

The effects of migration and pollution externality on cognitive skills in Caribbean economies: a Theoretical analysis

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Abstract

Caribbean Small Island Developing States (SIDS) face specific social, economic and environmental vulnerabilities. This paper provides a simple overlapping generations model with migration and intergenerational transfers in an economy where production generates pollution. This pollution hampers the cognitive skills of the children and thus the efficiency of human capital accumulation in the economy. Therefore, key features of Caribbean SIDS are introduced in the model in order to exacerbate the potential link between the demographic dynamics – *i.e.* migration and human capital accumulation – and pollution. Results reveal that the usual gain from migration in terms of human capital was no longer possible because of the environmental externality. Indeed, in most of the cases, in presence of the environmental externality, per capita variables (utility, production and capital) are decreased by migration, while the aggregated production can be enhanced thanks to the demographic growth that occurs with migration. Moreover, it has been shown that the conditions to have a profitable environmental tax depend on the environmental features of the economy. Finally, the interactions between the emigration rate and the form of intergenerational transfers – *i.e.* solidarity from the domestic area and/or from the diaspora – have an impact on the scale of the reduction of human capital due to migration. Thus, in this model a gain from an increase in the rate of emigration is still possible but only if migration is already very high.

Keywords: Pollution, Demographics, Economic Development, Migration, Caribbean, Small Island Developing States

JEL classification: Q01, Q56, F24, J24

1. Introduction

There are several interrelated determinants of welfare and development, as social and economic status, education, health and environmental exposures. Development agencies as the World Bank, the United Nations Educational, Scientific, and Cultural Organization (UNESCO), the United Nations Childrens Fund (UNICEF) and others have argued that human capital accumulation through, education and demographic transition could trigger sustainable economic development in a finite world (World Bank (2007), UNESCO, UNDP, UNFPA, UNHCR, UNICEF, UN Women, WB (2016), Bocquet et al. (2016)). In the context of a small isolated territory, the question of a sustainable development seems even more stringent, because of the numerous physical constraints and economic backwardness.

Caribbean Small Islands Developing States (SIDS) are defined as a group of developing countries facing specific social, economic and environmental vulnerabilities (UN-OHRLLS (2015))¹. In response of these vulnerabilities different strategies have been adopted by these countries, however most of the Caribbean SIDS are struggling to achieve an economic development that generates growth and employment for all. One of the inhabitants' response to this situation is to leave the country in order to increase their wealth and their well-being abroad. Therefore, the demographic dynamics of these countries is explained largely by the scale of emigration. Several authors such as Connell and Conway (2000) or Thomas-Hope (1992) study the impact of these features on the island economies. However, there is not a consensus on the

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¹There are 23 countries or territories in this group: Anguilla, Antigua and Barbuda, Aruba, Bahamas, Barbados, Belize, British Virgin Islands, Cuba, Dominica, Dominican Republic, Grenada, Guyana, Haiti, Jamaica, Montserrat, Netherlands Antilles, Puerto Rico, Saint-Kitts and Nevis, Saint-Lucia, Saint Vincent and the Grenadines, Suriname, Trinidad and Tobago and United States Virgin Islands.

effect of migration for developing economies. On the one hand, positive effects have been identified: as a potential *brain gain* – *i.e.* an increase in human capital – or the existence of remittances – *i.e.* cash transfers between diaspora and the native country. On the other hand, migration could lead to a depletion of skilled agents or to a reduction of the investments in the domestic area.

In this context, this paper provides a theoretical model for Caribbean economies to draw qualitative conclusions regarding the impact of socio-demographic variables and environment on economic growth. More particularly, this work aims to evaluate the influence of migration and remittances if an environmental degradation leads to a reduction of the efficiency of human capital accumulation. Indeed, is a *brain gain* from migration still possible in this context ?

The idea is to introduce this externality because of the known effect of many local pollutants on the cognitive skills of the population. Moreover, the question of the economic development of these islands is addressed while the space constraint is included with a congestion effect due to the land scarcity. Therefore, the usual trade-off between quality and quantity of children is introduced while taking into account the migration. Finally, an environmental policy is tested and a special focus is given to the evolution of the productive capital stocks – *i.e.* human capital level, and physical capital – to evaluate the policy and the migration effects.

This work arises from the observation that few papers include demographic features through migration and natality choices while dealing with congestion and pollution effects on human capital accumulation. However, stylized facts on Caribbean SIDS point out to the necessity to increase the understanding of the interactions between migration, demographic features and environmental issues altogether ; because it explains in a large extent the accumulation of human capital and physical capital. This work aims to do so thanks to an overlapping generations (OLG) model. The households decisions in terms of savings and education are scrutinized, knowing that an environmental externality reduces the ability to accumulate human capital by the next generation. Therefore, in this work, the impacts of migration on economic development and consumption per capita are different than in the situation without the environmental constraint and it will depends on the one hand on the form of the intergenerational transfers – *i.e.* if they are funded by an increase in the intergenerational solidarity or in the gain from migration – and on the other hand on the emigration rate. An unique steady-state has been found and the conditions that control the optimality of the public policy identified. They depend strongly on the pollution intensity of the production and the stability of the steady-state values is linked mostly to the damage function in the human capital. To clarify some trends of this model, a numerical analysis for a representative economy is conducted in order to evaluate the optimal level of the environmental tax as well as the impacts of an increase in emigration or in the gain from migration.

Most of the theoretical models find a positive effect of emigration or remittances on human capital or production until a threshold, where the investments in physical capital are too small to sustain the economy (Beine et al. (2006), Docquier et al. (2008)). This results from a substitution effect between savings and human capital investments if there is migration. In the present article, the relation between production or human capital and emigration rate is described by an U-shaped curve. Indeed, if migration is low, an increase in the probability of leaving the country leads to an increase in the population size as well as the pollution stock. Therefore the production per capita and the utility are reduced by migration until a threshold after which the pollution stock decreases thanks to the reduction of the population size. Moreover, a rise in the remittances or the domestic intergenerational transfers always leads to a decline in the human capital because of the combined effects of the augmentation of the pollution stock and/or the decrease in physical capital. In every cases, the environmental tax improve the situation if the pollution intensity of production is small enough. If the emissions are too high, the pollution dynamics during the transition leads to a decrease in the human capital and in the capital stock in the first stages of development which reduces the steady-state values.

The rest of this paper is structured as follows. Section 2 is a literature review on migration and pollution effect on health. Section 3 gives some stylized facts for the Caribbean economies. Section 4 describes the OLG model and the equilibrium. Section 5 is a discussion on the effects of migration and environment on steady-state values. Finally, the last section draws conclusions and defines a roadmap for future research.

2. Literature Review

There is an extensive economic literature on one or several aspects of economic development. Specifically, on one hand economists have built models to describe the demographic features of an economy and their impacts on economic growth (Schoonbroodt and Tertilt (2014), Del Rey and Lopez-Garcia (2016), Cardia and Michel (2004), Docquier et al. (2007)). On the other hand, researchers have developed models that take into account the environmental issues and their economic effects. However, it is widely acknowledged that to understand the roots of sustainable development in a finite world, it is useful, if not necessary, to study the interplay between economic activities, the environment, and the demography. A growing literature, as in Jouvét et al. (2000), Pautrel (2009), Jouvét et al. (2010), Mariani et al. (2010), Aloï and Tournemaine (2011), Constant et al. (2014), tries to link these two elements. In this regard, OLG models are particularly useful and this work follows this literature. The aim is to develop a model with intergenerational choices and solidarity, as well as their impacts on production, the environment and the population dynamics.

Population dynamics and especially the demographic transition has been extensively studied in the economic literature. It is well accepted that economic development of industrialized economies has been accompanied by a reduction in fertility rates. Therefore, various growth models with endogenous fertility have been developed to describe the economic reasons behind this transition. First, some researchers plead that the technological progress has led to an increase in the demand for skilled workers instead of unskilled ones (Galor and Weil (2000)). In this theory, the reverse relationship between income and population growth has resulted in a transition from Malthusian regime to classical regime. A second explanation is the increasing opportunity cost of raising children when the wages rise. In that context, a diminution of the fertility rates increase the capital stock (Galor and Weil (1993)). Third, trade-offs between quantity and quality of children could have been modified for different reasons: the preferences of the parents (Mulligan (1997)), the forces of natural selection (Galor and Moav (2002)), the increased longevity (Ehrlich and Lui (1991)) or unobservable skills of children (Becker and Becker (2009)). Finally, the Caldwell hypothesis pleads that development induces a change in the direction of intergenerational transfers (Caldwell (1982)). In the first stages of development parents receive transfers from the children and then increase the size of their progeniture, while in developed economies parents give to the children and then reduce their number. In the present model the fertility choices are not analyzed exactly in the scope of one of these explanations, because in terms of monocausal effects they all seem to fit one of the aspects of population dynamics of the island economies. Here, a simple trade-off between quality and quantity of children is introduced thanks to an opportunity cost which depends on the wages, the migration gain with human capital and the land scarcity. This last aspect has been introduced as in De la Croix and Gosseries (2012). Thus, the demographic features represent approximately the Caribbean population dynamics. In these islands, high population density should lead to a decrease in fertility but with the migration possibility this decline is often compensated. Therefore the potential gain from migration through remittances and especially human capital remuneration abroad, could lead to an increase of the population size, even if there is an incentive to decrease the fertility because of the overcrowding effect or the increasing cost of children.

