Cheap Talk under Ambiguity:

The IPCC in Climate-Change Agreements

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Abstract

This paper shows that the Intergovernmental Panel on Climate Change (IPCC) can take advantage of her position over the report of climate knowledge to mitigate the inefficiency of the level of green house gases emissions in international environmental agreements (IEA). I model an IEA as a game of contribution to a public bad. In reporting confidence levels over the theories that could predict the impact of emissions, the IPCC plays a cheap talk game with IEA participants. The presence of ambiguity over these theories allows to account for a unique sequential equilibrium in the game under standard assumptions. This result sheds new light upon the role of scientific advising in uncertainty, supporting the view that it can contribute to restore economic efficiency.

Keywords : Ambiguity, contribution to a public bad, cheap talk, climate change, international environmental agreements

1 Introduction

This paper explores the strategic influence of the Intergovernmental Panel on Climate Change (IPCC) on international environmental agreements (IEA). The IPCC is an intergovernmental body of scientists, under the supervision of the United Nations, dedicated to establishing the state of scientific knowledge on climate change. Among her tasks, she has to convey all scientifically certified models of climate change, allowing to understand the impact of green house gas (GHG) emissions over the global temperature increase. IEA are where emission policies regarding climate change are negotiated between countries, and can formally be seen as a game of contribution to a public bad (GHG) under uncertainty. More precisely, countries face a situation of *model uncertainty*, a situation in which different data generating mechanisms are plausible regarding global warming. This corresponds to a case of *Knightian* uncertainty, or ambiguity, where events do not have a precise, objective probability assignment. Consequently, the IPCC can have an important influence on the resulting IEA. Her task is one of strategic communication with the countries, through which she can act as a social planner.

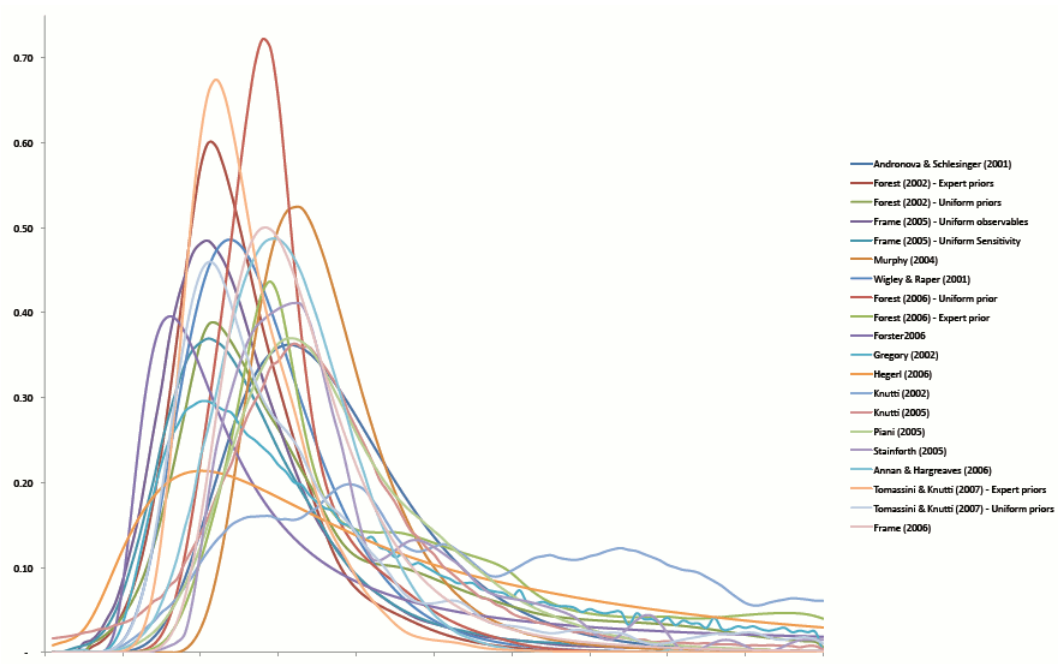

Figure 1: Main probability densities over temperature increase.
Source: Millner et al. 2013

One of the most peculiar aspects of scientific knowledge over climate change is its degree of uncertainty. As overall consensus exists, no unique description of the physical phenomena we are facing has emerged, leading instead to various possible scenarios for the future. Scenarios have multiple possible probabilities, corresponding to the different models which could explain climate change, as illustrated by figure 1. In addition of reporting scenarios and models to the public and the global decision makers, the IPCC also provides a confidence ranking of the theories it conveys. This is done through the use of a specific nomenclature (see Mastrandrea et al. (2010)). Confidence assessments, a self-determined qualitative notion, are used to rank statements of probability. The question I examine here is how such a tool can serve the IPCC in a game of strategic communication with the negotiating countries. While the informational clarity of this nomenclature has been discussed in the literature (see Risbey and Kandlikar (2007), Budescu et al. (2009)), this question has not yet been addressed.

