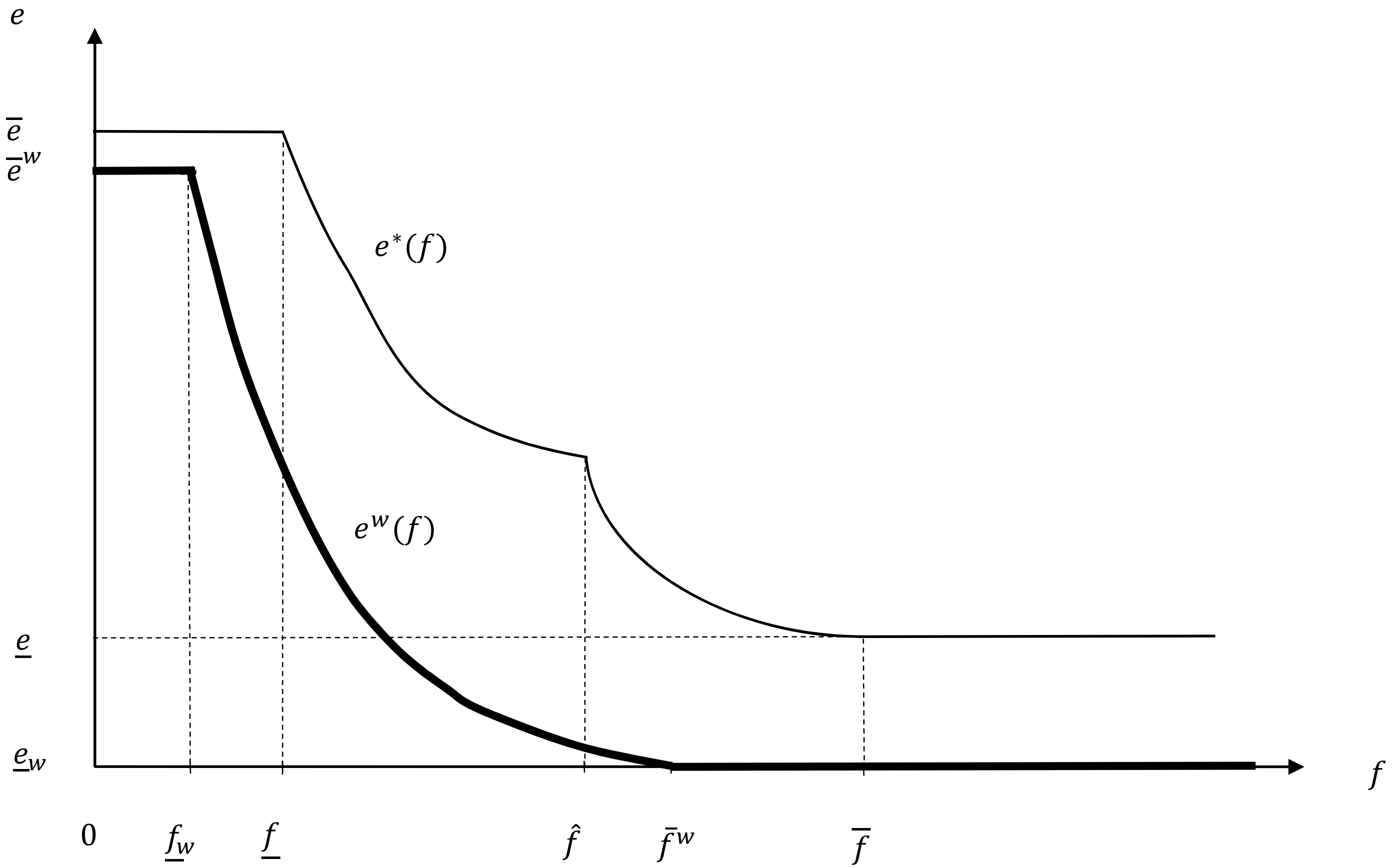

Figure 3 Free-entry equilibrium vs. socially optimal emissions: large externality.
$[\beta(e) = (e_0 - e)^2/2, e_0 \geq \theta_w]$