Intergenerational solidarity is introduced in the model but not altruism, on the contrary of works such as Cardia and Michel (2004), Thibault (2008), D'Albis and Decreuse (2009), Cremer and Roeder (2014), Soares (2015). An altruistic individual is defined as someone who cares about someone else's welfare or characteristics (*e.g.* consumption, human capital or bequests) even if it is costly for her. The literature on altruism studies its impact on the one hand, on economic growth, stability or optimality of the equilibrium and on the other hand, on variables such as human capital accumulation or bequests. In the present work, the solidarity is introduced through a compulsory intergenerational transfer and the optimality of the amount transferred is not studied. Therefore, the emphasis is put on two trade-offs, a first one between savings and human capital investments for parents as well as a second one between quality and quantity of children, knowing that a part of the children income is received through remittances.

This family solidarity occurs in a context of strong emigration. The effect of migration on the sending country has been well studied in the economic literature, however there is not a consensus for the developing economies. First, contributions such as Beine et al. (2006) or Docquier et al. (2008), defend the idea that brain drain and migration in general could enhance economic development ; thanks to an increase in the mean human capital in the domestic country. Indeed, migration is used as a way to increase revenues. Knowing that human capital enhances the probability of migration or the remuneration abroad, the possibility to migrate creates an incentive to invest in education and thus could lead to a *brain gain*.

That means that migration could trigger a rise in the mean human capital, because the proportion of new skilled individuals staying in the domestic country could be larger than the loss of human capital due to the departures. This is particularly true, if the barriers to migration (e.g. cost or procedures) are important. Nevertheless, the conditions to have a *brain gain* depends on the characteristics of the countries, and therefore a large part of the literature tries to evaluate them (Yellen (1975), Stark et al. (1997), Beine and Docquier (2011), Sampson (2013)). In the present model, the conditions which allow to have a *brain gain* are scrutinized according to the environmental externality on human capital accumulation.

Second, remittances – *i.e.* cash transfers between the diaspora and the domestic country – can promote economic growth especially if they are used to increase the investments in human capital through informal loans between family members (Poirine (1997)) or the capital stock through the savings in the domestic country (Osili (2007)). However, most of the time, remittances fund consumption and thus unproductive investments. Therefore, according to some economists, it is not possible to rely on them to trigger the economic development of a country (Frucht (1968), Hill (1977)). Theoretical works reveal that the positive or the negative impact of remittances depends on the characteristics of the studied area. Several authors such as Connell and Conway (2000) or Thomas-Hope (1992) study the impact of migration and remittances, in particular in the island economies and they conclude that it could be positive. Nevertheless, two empirical studies focus on the impact of remittances for the independent states of the Caribbean. Both of them conclude that they do not impact long-term growth but long term consumption (Mishra (2006), Lim and Simmons (2015)). In the present work, the objective is to understand what are the effects of emigration, if the human capital accumulation depends also on the pollution stock. Indeed, in this context, migration and remittances could deteriorate environmental quality – through an increase in the production – and hamper human capital accumulation as well as reduce economic growth.

Finally in this work, pollution damages are introduced through the effect of local pollution on cognitive skills. The mechanism considered here is that the ability to learn depends partly on the children's health, and thus on the quality of the natural environment in which they are reared. This comes from the direct impact of the environment on food, water and air in terms of quality and quantity which are sensitive criteria for the health of individuals and especially children. Two reasons can be emphasized here. The first one is the direct effect of pollution exposition on the ability to learn. Indeed many studies have showed that the cognitive skills, and more especially that the IQ of children were lessened in a polluted area (Tzivian et al. (2015), Marcotte (2016), Power et al. (2016), Pujol et al. (2016), Lett et al. (2017), Omanbayev et al. (2018)). The second reason why pollution could affect the human capital accumulation is the health impact on school attendance. Indeed, there is a strong consensus on the effect of air pollution on the occurrence of asthma or on the health of the children when young (Beasley et al. (2015), Arroyo et al. (2016), José et al. (2017), Liu et al. (2017)). Moreover, the effect of pesticides is clearly negative and many studies call for taking into account the effect of their presence on health, whether these particles are in water, soil or air (Lai (2017), Lammoglia et al. (2017), Valcke et al. (2017)). Finally, even if there is no consensus on the quantities of pollution or the level of exposure that lead to a damage on human capital accumulation, it is reasonable to say that local pollution has an effect on health and cognitive skills.

The present work is linked to the first effect, this means that the environmental externality impacts directly the ability to acquire human capital, rather than just the efficiency of education expenditures. In a future work it will be interesting to test the other hypothesis. Here, the dynamics between pollution and human capital are not introduced precisely. Indeed, the emphasis is put on the link between human capital and environmental degradations because literature shows that human capital gain is one of the main channels to plead that migration is an economic opportunity for developing states. Therefore, it seems crucial to evaluate the new conditions to insure that there is a *brain gain* thanks to migration in presence of an environmental externalities on human capital accumulation, which is highly plausible.

In the economies modeled here, the local pollution stock is generated by the economic production. Different sources of local pollution emissions could be in question here. Indeed, in many developing countries, inefficient governance and informal dumping or recycling lead to inadequate environmental policies of waste management (Thonart Philippe et al. (2005), Wilson et al. (2006), Barton et al. (2008)) or wastewater. A review of this topic has been done by Mohee et al. (2015) for SIDS. Second, the close interaction between ecosystems implies that pollutants may easily travel through all the natural environment, ecosystems and vital stock of water. For example, the proximity of waste facilities and their lack of efficiency are directly responsible of soft water contamination and coasts degradation through

anthropogenic seafloor debris. Domestic waste is not the only source of pollution in the Caribbean. In fact agricultural use of pesticides and fertilizers were considered as the main local pollution source between 1980 and 2000 (Rawlins et al. (1998)), knowing that many pesticides are very persistent in the ecosystems. Therefore, even if the management of agricultural pollution has improved since the 1990s, the pollutants tend to accumulate in the sea (this is due to the oceanographic features of the Caribbean region), and are still impacting the health of the inhabitants (Ross and de Lorenzo (1997)).

3. Stylized Facts: the demographic features of Caribbean SIDS

Countries	Natural Balance ^a (total per 1000 pop.)			Migratory sold (percent)		
	1972	1992	2012	1972	1992	2012
Caribbean Small States	24,07	16,92	9,71	-5,78	-4,13	-1,78
Antigua and Barbuda	20,38	11,8	10,41	-4,28	4,13	-0,06
Bahamas	20,16	17,97	9,45	0,5	-0,15	2,6
Barbados	10,49	6,04	1,84	-2,25	-1,24	0,77
Cuba	20,37	8,73	3,01	-2,11	-1,12	-0,71
Dominican Republic	29,98	22,74	15,51	-1,49	-1,93	-1,51
Haiti	21,24	23,31	16,82	-1,8	-1,9	-1,46
Jamaica	26,06	17,77	10,88	-5,35	-4,54	-3,51
St. Vincent and the Grenadines	30,26	17,37	9,53	-9,51	-8,04	-4,57
St. Lucia	28,93	19,74	8,71	-9,06	-3,39	0,02
Trinidad and Tobago	19,15	11,01	5,64	-3,03	-2,58	-0,37
United States of America	6,2	7,3	4,5	1,35	1,78	1,59
United Kingdom	2,9	2,6	3,9	0,19	0,36	1,41
Spain	11,3	1,6	1,1	0,28	0,82	-1,27
France	5,6	3,9	3,9	1,52	0,52	0,5

Table 1: Caribbean SIDS: Demographic dynamics
Source: World Development Indicators, World Bank

^aNatural Balance is define as the number of births minus the number of deaths in a year

This work scrutinizes the interactions between demographic features and environmental quality for Caribbean islands. In this section, some elements are given in order to evaluate the scale of these features. For the demographic dynamics there are three main characteristics in the Caribbean SIDS: first a rapid demographic transition, second a negative migratory sold and finally a strong heterogeneity in the scale of these phenomena in the different territories. The Caribbean demographic transition started soon after the period of independences in the 1960s. The mean number of children per woman in the Caribbean independant countries decreased from 5.3 in 1960 to 2.83 in 1990, and it is now at 2.4. Over the same period, the life expectancy at birth in the area increased from 52 years to 67.7 years, reaching 72.4 in 2010. This demographic transition was accelerated by the emigration of these countries which is very high, even compared to other developing countries (figure 1)². Therefore, in 2002, the demographic growth in Caribbean SIDS (CSS) was the same than in OECD countries (OED), even if the natural sold was higher. Migration does not only impact the population evolution but also the economic features. In figure 3 remittances are displayed as a percentage of GDP for the Caribbean islands (CSS), the Latin American and Caribbean group (LCN) as well as the Low and Middle income countries (LMY). It is clear that for Caribbean economies, remittances are non negligible, and this is particularly true if compared to savings (figures 2 and 3). Indeed, savings in CSS are lower than in other developing countries, with an average of 24% percent during the period considered versus 32 % in other developing countries. While remittances are higher than in other regions. Moreover the variability of the savings is much more important than the one of the remittances for CSS. Indeed, in many developing countries, remittances reduce the variability of income for the domestic workers.