The choice of a confidence ranking can be modelled as a game of communication played by the IPCC, as a sender, and the countries, as receivers. As argued by Tol (2011), the IPCC has a position of monopoly on the report of climate knowledge, and can be considered as the only sender in that context. While climate models assessing the probability of various scenarios are known by all, the IPCC's confidence ranking, as second order beliefs over prospective theories, is a non certifiable information. Yet, it can serve her interest if properly chosen. In particular, the IPCC can behave as a social planner, who aims at maximising the social welfare. Although formally equivalent to a game of contribution to a public good, IEA are a problem of contribution to a global public bad. Their driving aspect is free-riding: all countries have an individual interest in emitting green house gazes (GHG) as it ultimately represents economic growth. At the same time, they are all better off in having the total level of emission as low as possible as it directly impacts global warming and ultimately their welfare. Overall, the total level of emission will be higher than the one chosen by a social planner. For the IPCC, maximising the social welfare means compensating for the inefficiency of the IEA. In doing so, she will systematically be biased with respect to the countries. This asymmetry of interests and the nature of the information conveyed makes the game played one of cheap talk *à la* Crawford and Sobel (1982).

In the literature on social planning regarding emission policies, the specific impact of the scientific uncertainty surrounding climate change on the chosen emission policies has been pointed out as essential (a review of it and of its economic modelling can be found in Pindyck (2007)). In particular, it has been argued that model uncertainty and the resulting ambiguity calls for the use of non-expected utility frameworks. In particular, models of ambiguity aversion have been proposed. An individual is exposed to ambiguity when the expected payoff to his strategy varies with probabilities over which he is uncertain. An ambiguity averse individual will tend to favour strategies that reduce that exposure. Application to climate social planning can be found in Lange and Treich (2008) and Millner et al. (2013) who both use the smooth ambiguity model of Klibanoff et al. (2005). In this paper I will assume that countries are *max-min* expected utility maximisers (MEU) *à la* Gilboa and Schmeidler (1989), the most widespread decision criteria capturing the idea of ambiguity aversion. In the following, I will argue this assumption to be both normatively and positively grounded. To the best of my knowledge, this definition of the countries preferences is new in the IEA literature. Recent research focuses on the impact of uncertainty in IEA, but assumes expected utility preferences under risk. Na and Shin (1998) were the first to take this route. Finus and Pintassilgo (2013) added risk behaviour to the picture. The MEU assumption on the countries preference makes the *cheap talk* game over climate models relatively new. Ambiguity in general *cheap talk* has already been introduced by Kellner and Le Quement (2018) regarding Ellsbergian communication strategies. In my case, communication is standard, and the ambiguity is over the type of the sender.

In the following, I consider a two period game with two countries with heterogeneous preferences and a third party, the IPCC. A cheap talk game is played in the first period and a contribution to a public bad game consequently in the second. I call a Bayesian Nash equilibrium on the countries sub-game an agreement and the resulting total emissions level the outcome of the game. I show that there is a unique sequential equilibrium of the game. Whatever her type, at equilibrium, the IPCC will be credible in pointing out a subset of climate theories as the most likely. As a result, the agreement between countries will be reached on that basis. My results are to be read in normative terms. They show that it is possible for the IPCC to better the social welfare through the choice of her communication.

2 A game of contribution to a public bad under ambiguity

Setup. I consider a emission game where two countries have to decide what level of GHG they want to emit. Let $\mathcal{P} = \{1, 2\}$ be the set of countries. $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2$ is the set of strategies and $\forall i \in \mathcal{P}, \mathcal{S}_i = [0, 1]$, is a level of emission for player i. $s = 1$ corresponds to the maximum conceivable level of emissions and $s = 0$ to the minimum conceivable emission level. Both countries thus have the same bounds on their emission choice, which corresponds to countries of equivalent economic importance. One could think of China and the USA, who's decision often drives IEA. Note that this assumption is not necessary for the results of this paper (the set of strategy only needs to be bounded). Yet, by focusing on major players, it has the merit of simplicity.

I further define $L = \{l \in [0, 2] | s_1 + s_2 = l\}$, the set of total level of emission of all countries. Each player's payoff is positively function of their emissions (which generates wealth) and negatively function of the total amount of emissions due to global warming. The exact impact of total emissions over each player's welfare is unknown to all countries. Let $d_i : L \times \Omega \rightarrow \mathbb{R}^+$ be the function that maps global emissions into the well-being of player i, where Ω is the finite set of possible states of nature. It represents the commonly accepted set of possible scenarios that the scientific community has established for global warming. As a consequence Ω is assumed to be finite and equally distinguished by all countries.

Note that damages are heterogeneous among countries: for a given scenario of say rising water, the impact on China's economy, which is mostly concentrated on its coastline activities, is presumably not the same than on the USA where wealth rests more on the tertiary sector. As it is usually done in the IEA literature (see Eichberger and Kelsey (2002) for an example), I will assume d_i to be increasing and strictly convex in l. It corresponds to the assumption that whatever the scenario you consider, the higher the emission, the higher the damages. I also impose $\forall \omega \in \Omega$, $d_i(0, \omega) = 0$ such that whatever the state of the world, the minimal level of emission of all countries lead to no damage. A direct consequence is that d_i is always positive.

The utility functions of player i is defined as follows :

$$u_i(s_i, s_{-i}) = s_i - d_i(l, \omega)$$

For convenience, I will assume the utility of countries to be always positive, such that: $\forall l \in L$ and $\forall \omega \in \Omega$, $l \geq d_i(l, \omega)$.