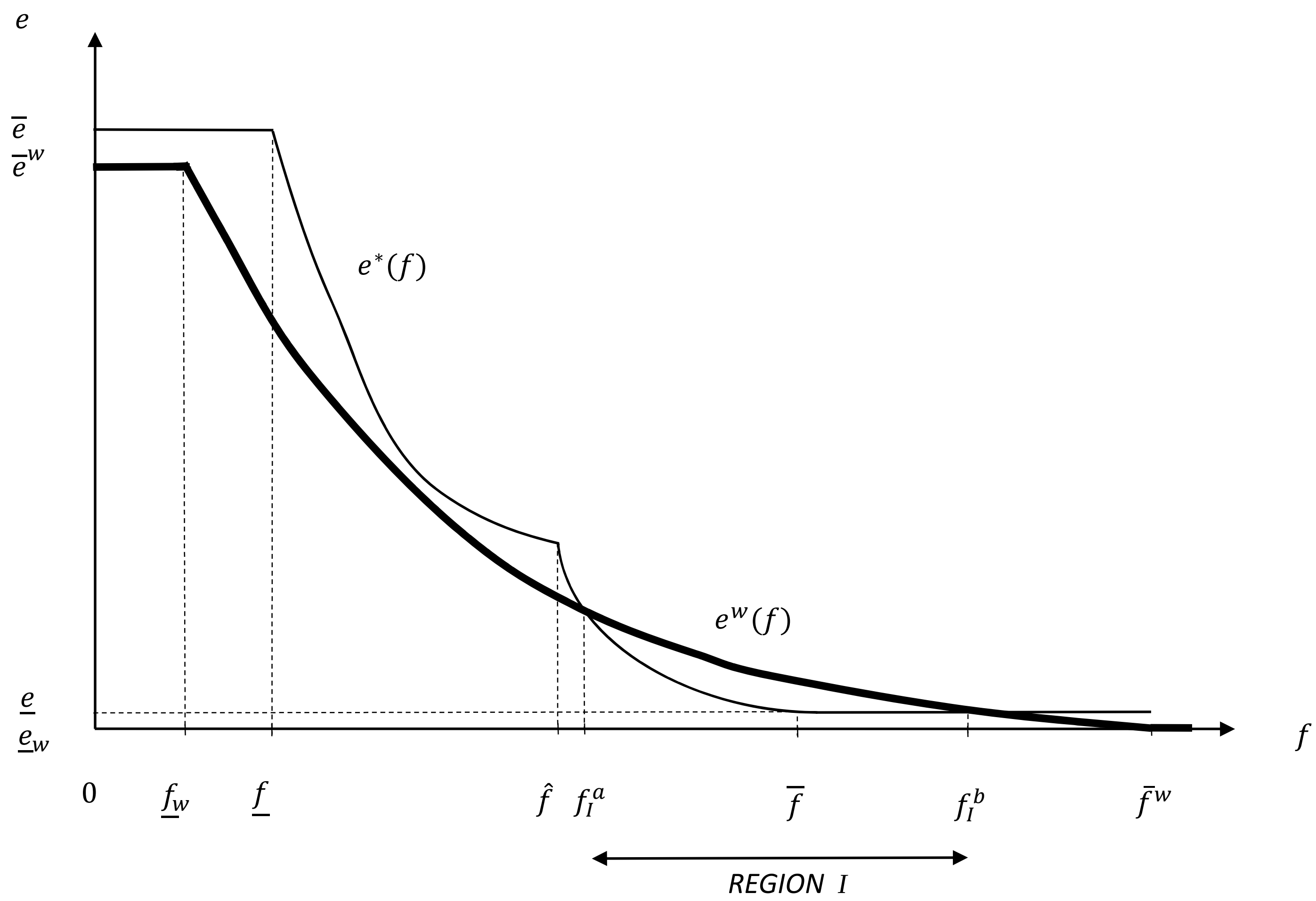

Figure 4 Free-entry equilibrium vs. socially optimal emissions: intermediate externality. $[\beta(e) = (e_0 - e)^2/2, e_0 \geq \theta_w]$