²There are countries where the migration sold is positive – *i.e.* the Bahamas, Aruba and Antigua – nevertheless, in these countries the fluctuations of immigration are very high especially if they are considered as a percentage of population. Therefore, most of these effects are explained by the size of the countries.

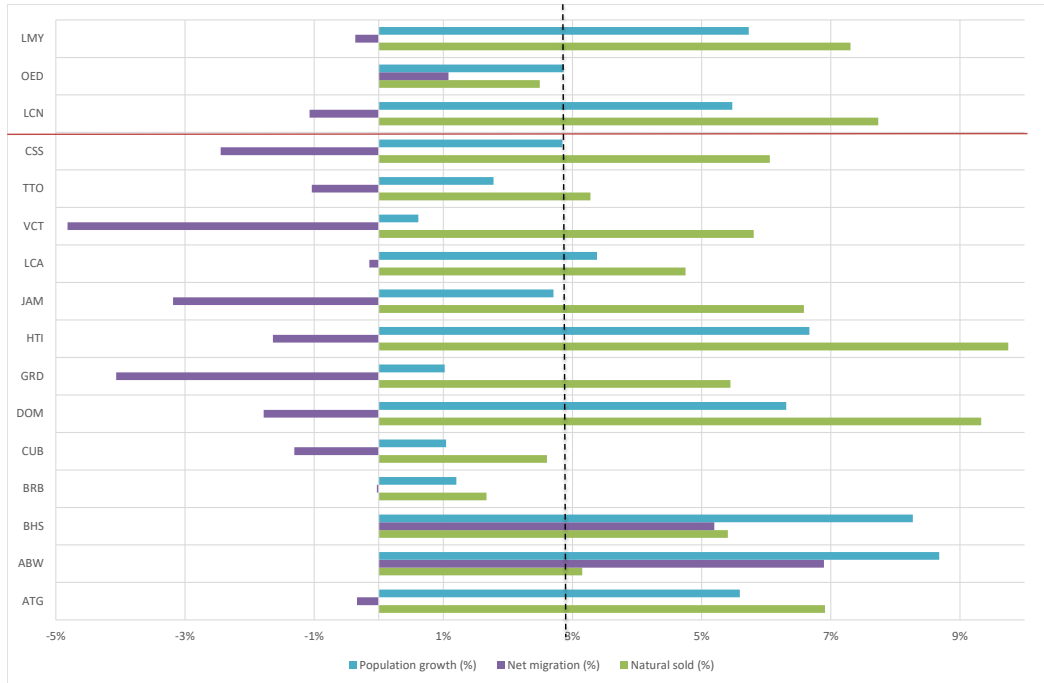


Figure 1: Demographic features of Caribbean SIDS

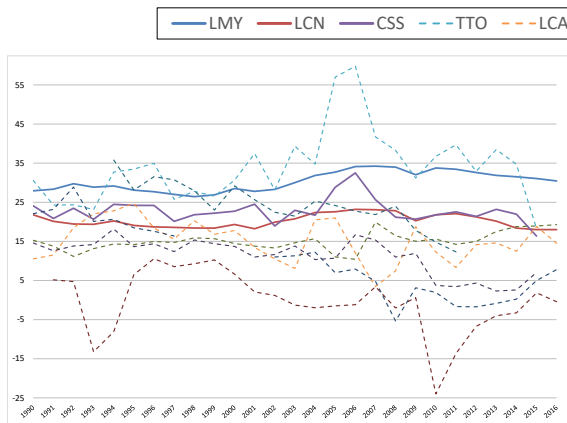


Figure 2: Comparison of savings in Caribbean SIDS and Developing countries (% of GDP)

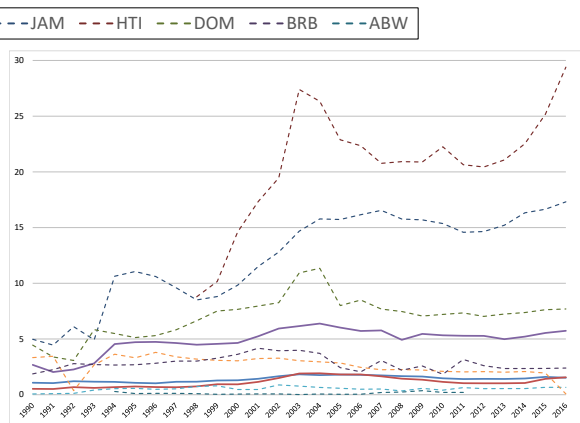


Figure 3: Comparison of remittances in Caribbean SIDS and Developing countries (% of GDP)

LMY, Low and medium income countries, OED, OECD countries, LCN, Latin American countries, CSS, Caribbean Small States, TTO, Trinidad and Tobago, VCT, Saint-Vincent and The Grenadines, LCA, Saint-Lucia, JAM, Jamaica, HTI, Haiti, GRD, Grenada, DOM, Dominican Republic, CUB, Cuba, BRB, Barbados, BHS, The Bahamas, ABW, Aruba, ATG, Antigua

4. The Model

To analyze the trend of capital intensity in SIDS with pollution, migration and intergenerational transfers, an overlapping generations (OLG) model is used, with discrete time indexed by $t = 0, 1, 2, \dots, +\infty$ ³. OLG models are extensively described in De La Croix and Michel (2002) and are really convenient to study intergenerational transfers – as in Thibault (2008) or Del Rey and Lopez-Garcia (2016) – and human capital dynamics. The present model is kept as simple as possible in order to exacerbate the households' behavior toward savings, consumption, fertility and education.

4.1. Production and the environment

Production of the composite good is carried out by a representative firm. The output is produced according to a constant returns to scale technology:

$$Y_t = AK_t^\alpha (L_t h_t)^{1-\alpha} \quad (1)$$

where K_t is the aggregate stock of physical capital, L_t is the aggregate labor supply to production, h_t is the mean human capital, $A > 0$ measures the technology level, and $\alpha \in (0, 1)$ is the share of physical capital in the production. The government collects revenues through a tax rate $\tau \in (0, 1)$ on production, which is the source of pollution. The tax revenue is used to fund pollution abatement m_t , with $m_t = \tau Y_t$, in order to correct the environmental externality on human capital accumulation. The firm profit is:

$$\Pi_t = A(1 - \tau)K_t^\alpha (L_t h_t)^{1-\alpha} - w_t h_t L_t - R_t K_t \quad (2)$$

where w_t is the wage for one unit of efficient labor and $R_t \equiv 1 + r_t$ is the interest rate of capital.

Assuming that the capital fully depreciates in one period, factor prices are:

$$w_t = A(1 - \alpha)(1 - \tau)K_t^\alpha (L_t h_t)^{-\alpha} \quad (3)$$

$$R_t = A\alpha(1 - \tau)K_t^{\alpha-1} (L_t h_t)^{1-\alpha} \quad (4)$$

The production sector generates a pollution flow. According to the pollution intensity of production, Ω , the natural absorption rate of pollution, a , and the cleanup effort, m_t , the dynamics of the pollution stock is such that :

$$Z_{t+1} = (\Omega - \tau)AK_t^\alpha (L_t h_t)^{1-\alpha} + (1 - a)Z_t \quad (5)$$

4.2. Family's behavior

Households live three periods, childhood, adulthood, and old age. At $t + 1$, a new generation of $n_t N_t$ homogenous agents is born, where n_t is the growth rate of the adult generation between period t and $t + 1$. As in De la Croix and Gosseries (2012) the value of n_t is chosen by the adults of period t , knowing that raising n_t children has a cost in time linked to the available land per household T/N_t . Use of this specification allows to introduce the space constraint that exists in all countries and which is exacerbated in islands:

$$\frac{1}{\lambda} \left(\frac{N_t}{T} \right)^\delta n_t = \sigma N_t^\delta n_t \quad (6)$$

where $\sigma = \frac{1}{\lambda} \left(\frac{1}{T} \right)^\delta$, the parameter $\lambda > 0$ is a measure of the total factor productivity (TFP) of the procreation activity, $\delta > 0$ captures the importance of space to have children and $T > 0$ is the available land.