It thus appears clearly that the beliefs countries hold over the possible states of nature is central in their decision. Yet, knowledge over Ω is scarce. Various theories exist among climate scientist regarding the likelihood of each scenario and it is not possible to discard one of them by lack of sufficient scientific understanding. Yet, they can be formalised under the form of a probabilistic estimation over the possible states of nature. Climate theories correspond to a finite set of probability distribution over states of the world, $\{p_1, ..., p_m\}$, such that no theory predicts the same expected damage. Formally: $\forall i \in \mathcal{P}$ and $\forall p, q \in \{p_1, ..., p_m\}$, $p \neq q \Rightarrow \sum_{\omega \in \Omega} p(\omega) d_i(l, \omega) \neq \sum_{\omega \in \Omega} q(\omega) d_i(l, \omega)$. I call the convex hull of that set $\mathcal{C} = co(\{p_1, ..., p_m\})$ the available common knowledge on climate change.

Countries' decision making. In order to model the way countries act towards ambiguity I choose to assume that the countries are max-min decision maker (MEU) *à la* Gilboa and Schmeidler (1989). This can be justified both on positive and normative terms. Since Ellsberg (1961), experimental results have supported the view that individuals exhibit ambiguity aversion. MEU preferences are the most used formal representation of that behaviour. Equally, on the normative side, many authors, including Gilboa and Marinacci (2016) and Gilboa et al. (2009), have argued that ambiguity aversion is not necessarily less rational than Savage's subjective expected utility axioms.

I call $\mathcal{B}$ the Borel σ-algebra on $\mathcal{C}$ and $B \in \mathcal{B}$ the beliefs of player i at equilibrium. To evaluate a strategy, player i chooses the probability distribution in B that minimises its expected utility. Thus, for $s_i \in \mathcal{S}_i$:

$$V_i(s_i, s_{-i}) = min_{p \in B} s_i - \sum_{\omega \in \Omega} p(\omega) d_i(l, \omega)$$

I call $p_i \in \Delta(\Omega)$ the corresponding probability distribution for player i. Note that as d_i and d_j might differ, countries need not to select the same distribution in B. I then have:

$$V_i(s_i, s_{-i}) = s_i - \hat{d}_i(l)$$

where $\hat{d}_i(l) = min_{p \in \mathcal{B}_i} \sum_{\omega \in \Omega} p(\omega) d(l, \omega)$ is the expected value of d for player i.

In the next section, I study as a benchmark situation the sub-game played by the countries, for a common set of beliefs $B \in \mathcal{B}$.

3 Countries agreement under ambiguity

3.1 Agreements with homogeneous damages

I first assume that countries have the same damage function $d_1 = d_2$. For a given set of beliefs $B \in \mathcal{B}$, the MEU decision rule leads countries to select the same probability distribution to evaluate a given act, such that $\hat{d}_1 = \hat{d}_2$.

I define $l_1^*(B)$ such that $l_1^*(B) = argmax_{l \in L} l - \hat{d}_1(l)$ and for any $s_i \in \mathcal{S}_i$, I call $BR_j(s_i) \in \mathcal{S}_j$ the best response of player j to s_i. I start by deriving the best response function for a given player.

Lemma 1. *$\forall s_i \in \mathcal{S}_i$, $s_i > l_1^*$ is a strictly dominated strategy and the best response of player i to $s_j \in \mathcal{S}_j$ is*

$$BR_i(s_j) = \begin{cases} 0 & \text{if } l_1^*(B) - s_j < 0 \\ l_1^* - s_j & \text{if } 0 \leq l_1^*(B) - s_j < 1 \\ 1 & \text{if } l_1^*(B) - s_j \geq 1 \end{cases}$$

Note that the best response function is entirely driven by the optimal level of total emission he believes in. This result comes from the convexity of the damage function.

Proposition 2. *When $p_1 = p_2 = p$, the set of all possible agreements is :*

$$\{(s_1, s_2) \in [0,1]^2 | s_1 + s_2 = l_1^*(B)\}$$

The outcome of the game is thus always l_1^*. Thus, $G(\mathcal{C}) = l_1^*(B)$.

Under homogeneous damage functions, an infinity of agreements can be reached and a unique symmetric agreement among them as predicted by Olson (1965). The outcome of the game is always the same. It thus falls under the more general result stated in Warr (1983) which shows that when a single public good is provided at positive levels by private individuals, its provision is unaffected by a redistribution of income. Yet, this is also a quite inflexible result: the presence of multiple countries, if they have the same damage function, does not influence the outcome of the game. They will collectively emit as much as if only one player (or a centralised decision maker) was choosing an emission level for the both of them.

3.2 Agreements with heterogeneous damages

I now turn to the more general case where countries have heterogeneous damage functions: for $(i, j) \in \mathcal{P}^2$, $d_i \neq d_j$. For $B \in \mathcal{B}$ the set of beliefs of the players at equilibrium, I further define $l_i^*(B) = argmax_{l \in L} l - \hat{d}_i(l)$ and $l_j^*(B) = argmax_{l \in L} l - \hat{d}_j(l)$ the optimal levels of emissions of both countries. In generality, $l_i^*(B)$ and $l_j^*(B)$ might not necessarily differ.