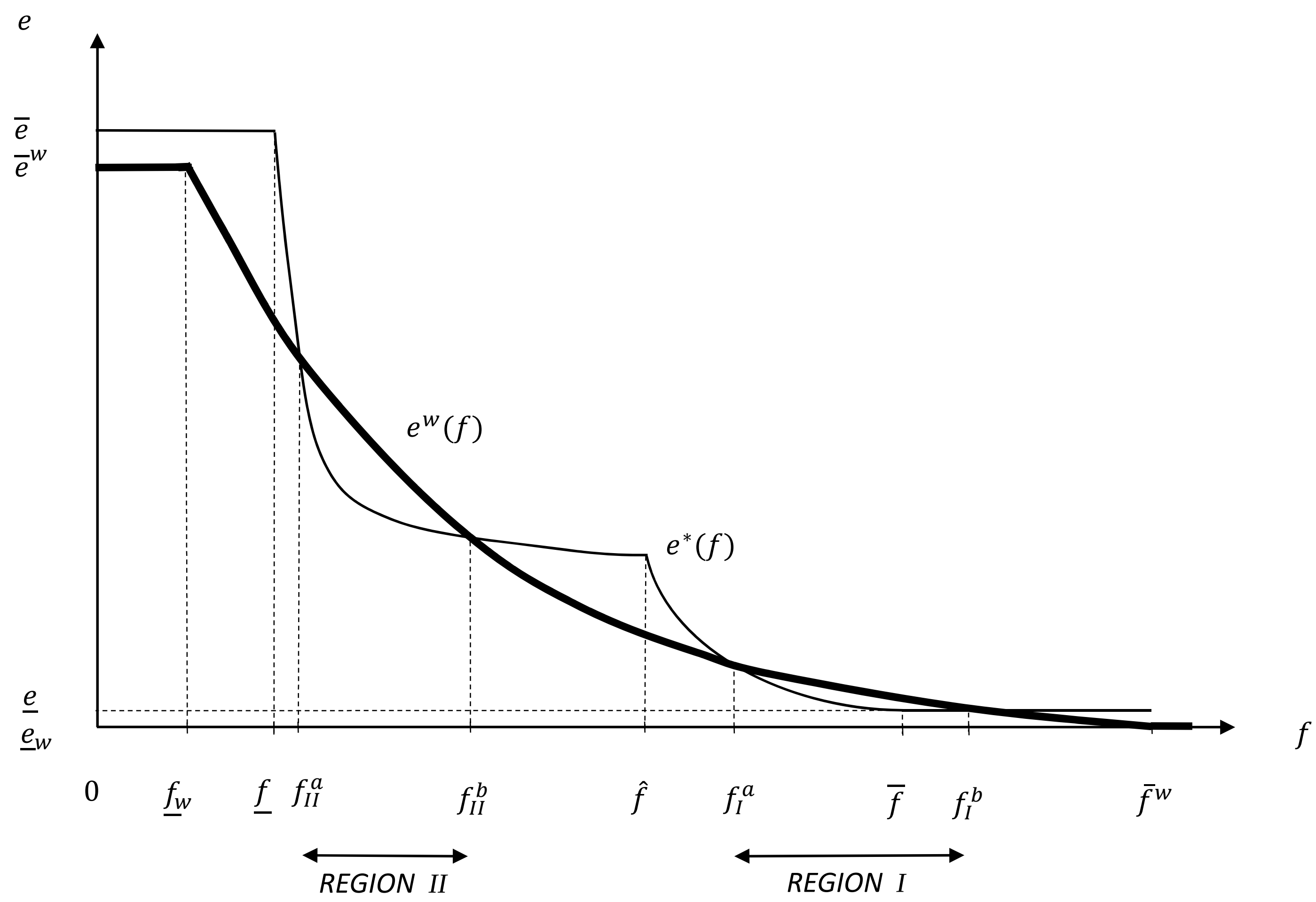

Figure 5 Free-entry equilibrium vs. socially optimal emissions: small externality.
$[\beta(e) = (e_0 - e)^2/2, e_0 \geq \theta_w]$