The probability of migration is denoted by $\rho \in [0, 1[$. Migration implies that only $(1 - \rho)n_t N_t$ children stay in the domestic country after childhood, the others $\rho n_t N_t$ children migrate to countries where wages are higher. The evolution of the size of the adult generation writes:

$$N_{t+1} = n_t N_t (1 - \rho) \quad (7)$$

³Each period is assumed to last twenty to thirty years

Individual born in $t - 1$ cares about her adult consumption c_t and her old-age consumption d_{t+1} , according to the preference factor for the future β . Agent's preferences are represented by this utility function:

$$U(c_t, d_{t+1}) = \ln(c_t) + \beta \ln(d_{t+1}) \quad (8)$$

During childhood, individuals are reared by their parents and do not make any decisions. When adult, if they stay in the domestic territory, they supply inelastically one unit of labor remunerated at wage w_t per unit of human capital h_t . They allocate their income to consumption c_t , savings s_t and children education $n_t e_t$. Besides, they transfer a part γ of their revenue to their parents. Agents who have migrated send the same part γ of their revenue to their parents, but they can have a higher wage abroad which is proportional to the domestic one: $w_t^F \equiv \varepsilon w_t$, where $\varepsilon \geq 1$ is the net gain from migration. Cashflows from the migrants are remittances in the domestic economy, while transfers from domestic workers are simply intergenerational transfers. In this model, migrants are not economically active in the domestic country, except for the remittances sent to their parents. Therefore here, the decision of migration by the children or the remittances level are not studied. The emphasis is put on the decisions made by the parents and thus on their trade-off between savings, children education and their quantity, knowing that children will leave the country with a probability ρ and will remit more in this case according to the value of ε .

$$c_t + s_t + n_t e_t = w_t h_t (1 - \gamma - \sigma N_t^\delta n_t) \quad (9)$$

Finally, when old, agents only consume their savings remunerated at the rate R_{t+1} and the "inter-generational transfers" sent by their children, wherever they live.

$$d_{t+1} = s_t R_{t+1} + n_t \gamma (1 - \rho + \rho \varepsilon) w_{t+1} h_{t+1} \quad (10)$$

The in household's children acquire a certain level of human capital which depends on the total investments in education e_t , the current human capital h_t and the pollution level which has an impact on the efficiency of human capital accumulation $\theta(Z_t)^4$. This function is defined for positive or null real values of Z_t and decreases with the pollution stock, such that $\theta'(Z_t) < 0$. Therefore, here, a polluted environment deteriorates the children's ability to accumulate human capital whether it comes from their parents' human capital or the investments in education. Consequently the human capital of the children h_{t+1} writes:

$$h_{t+1} = \theta(Z_t) h_t^{1-\mu} e_t^\mu \quad (11)$$

where $0 > \mu > 1$ represents the education efficiency. Note that in this model, corner solutions are not considered, thus a condition for interior solutions is set to $e_t > 0$.

The consumer program is summarized by:

$$\begin{aligned} \max_{c_t, s_t, e_t, n_t} U(c_t, d_{t+1}) &= \ln(c_t) + \beta \ln(d_{t+1}) \\ s.t \quad c_t + s_t + n_t e_t &= w_t h_t (1 - \gamma - \sigma N_t^\delta n_t) \\ d_{t+1} &= s_t R_{t+1} + n_t \gamma (1 - \rho + \rho \varepsilon) w_{t+1} h_{t+1} \\ h_{t+1} &= \theta(Z_t) h_t^{1-\mu} e_t^\mu \\ Z_{t+1} &= (\Omega - \tau) Y_t + (1 - a) Z_t \end{aligned}$$

The first order condition (FOC) of the household's problem with respect to s_t shows the following standard trade-off between adult consumption and old-age consumption according to the preference factor for the future, β and the interest rate R_{t+1} :

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{d_{t+1}} \quad (12)$$

Similarly, the FOC of the household's problem with respect to education suggests that the remuneration from education and savings should be equal on the equilibrium, it leads to:

$$\frac{1}{c_t} = \frac{\beta \gamma \mu (1 - \rho + \rho \varepsilon) w_{t+1} h_{t+1}}{e_t d_{t+1}} \quad (13)$$

⁴A more precise analysis of the characteristics of this function is conducted in Appendix B

Finally, the FOC of the household's problem with respect to n_t , the fertility choice, writes:

$$\frac{1}{c_t} = \frac{1}{\sigma N_t^\delta w_t h_t + e_t} \frac{\beta \gamma (1 - \rho + \rho \varepsilon) w_{t+1} h_{t+1}}{d_{t+1}} \quad (14)$$

Combining equations (13) and (14) leads to the optimal choice of education:

$$e_t^* = \frac{\mu \sigma}{1 - \mu} w_t h_t N_t^\delta \quad (15)$$

Substituting the education choice into equation (14) and combining with equation (12) give a relation between the future prices of the production factors:

$$R_{t+1} = \frac{\gamma (1 - \mu) (1 - \rho + \rho \varepsilon)}{\sigma N_t^\delta} \frac{w_{t+1} h_{t+1}}{w_t h_t} \quad (16)$$

The income allocated to future consumption is denoted x_t . Its value differs from d_{t+1} , the consumption when old, because it does not include the remitted share of the children income and the remuneration of the savings. Therefore, x_t is the part of the adult income which is invested to finance old-age consumption, whether it is through savings or human capital investments. Two expressions of x_t are obtained. The first one is given by equations (15) and (16), while the second one results from the budget constraints:

$$x_t = s_t + \frac{\sigma N_t^\delta}{1 - \mu} w_t h_t n_t \equiv \frac{\beta (1 - \gamma)}{1 + \beta} w_t h_t \quad (17)$$

At this point, the respective shares of investments in the children transfers or savings to fund old-age consumption are undetermined. Therefore, the market clearing conditions (MCC) (given in equations (18) to (20)) are used as well as the prices of factors (given by equations (3) and (4)).

$$K_{t+1} = s_t N_t \quad (18)$$

$$L_{t+1} = N_{t+1} = n_t N_t (1 - \rho) \quad (19)$$

$$h_{t+1} = \theta(Z_t) e_t^\mu h_t^{1-\mu} = \theta(Z_t) \left(\frac{\mu \sigma}{1 - \mu} \right)^\mu h_t (N_t^\delta w_t)^\mu \quad (20)$$

A relationship between s_t and n_t appears and leads to the optimal choices for savings and natality if replaced in x_t (equation (17)):

$$s_t^* = \frac{\beta \alpha (1 - \rho) (1 - \gamma)}{(1 + \beta) [\alpha (1 - \rho) + (1 - \alpha) \gamma (1 - \rho + \rho \varepsilon)]} w_t h_t \quad (21)$$

$$n_t^* = \frac{\beta (1 - \gamma) (1 - \alpha) (1 - \mu) \gamma (1 - \rho + \rho \varepsilon)}{\sigma (1 + \beta) [\alpha (1 - \rho) + (1 - \alpha) \gamma (1 - \rho + \rho \varepsilon)]} N_t^{-\delta} \quad (22)$$

First of all, household's choices are totally independant of the pollution stock and depend exclusively on the migration features, the intergenerational rate and their intertemporal preferences. Therefore, the interpretation of the choices of the households is twofold. First, there is a trade-off between adulthood and old-age consumptions – *i.e.* respectively c_t and x_t – according to β , the preference for the future. Second, there is a choice between savings and transfers from the children to fund the old-age consumption. Both adult consumption and investments in old-age consumption are positively correlated to the income $w_t h_t$ and negatively to γ – the share of intergenerational tranfers. Indeed, this negative impact of γ is explained by the income effect at the first period, because it decreases the income available for present consumption, but also for savings and human capital investments. In the present case, the reduction of x_t seems to be compensated by high transfers from the children. Therefore, when γ is large it is less useful to invest in future consumption which depends in a large extent on the future revenue of the children.

Regarding the second trade-off between savings and human capital investments, savings – given by equation(21) – increase with respect to $w_t h_t$, β and the share of capital in the production, α . They decrease with ρ , γ and ε . This reveals that if there are highest gains from the investments in human capital, households will save less in order to raise the education expenditures. The effect of ε or ρ is only

due to this substitution effect. The effect of γ is twofold. In addition of the substitution between human capital investments and savings, there is the income effect described earlier.

In this model the amount of transfers from children, received at the second period depends on two choices. Indeed, households can choose between education or natality to increase their future consumption through intergenerational transfers. Education expenditures increase with the income $w_t h_t$, with the efficiency of education μ , with the opportunity cost of raising a child σ and the population size N_t . Therefore, a high density of population leads to a decline in natality, but the level of education given to each child will be higher. Moreover, the education expenditures can be rewritten according to equation (16) ; which exhibits positive correlations between e_t and ρ as well as ε . Thus, migration boosts the education expenditures, because the net gain from human capital is higher abroad. This is a traditional effect of migration described in the literature. However here the human capital accumulation is not exclusively linked to the investments in human capital but also to the environmental quality.

The interpretation of n_t^* according to the parameters is less intuitive. First, the transitional dynamics of natality is explained exclusively by the population size. The introduction of the congestion in the model leads to a constant population on the steady-state. In this model, natality is increasing with respect to β , ρ and ε and decreasing with α as well as σ . More interestingly, the impact of the transfer ratio γ on n_t is ambiguous and depends on the condition following:

$$\gamma < \frac{\sqrt{\alpha(1-\rho)} \left[\sqrt{\alpha(1-\rho) + 4(1-\alpha)(1-\rho + \rho\varepsilon)} - \sqrt{\alpha(1-\rho)} \right]}{2(1-\alpha)(1-\rho + \rho\varepsilon)} \quad (23)$$

This comes from the income effect described earlier which is competing with the substitution effect between savings and natality. It is interesting to note that in this case, the effect of γ is always positive for education expenditures, however its effect depends on a condition for natality. Therefore, if γ leads to an increase in the stock of units of efficient labour – *i.e.* $L_t h_t$ – it is more likely that it will be through a higher level of human capital (h_t).