Remark 3. *When damage functions and probability distribution among countries vary the optimal level of total emission might be the same.*

Intuitively, MEU countries might end up with $l_i^*(B) = l_j^*(B)$ when the differences in the damage function are offset by the choice of the probability distribution in $\mathcal{B}$. This corresponds to the case of *spurious unanimity* studied in Mongin (1995). It then follows from lemma 1 that the outcome of the game is the same under *spurious unanimity* than under homogeneous damage functions.

In the following I will focus on the case $l_i^*(B) < l_j^*(B)$:

Proposition 4. *When damages are heterogeneous a unique agreement is possible. The corresponding outcome is the optimal level of emission for one or the other player if both are on the same side of 1. If not, the outcome is 1.*

The outcome of the game is $l_i^*(B)$ *when* $l_i^*(B) \geq 1$. *The corresponding agreement is* $(l_i^*(B) - 1, 1)$.

The outcome of the game is $l_j^*(B)$ *when* $l_j^*(B) \leq 1$. *The corresponding agreement is* $(0, l_j^*(B))$.

The outcome of the game is 1 *when* $l_j^*(B) \geq 1$ *and* $l_i^*(B) < 1$. *The corresponding agreement is* $(0, 1)$.

Thus, overall, if countries have different belief, there are three possible cases. In each of them, a unique agreement can be reached, different in every case and leading to a different outcome. The outcome of the game might be the optimal level of global emission for one or the other player, or can be equal to one. The game boils down to a high outcome when both countries belief is that optimal emissions are high and to a low outcome when both beliefs over optimal emissions are low. When the beliefs of the countries are contradictory, in the sense that the optimal level they believe in is on both sides of one, the game can boil down to one as an outcome.

Note that the important reduction of Nash equilibrium - from a continuum to a unique one - is made possible by the presence of uncertainty and the assumption that both countries have a bounded strategy set. Would the strategy set be unbounded, there would still be a continuum of equilibrium.

4 The IPCC's strategic information transmission

Let us now introduce the IPCC as a strategic party in charge of summarising and reporting the state of knowledge over Ω. It has no direct leverage on emissions, but can choose to convey a message concerning the available knowledge on climate change, by pointing out which theories are the most likely. In practice, this is done by the use of the IPCC's confidence nomenclature (Mastrandrea et al., 2010). The current guidance offers five qualifiers for expressing confidence: very low, low, medium, high, and very high. For each theory, the choice of the appropriate qualifier is left to the authors of the IPCC report. Although the justification for a level of confidence in a theory has scientific foundation, I will assume that in the eyes of the countries, second order beliefs over prospective scenarios are non certifiable claims.

Let M be the set of messages the IPCC can send to the countries and $G : M \to L$ that maps a given message to the outcome of the countries' sub-game. I deduce from the study of the countries' sub-game that G is a function, as, for any set of beliefs $B \in \mathcal{B}$, there is a unique outcome of the game. I write $\mathcal{C} = \{p_\theta\}_{\theta\in[0,1]}$ the reordering of $\mathcal{C}$ such that G is an increasing function of θ. Thus, $M = [0, 1]$. Intuitively, I have arranged the climate theories from the one which derives the lowest final emission level to the one who derives the highest. I deduce from the previous section that :

$$G(\theta) = \begin{cases} l_j(\{p_\theta\}) \text{ when } l_j(\{p_\theta\}) \leq 1 \\ 1 \text{ when } l_i(\{p_\theta\}) \leq 1 \leq l_j(\{p_\theta\}) \\ l_i(\{p_\theta\}) \text{ when } 1 \leq l_i(\{p_\theta\}) \end{cases}$$

By construction G is an increasing function of θ. Countries being MEU, I can extend the domain of G to $\widetilde{M}$, the algebra of all subsets on $[0,1]$, such that I get that for $(\underline{\theta}, \overline{\theta}) \in [0,1]^2$:

$$G([\underline{\theta}, \overline{\theta}[) = \begin{cases} l_j(\{p_{\underline{\theta}}\}) \text{ when } l_j(\{p_{\underline{\theta}}\}) \leq 1 \\ 1 \text{ when } l_i(\{p_{\underline{\theta}}\}) \leq 1 \leq l_j(\{p_{\underline{\theta}}\}) \\ l_i(\{p_{\underline{\theta}}\}) \text{ when } 1 \leq l_i(\{p_{\underline{\theta}}\}) \end{cases}$$

By a slight abuse of notation I will write $G(\theta)$ the outcome of the countries' sub-game when the IPCC's message is $\{\theta\}$. I will further assume, without lost of generality (as level of emissions are normalised), that $\mathcal{C}$ is such that there is a theory that leads to a non emission outcome of the game and one that leads to a maximal emission outcome. Formally $G(0) = 0$ and $G(1) = 2$.

IPCC's decision making I will assume that the IPCC cares only of what it thinks to be the social optima. Yet, to the contrary of a problem of decision making under certainty or risk, no *true* probability distribution over Ω is available. As a consequence, in order to determine what it thinks to be the social optima, the IPCC has to reach some internal consensus on the probability distribution on which to rely. The explicit modelling of the formation of a consensus being beyond the scope of this paper, I will assume that the IPCC behaves as if it had learned the true probability distribution $\theta \in \mathcal{C}$. This knowledge is her private information. As a consequence, in the eyes of the countries, $\mathcal{C}$ is also the set of types T of the IPCC.