Finally the education expenditures and the fertility choices are directly linked to the size of the population. While this element has no impact on the savings. Therefore, the first choice is made between savings and human capital according to the remuneration of these two types of investments. Secondly, another choice among natality and education is made with respect to the population size ; which has a positive impact on education expenditures. Here there is a standard Beckerian trade-off between quantity and quality (*i.e.* formation of the children), and congestion acts as a catalyst of the incentives to increase the children's skills, rather than their number.

4.3. Intertemporal equilibrium

The market-clearing conditions for capital, adult generation size and human capital level are given respectively by the equations (18) to (20). The values of s_t , n_t and e_t come from the optimal choices of the households in equations (21), (22) and (15). The wage and the rate of interest correspond respectively to (3) and (4). After some computations, the intertemporal equilibrium can be defined.

Definition 1. *Given the initial conditions $K_0 > 0$, $L_0 > 0$, $h_0 > 0$ and $Z_0 > 0$, the intertemporal equilibrium is the sequence (N_t, K_t, h_t, Z_t) such that the following system is satisfied for all $t \geq 0$:*

$$\begin{cases} N_{t+1} &= \frac{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}{\sigma} \Psi N_t^{1-\delta} \\ K_{t+1} &= \alpha A(1-\tau) \Psi K_t^\alpha N_t^{1-\alpha} h_t^{1-\alpha} \\ h_{t+1} &= \theta(Z_t) \left[\frac{\mu A \sigma (1-\alpha)(1-\tau)}{1-\mu} \right]^\mu K_t^{\alpha\mu} N_t^{\mu(\delta-\alpha)} h_t^{1-\alpha\mu} \\ Z_{t+1} &= (\Omega - \tau) A K_t^\alpha (N_t h_t)^{1-\alpha} + (1-a) Z_t \end{cases} \quad (24)$$

where $\Psi \equiv \frac{\beta(1-\gamma)(1-\alpha)(1-\rho)}{(1+\beta)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)]}$

Definition 2. *A Steady-State (SS) is an equilibrium satisfying **Definition 1** and where N_t , h_t , K_t and Z_t are constant.*

Proposition 1. According to the *Definition 2* there is a unique equilibrium⁵, for which the values of N^* , K^* , h^* and Z^* are:

$$N^* = \left[\frac{\gamma(1-\mu)(1-\rho+\rho\varepsilon)}{\sigma} \Psi \right]^{\frac{1}{\delta}} \quad (25)$$

$$K^* = \alpha(1-\tau) \frac{a\theta^{-1}(\chi)}{\Omega-\tau} \Psi \quad (26)$$

$$h^* = \frac{a\theta^{-1}(\chi)}{(\Omega-\tau)A} [\alpha A(1-\tau)]^{-\frac{\alpha}{1-\alpha}} \left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)} \right]^{\frac{1}{\delta}} \Psi^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}} \quad (27)$$

$$Z^* = \theta^{-1}(\chi) \quad (28)$$

where $\chi = [\gamma\mu(1-\rho+\rho\varepsilon)]^{-\mu} \left[\frac{\beta A \alpha^\alpha (1-\tau)(1-\rho)(1-\gamma)(1-\alpha)(2-\alpha)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \right]^{-\frac{\mu}{1-\alpha}}$ is the efficiency of human capital accumulation in the steady-state and $\theta^{-1}(\cdot)$ is the inverse function of $\theta(Z_t)$.

Proof of proposition 1. see Appendix A □

Next, an analysis of the effects of the various parameters on the steady-state values N^* , h^* , K^* and Z^* is conducted. Note that in the steady-state, the evolution of population size is considered to be equivalent to the adult generation size with a factor three, because the size of every generation will be the same – except for the number of children before the migration. Therefore in the rest of this work, population size and adult generation size present exactly the same features and both terms can be used indifferently to describe the demographic growth.

5. Migration and Environmental impacts analysis

In this section the focus is on the environmental features and on the parameters that control the migration impact. A static comparative analysis and numerical simulations are used in this discussion which is conducted in two steps. Most of the results are derived from the numerical simulations because of the complexity of the expressions obtained from the static comparative exercise. However, the conclusions from the simulations are very consistent to changes in the parameters or in the damage function. Moreover, all the results are not discussed in the present section but they are given in Appendix C.

Table 5 gives the parameters values for the benchmark economy. They have been set to average values plausible for the Caribbean economies. Note that the aim of this paper is not to scrutinize the values of the parameters but only to exhibit the impacts of their variations on the economies. In a precedent work⁶, a precised calibration for several islands has been conducted and is not reproduced in the present work. Consequently, the variations induced by changes in the parameters will be described but not the levels reached by the different aggregates in the economies.

5.1. The environment and the public policy

According to the static comparative analysis, the pollution intensity, Ω , has a negative effect on the human capital level and the capital stock, while it has no effect on the population size and the pollution stock. The productive capital stocks are positively correlated to the absorption rate, a , while the pollution stock is negatively correlated to a . Finally the tax reduces the pollution stock, while it has a positive effect on K^* and h^* under the following conditions :

$$\frac{\partial K^*}{\partial \tau} > 0 \iff \zeta_\tau > \frac{\tau}{1-\tau} \left[\frac{1-\Omega}{\Omega-\tau} \right]$$

$$\frac{\partial h^*}{\partial \tau} > 0 \iff \zeta_\tau > \frac{\tau}{1-\tau} \left[\frac{1-\tau-\alpha(1-\Omega)}{(\Omega-\tau)(1-\alpha)} \right]$$

where $\zeta_\tau = \frac{\partial \theta^{-1}(\chi)}{\partial \tau} \frac{\tau}{\theta^{-1}(\chi)}$

⁵The stability of the equilibrium could not be proven analytically, however its robustness has been checked by numerical analysis. The equilibrium is stable for a large range of parameters values and initial conditions (cf. Appendix B).

⁶available upon request

Parameters		Benchmark
Preference factor for the future	β	0.961
Capital intensity in production	α	0.33
Technology level	A	1.014
Education efficiency	μ	0.2
Emigration rate	ρ	0.2
Net gain from migration	ε	20
Share of income remitted	γ	0.15
Land size	T	10
TFP of reproduction function	λ	0.8
The weight of land for raising children	δ	0.7
Natural absorption	a	0.5
Efficiency of human capital accumulation	$\theta(Z_t)$	$\frac{4}{1+Z_t}$

Table 2: Parameters of the model

First of all, the environmental degradation does not lead to a reduction of the population size, and thus, the public policy does not have a natalist effect, unlike the results found by De la Croix and Gosseries (2012). Indeed, in the present work the households' choices are totally independent of the level of pollution or the environmental tax. Here, only the human and physical capital stocks decrease according to the level of pollution – *i.e.* $\theta^{-1}(\chi)$ and increase with the value of χ . This is due to the direct impact of the externality.

The pollution stock in the steady-state is directly given by the value of χ which is the level of efficiency of human capital in the steady-state. Hence, in this model, Ω impacts strongly the transitional dynamics of the pollution stock and thus the accumulation of the productive capital stocks in the first periods. If the emissions of pollution from the production are high, the efficiency of human capital accumulation will be lessened in the first stages of the economies' development. That will lead to a reduction in the human capital level that can be attained in the steady-state, even if the value of $\theta(Z^*)$ is the same for all values of Ω . Similarly, the capital stock will be lessened even if the share of savings is independent of the pollution stock. This is due to the loss income that occurs with the decline in the human capital in the first stages of development when Ω is high.

In that context, an improvement of the human capital and the physical capital stock occurs thanks to the environmental policy which consists in a tax of the production and a clean-up effort. A first intuition would be that a reduction of the pollution stock thanks to the tax will lead in every cases to an increase in the productive capital stocks and then to an increase in the production as well as the household utility, until a threshold. However conditions to have a positive effect from the environmental policy are found and it appears that they are largely linked to the level of pollution intensity.

The numerical simulations simplify the analysis of this effect. In the figure 4 steady-state values for the production per capita (Y/L), the capital stock (Y), the pollution stock (Z), the utility (U), the human capital level (h) and the population (L) are displayed according to the tax level, $\tau \in [0; 0.5]$. In every graphics there are two curves, the dotted line is for the variables evolution when $\Omega = 0.5$, while the other one represents the effect of the tax if $\Omega = 0.8$.

In these graphics it appears that if Ω is small, it is very profitable to implement a strong tax – *i.e.* $\tau \rightarrow \Omega$ – because it always leads to an increase in human capital level, capital stock, production and then utility, while the pollution is lessened. However it is not possible to compensate completely the pollution emissions and to have a steady-state for human capital or capital stock. If pollution was totally abated a balanced growth path would appear, with a constant growth rate for physical capital and human capital. In a context of high emissions, the environmental policy leads in every cases, to a decrease in the physical capital stock and has a negative impact on utility. One explanation is that the effects of the environmental policy on the environmental externality are too small if the tax is low. However, if τ is high enough to overcome the pollution's effect on human capital, the negative income effect induced by the tax is too large to lead to an increase in the capital stock. Therefore even if the human capital and the production per capita increase slightly, it is not enough to compensate the income loss due to the capital stock, and thus the utility decreases. Indeed, the production per capita is not used exclusively in consumption, but also in education expenditures and in savings. Hence an increase in production per capita without utility improvement is possible.