Once she has learned her type, the IPCC has to choose which elements of $\mathcal{C}$ to point out as being the most likely. A strategy $\sigma : T \to \widetilde{M}$ for the IPCC consists in transmitting a message $m \in \widetilde{M}$ to the countries regarding $\mathcal{C}$.

As a consequence, the utility of the IPCC is given by :

$$U_0(m,\omega) = G(m) - d(G(m),\omega)$$

where $d = d_i + d_j$. Having learned her type θ, she evaluates her utility by:

$$V_\theta(m) = G(m) - \sum_{\omega\in\Omega} f_\theta(\omega) d(G(m),\omega) d\omega \tag{1}$$

I further define $l^*(\theta) = argmax_{l\in L} l - \int_{\omega\in\Omega} f_\theta(\omega) d(l,\omega) d\omega$ the optimal level of emission for the IPCC when it is of type θ. A consequence of equation (1) is that the IPCC is systematically biased with respect to the countries. It has always an interest of sending a message that would lead them to emit less, in order to compensate for the inefficiency of the IEA. As a consequence, she behaves as a sender in Crawford and Sobel's framework.

In the following I show that there is a unique sequential equilibrium (σ, s_i^*, s_j^*) of the game played by both the countries and the IPCC. In other words, whatever her type, the IPCC will always send an imprecise yet credible message to the countries that will influence the outcome of the IEA.

Proposition 5. *There is $(f_{\theta_1}, ..., f_{\theta_m}) \in T^m$, $C_k = [f_{\theta_k}, f_{\theta_{k+1}}[$, such that there is a unique sequential equilibrium in the game where any type $f_\theta \in C_k$ of IPCC sends a costless message $m_k = [\theta_k, \theta_{k+1}[$ to the countries, i.e $\sigma(f_\theta) = [\theta_k, \theta_{k+1}[$ leading to an outcome $G(\theta_k)$.*

Surprisingly the *cheap talk* game played by the IPCC and MEU countries on the set of climate theories happens to possess a unique sequential equilibrium. This is a strong result, as it constitutes a refinement compared to the canonical case of Crawford and Sobel (1982), which has several possible equilibrium. It is due to the MEU preferences of the countries. In addition, this assumption makes it possible to exhibit a constructive proof of the equilibrium.

What we learn from proposition 5 is that, at equilibrium, an institution in the position of the IPCC can act in order to reduce the inefficiency in which IEA are stuck. If countries are MEU, she can take advantage of the ambiguity over climate theories through cheap talk. While this claim is normative, one could wonder if the existence of a non-certifiable information device such as the IPCC's confidence nomenclature (Mastrandrea et al., 2010) can help us to extend it to positive terms. This question is still under investigation and will be left for future research.

Note that my result holds independently of the IPCC knowing what the true theory is. I have only assumed that she chooses one as being the true through deliberation or consensus. Because of the strategic interaction between the players, if the countries believe the IPCC to be rational, it is rational from them to respond to her message as if she knew the true distribution. Ultimately, it all relies on the rationality of the IPCC's choice over the available knowledge. One could argue that being herself uncertain over the true theory, she could behave as an MEU decision maker. This corresponds to a special case of our model where the IPCC's type is f_0. The same can be said for any decision rule under ambiguity, as long as it selects an element of $\mathcal{C}$ as the true distribution over the states of world.

5 Conclusion

This paper aims at understanding the influence the IPCC can have on the outcome of IEA through the use of its confidence nomenclature. I argue that the IPCC can act as a social planner that mitigates the IEA's inefficiency by playing a cheap talk game with the agreement participants. It could be argued that the IPCC's mission is not regulatory but only to convey scientific information to the decision makers. The argument made here is that she could act as such, in a way similar to a central bank that, by its announcements, oversees financial stability. In a similar spirit to this paper, Stein (1989) shows that through its imprecise communication, the Fed plays a cheap talk game that helps it to achieve its regulatory goals.

A central assumption for the results of my paper is that countries behave as having MEU preferences in the face of ambiguity. This assumption is responsible for the refinement of the cheap talk game equilibrium compared to the standard case of Crawford and Sobel (1982). I have argued that, for many authors, it is an assumption justified both normatively and positively in the context of climate change. Yet, implicitly, I have assumed that while holding a set of probability distributions as plausible, the countries would base their actions on the entire set. Nothing in the MEU axioms forces the countries to hold the entire set $\mathcal{C}$ as their belief - only a convex subset is imposed. An extension of the MEU model provides support to my claim. Hill (2013) suggests that a MEU decision maker will select the beliefs upon which he grounds his decisions in function of their stakes. The higher the stakes of the choices he faces, the wider the subset chosen among the plausible probabilities. Would one accept this idea, it would seem reasonable to think that while facing the potential extinction of the human species, a decision maker would rely on the entire set of information he has been provided with.

6 Appendix

Proof of lemma 1:

$\hat{d}$ is a linear combination of $(d(,\omega))_{\omega\in\Omega}$, a set of increasing and strictly convex functions. As a consequence $\hat{d}$ is increasing and strictly convex as well.