Note that these effects are very consistent to different values of parameters for migration – *i.e.* γ , ρ , ε – as well as for different values of absorption rate or parameters that control the overcrowding effect. Indeed a, δ and λ introduce a scale effect but they do not change the shapes of the curves of the steady-state values.

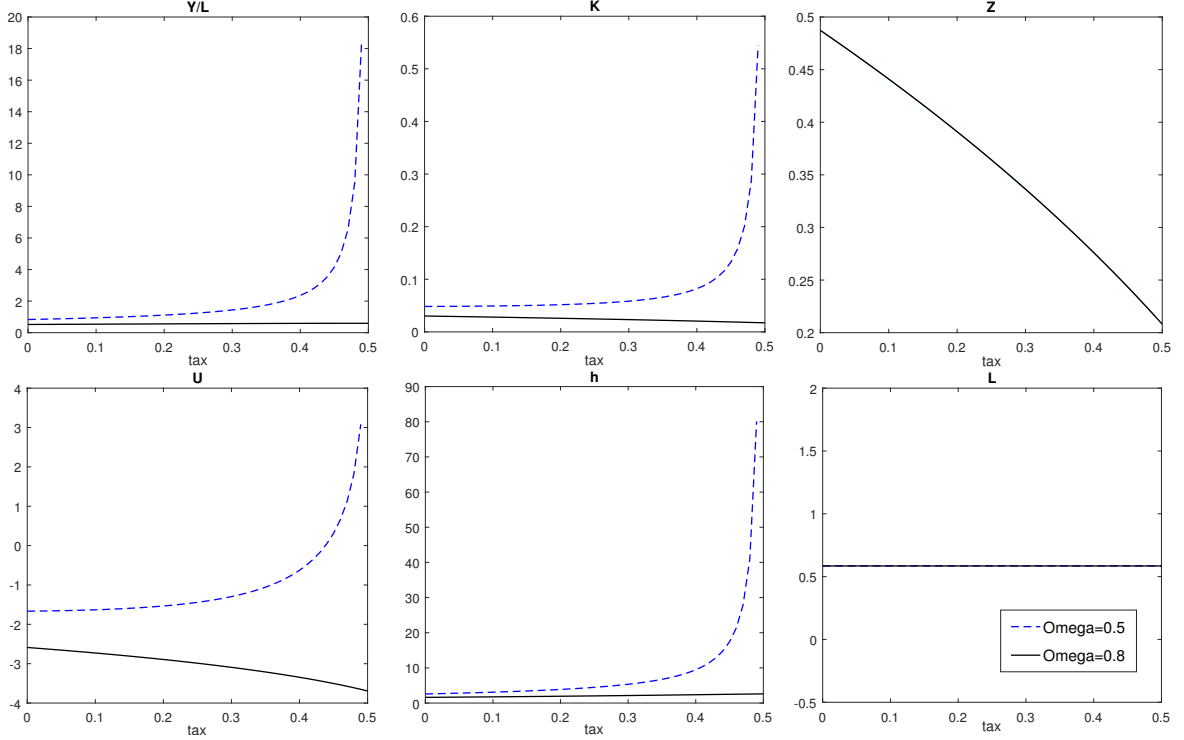


Figure 4: Effect of the tax on the steady-state values according to Ω

5.2. The migration impact

In the rest of this discussion the effects of migration features – *i.e.* ρ , ε and γ – are tested. Note that a raise in γ means that the intergenerational transfers are more important for all the children wherever they live, while an increase in ε induces a rise in the remittances received in the domestic economy. In the numerical simulations, values of emigration rate are chosen in order to exacerbate the differences between the various islands of the Caribbean. Indeed, there are some islands with extremes values of emigration, such as Jamaica (close to 50% over a 30 years period), some with average values like Dominican Republic or Haiti, and finally some with small values of emigration, as Trinidad and Tobago.

The effect of the emigration rate: ρ

The steady-state values of the population size, the stock of pollution, the capital and the human capital level are positively correlated to ρ under the following conditions:

$$\begin{aligned} \frac{\partial N^*}{\partial \rho} > 0 &\iff \frac{1 - \rho}{1 - \rho + \rho\varepsilon} > \left[\frac{\gamma(1 - \alpha)}{\alpha(\varepsilon - 1)} \right]^{1/2} \\ \frac{\partial Z^*}{\partial \rho} > 0 &\iff \frac{1 - \rho}{1 - \rho + \rho\varepsilon} > \frac{\gamma\varepsilon}{(\varepsilon - 1)[\alpha(1 - \rho) + \gamma(1 - \alpha)(1 - \rho + \rho\varepsilon)]} \\ \frac{\partial K^*}{\partial \rho} > 0 &\iff \zeta_\rho > \frac{\rho}{1 - \rho} \left[\frac{\gamma\varepsilon(1 - \alpha)}{\alpha(1 - \rho) + \gamma(1 - \alpha)(1 - \rho + \rho\varepsilon)} \right] \\ \frac{\partial h^*}{\partial \rho} > 0 &\iff \zeta_\rho < \frac{\rho}{1 - \rho} \left[\frac{(1 - \rho)(\varepsilon - 1)[\alpha(1 - \rho) + \gamma(1 - \alpha)(1 - \rho + \rho\varepsilon)] - \gamma\varepsilon[1 - \alpha(1 - \delta)](1 - \rho + \rho\varepsilon)}{(1 - \rho + \rho\varepsilon)[\alpha(1 - \rho) + \gamma(1 - \alpha)(1 - \rho + \rho\varepsilon)]} \right] \end{aligned}$$

where $\zeta_\rho = \frac{\partial \theta^{-1}(x)}{\partial \rho} \frac{\rho}{\theta^{-1}(x)}$

The impacts of the emigration rate on the pollution stock or the population size are given directly by conditions on the ratio controlling the gain from migration. While, the emigration rate effect on h^* and K^* depends on the elasticity of pollution with respect to ρ .

Emigration rate	ρ	$[0; 0.5]$
Pollution impact of production	Ω	0.3
Environmental tax	τ	0.2

Table 3: Parameters of the model to evaluate the effect of ρ

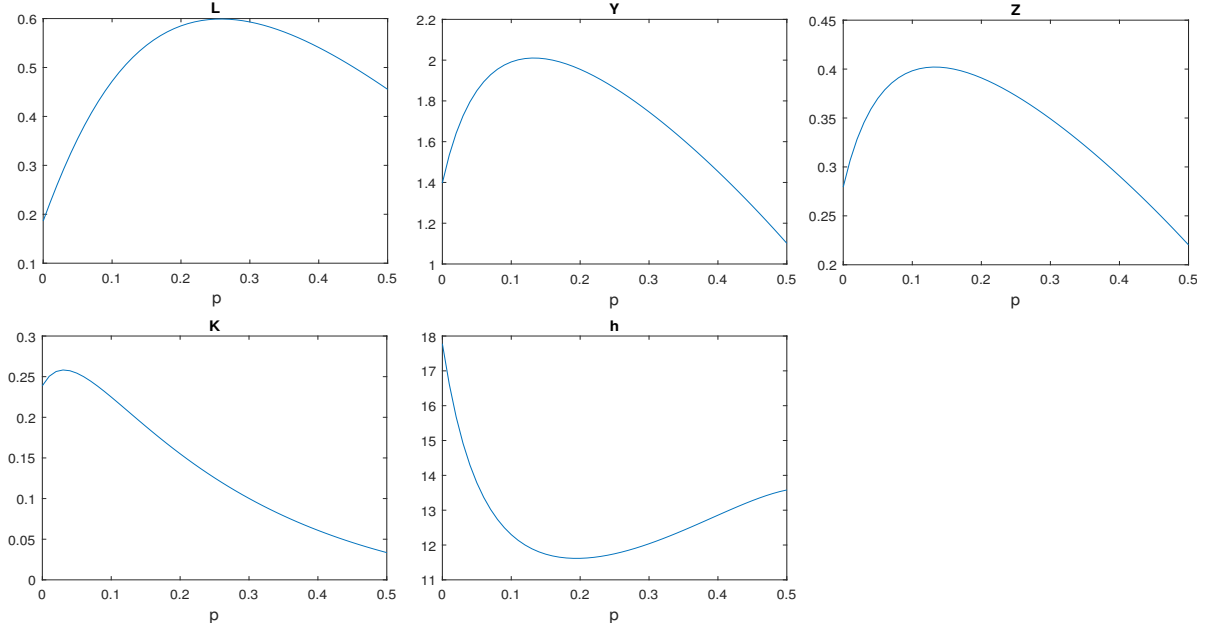


Figure 5: Effect of a variation of ρ on the economy

The numerical simulations allow us to specify the different mechanisms that explain the effect of migration rate. On figure 5 steady-state values for the adult generation size (L), the production (Y), the pollution stock (Z), the capital stock (K) and the human capital level (h) are displayed according to the emigration rate, $\rho \in [0; 0.5]$. U-shaped curves appear for the level of human capital, and inversed U-shaped curves are observed for the other variables. Indeed, under a certain level of ρ , the production stock increases thanks to the population size increase. This augmentation is due to the substitution effect between savings and natality induced by the emigration gain. Moreover, for small values of emigration, a positive effect is also observed for the capital stock because of the higher number of savers. This positive effect of migration on natality is usual. Nevertheless, an higher production leads to larger emissions of pollution, and thus to an increase in the environmental externalities on human capital accumulation. Consequently, the human capital per inhabitant decreases strongly with the emigration rate increase for small values of ρ .