It follows from the strict convexity of $\hat{d}$ that l^* is unique and that $l \to l-\hat{d}(l)$ is increasing on $[0, l^*]$ and decreasing on $[l^*, 2]$

Furthermore, for $s_j \in \mathcal{S}_j$:

$$max_{s_i\in\mathcal{S}_i}\mathbb{E}_p(u_i(s_i,s_j)) = max_{l\in[s_j,1+s_j]}\mathbb{E}_p(u_j(l-s_j,s_j)) = max_{l\in[s_j,1+s_j]}l - s_j - \hat{d}(l)$$

Thus overall:

$$argmax_{s_i\in\mathcal{S}_i}\mathbb{E}_p(u_i(s_i,s_j)) = \begin{cases} 0 & \text{if } l^* - s_j < 0 \\ l^* - s_j & \text{if } 0 \leq l^* - s_j < 1 \\ 1 & \text{if } l^* - s_j \geq 1 \end{cases}$$

and for $s_i > l^*$, $\mathbb{E}_p(u_i(s_i,s_j)) < \mathbb{E}_p(u_i(s_i^*,s_j))$, thus $s_i > l^*$ is a strictly dominated strategy.

□

Proof of proposition 2:

I set $i, j \in \mathcal{P}$, $i \neq j$ and assume player i plays $s_i \leq l^*$: I deduce from Lemma 1 that the best response function of player j to s_i is:

$$BR_j(s_i) = \begin{cases} l^* - s_i & \text{if } l^* < 1 + s_i \\ 1 & \text{if } l^* \geq 1 + s_i \end{cases} \tag{2}$$

Furthermore, by symmetry of the game I have that the best response function of player i to $\forall s_j \in \mathcal{S}_j$ is:

$$BR_i(s_j) = \begin{cases} l^* - s_j & \text{if } l^* < 1 + s_j \\ 1 & \text{if } l^* \geq 1 + s_j \end{cases} \tag{3}$$

Case 1: $l^* \geq 1$ and $l^* - 1 \geq s_i$.

Then, I deduce from (2) that $BR_j(s_i) = 1$ and from (3) that $BR_i(1) = l^* - 1$ and that $\{(s_i, 1)|0 \leq s_i \leq l^* - 1\}$ isn't a set of Bayesian Nash equilibrium.

Yet, by using (2) again I have that $BR_j(l^* - 1) = 1$. Thus $(l^* - 1, 1)$ is a Bayesian Nash equilibrium. By symmetry of the game $(1, l^* - 1)$ is one as well.

Case 2: $l^* \geq 1$ and $l^* - 1 < s_i < l^*$

Then, I deduce from (2) that $BR_j(s_i) = l^* - s_i$. Furthermore, I deduce from (3) that $BR_i(l^* - s_i) = s_i$. Thus $\{(s_i, l^* - s_i)|s_i > l^* - 1\}$ is a set of Bayesian Nash equilibrium and by symmetry of the game $\{(l^* - s_j, s_j)|s_j > l^* - 1\}$ is one as well.

Case 3: $l^* < 1$

Then, I deduce from (2) that $BR_j(s_i) = l^* - s_i$ and I deduce from (3) that $BR_i(l^* - s_i) = s_i$. Thus $\{(s_i, l^* - s_i)|s_i >$

$l^* - 1\}$ is a set of Bayesian Nash equilibrium and by symmetry of the game $\{(l^* - s_j, s_j)|s_j > l^* - 1\}$ is one as well.

Thus, the set of all possible Bayesian Nash equilibrium is $\{(s_1, s_2) \in [0,1]^2 | s_1 + s_2 = l^*\}$ and the only symmetric Bayesian Nash equilibrium is $s_1 = s_2 = \frac{l^*}{2}$

$\square$

Proof of proposition 4:

Assume player i plays $s_0 \in \mathcal{S}_i$.

I deduce from lemma 1 that $s_0 > l_i^*$ and $s_0 > l_j^*$ are strictly dominated strategies. Thus, for $s_i \leq l_i^*$, the best response function of player j to s_i is:

$$BR_j(s_i) = \begin{cases} l_j^* - s_i & \text{if } l_j^* - s_i < 1 \\ 1 & \text{if } l_j^* - s_i \geq 1 \end{cases} \tag{4}$$

as $l_j^* - s_i \geq 0$.

Equally, the best response function of player i to $s_j \in \mathcal{S}_j$ is:

$$BR_i(s_j) = \begin{cases} 0 & \text{if } l_i^* - s_j < 0 \\ l_i^* - s_j & \text{if } 0 \leq l_i^* - s_j < 1 \\ 1 & \text{if } l_i^* - s_j \geq 1 \end{cases} \tag{5}$$

Then, three possible cases arise:

Case 1: $l_j^* \geq 1$ and $l_j^* - 1 > s_0$

Then, I deduce from (4) that $BR_j(s_0) = 1$. Furthermore,

1. If $l_i^* < 1$, $BR_i(1) = 0$ and then $BR_j(0) = 1$. Thus, $(0, 1)$ is a Bayesian Nash equilibrium.

2. If $l_i^* \geq 1$, $BR_i(1) = l_i^* - 1$ and $BR_j(l_i^* - 1) = 1$ as $l_i^* < l_j^*$. Thus $(l_i^* - 1, 1)$ is a Bayesian Nash equilibrium.

Case 2: $l_j^* \geq 1$ and $l_j^* - 1 < s_0 \leq l_i^*$

I deduce again from (4) that $BR_j(s_0) = l_j^* - s_0$ and from (5) that:

$$BR_i(l_j^* - s_0) = \begin{cases} 0 & \text{if } l_i^* - l_j^* + s_0 < 0 \\ l_i^* - l_j^* + s_0 & \text{if } 0 \leq l_i^* - l_j^* + s_0 \leq 1 \\ 1 & \text{if } l_i^* - l_j^* + s_0 > 1 \end{cases}$$

and from (4) again that:

$$BR_j(l_i^* - l_j^* + s_0) = \begin{cases} 2l_j^* - l_i^* - s_0 & \text{if } 2l_j^* - l_i^* - s_0 \leq 1 \\ 1 & \text{if } 2l_j^* - l_i^* - s_0 > 1 \end{cases}$$

I define the following sequence:

$$U_n = (n+1)l_j^* - nl_i^* - s_0$$

I have that:

- $BR_j(s_0) = U_0$
- $\forall n \in 2\mathbb{N}$, $U_n \in [0, 1] \Rightarrow BR_i(U_n) = l_i^* - U_n$

- $\forall n \in 2\mathbb{N}+1$, $l_i^* - U_n \in [0,1] \Rightarrow BR_j(l_i^* - U_n) = U_{n+1}$

Thus, I take attention on the conditions under which $(U_n)_{n\in\mathbb{N}}$ and $(l_i^* - U_n)_{n\in\mathbb{N}}$ are in $[0,1]$. As long as they are the case no Bayesian Nash equilibrium can arise from the best response algorithm. Yet, once they leave $[0,1]$ - if they do- an equilibrium might form.

For $n \in \mathbb{N}$, the following cases arise:

$$BR_i(U_n) = \begin{cases} 0 & \text{if } l_i^* - U_n < 0 \\ l_i^* - U_n & \text{if } 0 \le l_i^* - U_n \le 1 \\ 1 & \text{if } l_i^* - U_n > 1 \end{cases}$$

Equally,

$$BR_j(l_i^* - U_n) = \begin{cases} U_{n+1} & \text{if } 0 \le U_{n+1} \le 1 \\ 1 & \text{if } U_{n+1} > 1 \end{cases}$$

Yet, I have that:

- When can $BR_i(U_n) = 0$?

$$\begin{aligned} l_i^* - U_n < 0 &\iff l_i^* - [(n+1)l_j^* - nl_i^* - s_0] < 0 \\ &\iff s_0 < (n+1)(l_j^* - l_i^*) \end{aligned}$$

Yet, $l_j^* - 1 < s_0 \le l_i^*$ and $l_j^* - 1 > (n+1)(l_j^* - l_i^*) \iff n(l_i^* - l_j^*) + l_i^* < 1$ which is always true. Thus there is no $s_0 \in [l_j^* - 1, l_i^*]$ such that $BR_i(U_n) = 0$

- When can $BR_i(U_n) = 1$?

$$\begin{aligned} l_i^* - U_n > 1 &\iff l_i^* - [(n+1)l_j^* - nl_i^* - s_0] > 1 \\ &\iff s_0 > (n+1)(l_j^* - l_i^*) + 1 \end{aligned}$$

As $s_0 \leq 1$, that $l_j^* - l_i^* > 0$ and that $l_j^* > 1$ this inequality is never true. Thus $BR_i(U_{2m}) = 1$ is never possible.

- As a consequence $BR_i(U_n)$ is always $l_i^* - U_n$

Equally, I have that:

- When can $BR_j(l_i^* - U_n) = 1$?

$$\begin{aligned} U_{n+1} > 1 &\iff (n+1)l_j^* - nl_i^* - s_0 > 1 \\ &\iff s_0 < n(l_j^* - l_i^*) + l_j^* - 1 \end{aligned}$$

Yet, $l_j^* - 1 < s_0 \leq l_i^*$, $l_j^* - 1 > 0$ and $n(l_j^* - l_i^*) \geq 0$ for all $n \in \mathbb{N}$. Thus, the above inequality is never true.

As a consequence $BR_j(l_i^* - U_n)$ is always U_{n+1}

Yet, $lim_{n \to +\infty} U_n = l_j^* - s_0$ and $lim_{n \to +\infty} l_i^* - U_n = l_i^* - l_j^* + s_0$. Thus, overall there is no Bayesian Nash equilibrium in this case.

Case 3: $l_j^* < 1$

I deduce again from (4) that $BR_j(s_0) = l_j^* - s_0$.

- As in case 2, $BR_i(U_n)$ is $l_i^* - U_n$ as long as $s_0 \geq (n+1)(l_j^* - l_i^*)$. As $0 < s_0 \leq l_i^*$, this is possible as long as $(n+1)(l_j^* - l_i^*) < l_i^* \iff n < \frac{2l_i^* - l_j^*}{l_j^* - l_i^*}$.

  Thus, for $n \geq \frac{2l_i^* - l_j^*}{l_j^* - l_i^*}$, $BR_j(l_i^* - U_n) = 0$.

- Yet, $BR_j(l_i^* - U_n) = U_{n+1}$ as long as $s_0 > n(l_j^* - l_i^*) + l_j^* - 1$. As $0 < s_0 \leq l_i^*$, this is possible as long as $n(l_j^* - l_i^*) + l_j^* - 1 < l_i^* \iff n < \frac{1-(l_j^*-l_i^*)}{l_j^*-l_i^*}$.

  Thus, for $n \geq \frac{1-(l_j^*-l_i^*)}{l_j^*-l_i^*}$, $BR_j(l_i^* - U_n) = 1$.

Yet, $\frac{2l_i^*-l_j^*}{l_j^*-l_i^*} < \frac{1-(l_j^*-l_i^*)}{l_j^*-l_i^*}$, thus the Bayesian Nash equilibrium of the game is $(0, l_j^*)$

Thus, overall, the following agreements are possible:

- $(0,1)$ when $(l_i^*, l_j^*) \in \mathcal{C}_1 = \{(l_i^*, l_j^*) \in L^2 | l_j^* \geq 1, l_i^* < 1\}$ and the corresponding outcome of the game is 1
- $(l_i^* - 1, 1)$ when $(l_i^*, l_j^*) \in \mathcal{C}_2 = \{(l_i^*, l_j^*) \in L^2 | l_j^* \geq 1, l_i^* \geq 1\}$ and the corresponding outcome of the game is l_i^*
- $(0, l_j^*)$ when $(l_i^*, l_j^*) \in \mathcal{C}_3 = \{(l_i^*, l_j^*) \in L^2 | l_j^* \leq 1\}$ and the corresponding outcome of the game is l_j^*

$\square$

Proof of proposition 5:

Assume any type $f_\theta \in C_k$ sends message $m_k = [\theta_k, \theta_{k+1}[$.

By construction, $G(m_{k-1}) \leq l(f_{\theta_k}) \leq G(m_k)$. For all C_k, type f_{θ_k} has the most incentive to deviate from sending m_k to sending m_{k-1}. Indeed, G is an increasing function, thus, $\forall \theta \in C_k$, $|l^*(f_{\theta_k}) - \int_{\omega\in\Omega} f_\theta(\omega) d(l^*(f_{\theta_k}), \omega) d\omega - V_{\theta_k}(m)| \geq |l^*(f_\theta) - \int_{\omega\in\Omega} f_\theta(\omega) d(l^*(f_\theta), \omega) d\omega - V_\theta(m)|$.

Thus, it is a necessary and sufficient condition for all types in C_k to send m_k is that $V_{\theta_k}(m_k) \geq V_{\theta_k}(m_{k-1})$.

Furthermore, it is also necessary that all types in C_{k-1} prefer message m_{k-1}. In particular it is the case for type f_{θ_k}, thus: $V_{\theta_k}(m_k) \leq V_{\theta_k}(m_{k-1})$. As a consequence, a necessary and sufficient condition for (σ, s_i^*, s_j^*) to be a sequential equilibrium is for all $k \in [\![2, n]\!]$:

$$V_{\theta_k}(m_k) = V_{\theta_k}(m_{k-1}) \tag{6}$$

$$\iff \quad G(m_k) - \int_{\omega\in\Omega} f_{\theta_k}(\omega) d(G(m_k), \omega) d\omega = G(m_{k-1}) - \int_{\omega\in\Omega} f_{\theta_k}(\omega) d(G(m_{k-1}), \omega) d\omega$$

For m_k such that $G(m_k) \leq 1$, G is strictly increasing. Thus $G(m_k) \neq G(m_{k-1})$ and as countries are MEU I have that :

$$
\begin{aligned}
& V_{\theta_k}(m_k) = V_{\theta_k}(m_{k-1}) && (7)\\
\iff \quad & \int_{\omega \in \Omega} f_{\theta_k}(\omega) \tfrac{d(G(m_k),\omega) - d(G(m_{k-1}),\omega)}{G(m_k) - G(m_{k-1})} d\omega = 1 \\
\iff \quad & \int_{\omega \in \Omega} f_{\theta_k}(\omega) \tfrac{d(G(\theta_k),\omega) - d(G(\theta_{k-1}),\omega)}{G(\theta_k) - G(\theta_{k-1})} d\omega = 1
\end{aligned}
$$

I have that $G(\theta_1) = l_i^*(f_{\theta_1}) = 0$ by assumption, and for $k \geq 2$, I deduce $G(\theta_k)$ by induction until θ_n such that $G(\theta_n) \leq 1$ and $G(\theta_{n+1}) > 1$.

Further I define θ_{n+1} and θ_{n+2} such that $l_j^*(f_{\theta_{n+1}}) = 1$ and $l_i^*(f_{\theta_{n+2}}) = 1$. When $\theta \in C_n$, (7) guarantees that m_n is an equilibrium and when $\theta \in C_n$, (6) does.

For m_k such that $G(m_k) \geq 1$, G is strictly increasing and $G(m_{k+1}) \neq G(m_k)$ again. Thus I define yet again $\{\theta_{n+3}, ..., \theta_{m-1}\}$ by induction through (7) as long as $G(\theta_k) \leq 2$. I finally set $\theta_m = 1$ such that $G(\theta_m) = 2$.

□

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