If the emigration rate exceeds a certain limit, the substitution effect is substantial and a strong decrease in savings occurs. In that case, the physical capital decreases too much to sustain the production. This is aggravated by the reduction of the population induced by the migration. Indeed, an increase in ρ leads to an higher natality but it decreases the number of adults staying in the territory. If migration is very large the loss of adults in the territory is higher than the rise in the number of children and the population size is reduced, as well as the number of savers. Ultimately, high values of ρ have also a negative impact on capital stock. The combination of these various mechanisms leads to a reduction in the production stock and in the pollution externality on human capital. Therefore in this case, the human capital rises.

In the figure 6 the production per capita (Y/L) and the household's utility (U) are displayed according to ρ . Simultaneously of the increase in human capital there is a rise in the income per inhabitant which leads to a higher consumption and thus utility (6). However the level which is attained is smaller than in the situation without any migration. Indeed, the increase in the emigration rate leads in any case to a reduction of the production per capita, however for high values of emigration rate, the gains from human capital increase are high enough to overcome the reduction of the production per capita.

These results differ largely from the literature. The consensual result is that an increase in migration leads to a gain if the value of the emigration rate is low and to an economic decrease when ρ is high.

Here, in most of the cases, with the environmental externality on human capital, migration leads to a decrease in per capita economic results. Therefore, a benefit from an increase in migration rate is possible exclusively if the initial value of migration is already high.

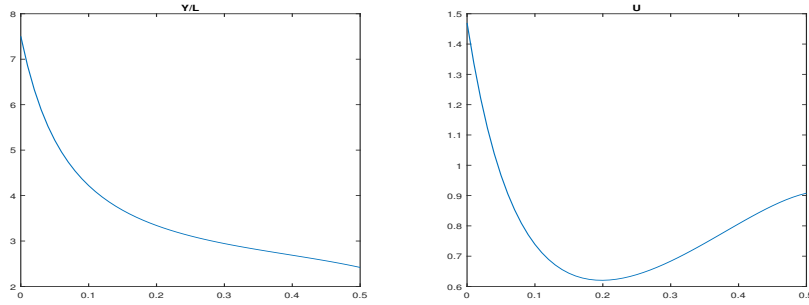


Figure 6: Effect of a variation of ρ on the production per capita and the utility

The effect of the net gain from migration: ε

The effect of ε on the population size is always positive, while its effects for the stock of pollution, the capital stock and the human capital level depend on the following conditions:

$$\begin{aligned} \frac{\partial Z^*}{\partial \varepsilon} > 0 &\iff \varepsilon > \frac{(1-\rho)(1-\gamma)}{\gamma\rho} \\ \frac{\partial K^*}{\partial \varepsilon} > 0 &\iff \zeta_\varepsilon > \frac{(1-\alpha)(1-\rho)(1-\gamma)}{\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)} \\ \frac{\partial h^*}{\partial \varepsilon} > 0 &\iff \zeta_\varepsilon > \frac{\varepsilon\alpha\rho}{\delta} \left[\frac{\gamma\delta\varepsilon\rho - (1-\rho)(1-\gamma\delta)}{[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)]} \right] \end{aligned}$$

where $\zeta_\varepsilon = \frac{\partial\theta^{-1}(\chi)}{\partial\varepsilon} \frac{\varepsilon}{\theta^{-1}(\chi)}$

The static comparative analysis shows that the effects of ε on Z^* and N^* depend directly on the value of this parameter, while its effect on h^* and K^* depends on the elasticity of pollution with respect to ε .

Emigration rate	ρ	$\{0, 0.1, 0.2, 0.3, 0.4\}$
Net gain from migration	ε	$[1; 50]$
Pollution impact of production	Ω	0.3
Environmental tax	τ	0.2

Table 4: Parameters of the model to evaluate the effect of ε

The numerical analysis has been conducted with values of $\varepsilon \in [1, 50]$. These values could seem very high, but, the economic situation of country as Haiti compared to OECD countries is pretty well described by a value of ε close to 50. In figure 7, the steady-state values for the adult generation size (L), the production (Y), the pollution stock (Z), the capital stock (K) and the human capital level (h) are displayed according to the net gain from migration. In each graphic, five curves are represented with respect to different values of emigration rate ρ : 0, 0.1, 0.2, 0.3 and 0.4. These features allow to exhibit that the effect of an increase in the net gain of migration depends on the amplitude of migration. Obviously, the net gain from migration does not have any impact if the migration rate is null.

The effects of ε on h^* and K^* are similar to the effects of the migration rate ρ on these variables. Indeed, the mechanisms that explain the effects of ε depends strongly on the substitution between children (in quality and quantity) and savings as well as on the increase in the pollution emissions due to the larger population.

However, in terms of production the effects of ε are not strictly the same according to the level of migration. If migration is low – *i.e.* close to 10% over a 30 years period – an increase in ε is always positive for the production, thanks to the larger population size. This occurs, even if the capital stock and the human capital level decrease. If migration is high, inversed U-shaped curves for production and pollution are observed. Indeed, with high values of emigration, an increase in ε leads to a strong decline

in the capital stock. For the lowest values of ε , this is compensated by the larger number of workers but not for the highest values. Note that, when ρ is high, even if Z^* is decreases strongly because of ε , the human capital level is never enhanced by the higher net gain from migration. Indeed, for the small values of ε the pollution is too large to allow for human capital accumulation, and for high values, the reduction of capital stock leads to a contraction in the income per capita and thus in the education expenditures.

Finally, utility depends solely on consumption which is partly linked to production per capita. These two variables for the economies described before, are displayed in figure 8. Whatever the level of migration (except for $\rho = 0$), the population size increases faster with ε than the production stock. Therefore, an increase in the net gain from migration leads in any case to a reduction in the utility per inhabitant, because of the increase in the pollution stock – if ε is small – or because of the decrease in capital stock – if ε is high.

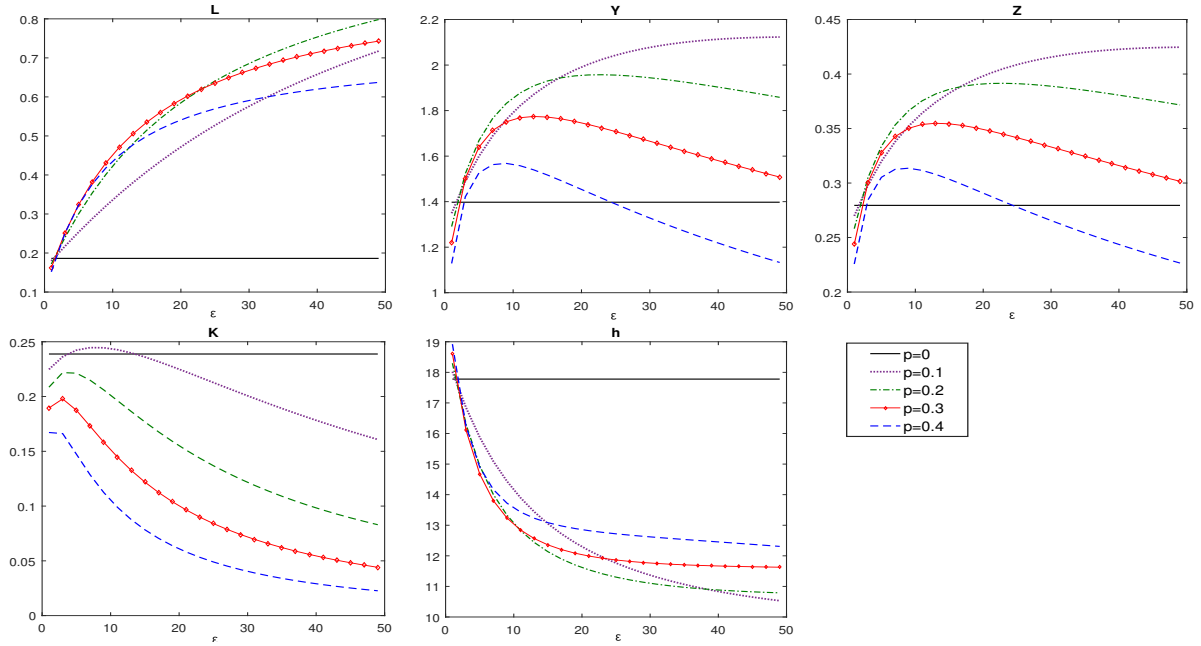


Figure 7: Effect of a variation of ε on the economy

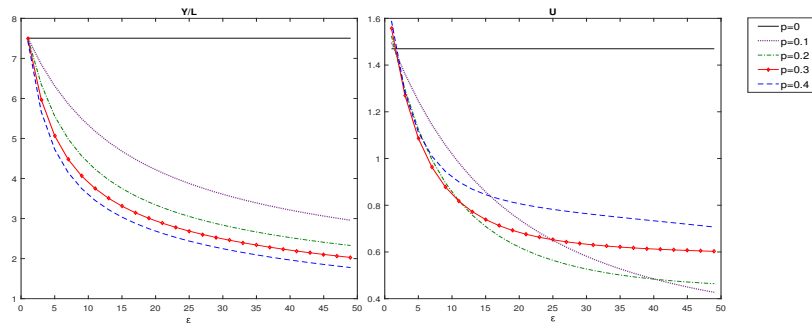


Figure 8: Effect of a variation of ε on production per capita and utility

The effect of intergenerational transfers: γ

The steady-state values of the population size, the stock of pollution, the capital and the human capital level are positively correlated to γ under the following conditions:

$$\frac{\partial N^*}{\partial \gamma} > 0 \iff \gamma < \frac{2\sqrt{\alpha(1-\rho)}[\sqrt{\alpha(1-\rho)} + \sqrt{1-\rho+\rho\varepsilon(1-\alpha)}]}{(1-\alpha)(1-\rho+\rho\varepsilon)}$$

$$\frac{\partial Z^*}{\partial \gamma} > 0 \iff \gamma < \frac{\sqrt{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]^2 + 4\alpha(1-\rho)(1-\alpha)^3(1-\rho+\rho\varepsilon)}}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)} - \frac{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)}$$

$$\frac{\partial K^*}{\partial \gamma} > 0 \iff \zeta_\gamma > \frac{\gamma}{1-\gamma} \left[\frac{(1-\rho+\rho\varepsilon)(1-\alpha)}{\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)} \right]$$

$$\frac{\partial h^*}{\partial \gamma} > 0 \iff \zeta_\gamma < \frac{\gamma}{1-\gamma} \left[\frac{\alpha(1-\rho)[\gamma(1-\alpha(1-\delta))-(1-\alpha)(1-\gamma)]+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)[\alpha(1-\rho)+p]}{\delta(1-\alpha)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \right]$$

where $\zeta_\gamma = \frac{\partial \theta^{-1}(x)}{\partial \gamma} \frac{\gamma}{\theta^{-1}(x)}$

Impacts of γ are linked, on the one hand to the income effect in the first period for the domestic households and on the other hand to the amount of transfers received during the second period. As said earlier, the negative income effect induced by γ leads to a decrease in present consumption and in investments for future consumption. However if γ is higher there is also an incentive to invest in education and it is possible to observe an increase in human capital and natality. Therefore, the positive effect of γ will depend on conditions. Here again, these conditions define directly the value of γ for Z^* and N^* , while the effect of γ depends on the elasticity of pollution with respect to this parameter for h^* and K^* .

Interpreting these conditions is not easy, therefore a numerical analysis is conducted for $\gamma \in [0.05; 0.3]$. In the figure 9, the steady-state values for the adult generation size (L), the production (Y), the pollution stock (Z), the capital stock (K) and the human capital level (h) are displayed according to the intergenerational transfer share. On each figure there are five curves corresponding to different level of migration – *i.e.* $\rho = \{0, 0.1, 0.2, 0.3, 0.4\}$ – in order to exhibit the combined effect of migration and intergenerational transfers.

Emigration rate	ρ	$\{0, 0.1, 0.2, 0.3, 0.4\}$
The intergenerational transfer	γ	$[0.05; 0.3]$
Pollution impact of production	Ω	0.3
Environmental tax	τ	0.2

Table 5: Parameters of the model to evaluate the effect of ε

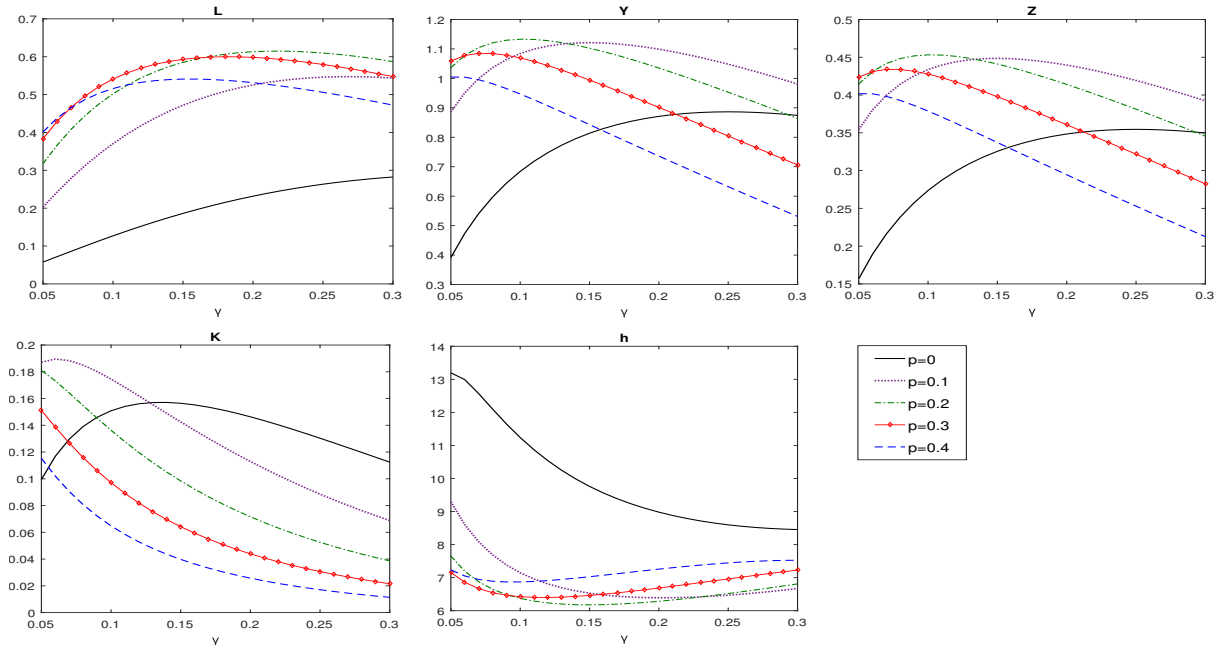


Figure 9: Effect of a variation of γ on the economy

The impacts of an increase in γ are similar to the effects generated by an increase in ε but not in the same scale. First if there is no emigration in this economy. An inversed U-shaped curve is observed for the capital stock. Therefore, for almost all the values of γ introduced here, there is a positive effect on the production stock. This results on the one hand from the increase in the number of workers and on the other hand from the capital stock. The increase in production leads to a rise in the pollution stock

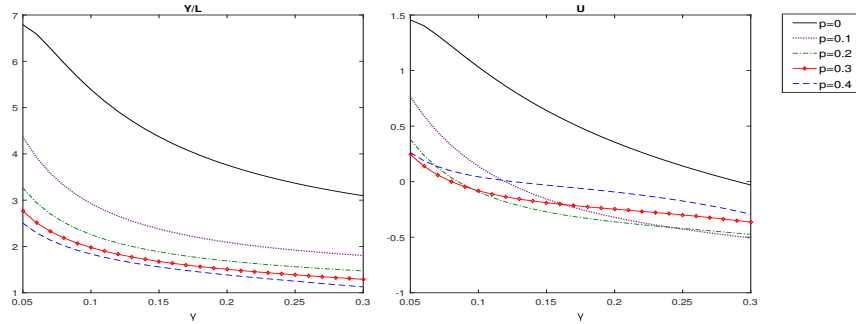


Figure 10: Effect of a variation of γ on the production per capita and the utility

and thus to a decrease in human capital because of the environmental externality.

If there is emigration, the increase in γ creates a substitution effect between natality and savings, and thus an increase in the population size, regardless of the emigration rate. However for high values of ρ and γ the size of the population can slightly decrease. Moreover, the effect of the increase in γ on capital stock is always negative if the emigration is non null. The combination of the positive effect on population size and the negative effect on capital stock leads to an inversed u-shaped curve for production and pollution if the emigration rate is non-null, and to a concave curve otherwise. Here, the human capital level is impacted in the same way than in the precedent case. Indeed, if γ is small it increases the production which leads to a strong externality and when γ is very high production decreases but the physical capital is too low.

As in the precedent case, the decrease in the human capital leads to a decrease in the income per capita, knowing that the population size increases faster than the production. Therefore, the utility per inhabitant decreases with γ even if the production is higher. Moreover, in that case utility – which is given in figure 10 – is always higher when migration is null, thanks to the higher human capital.

6. Conclusion

This paper presents a simple overlapping generations model, to explain the process of the interplay between economic activities, pollution emissions and investments in human and physical capital, according to the demographic structure of island economies. In this model, the demographic structure is strongly dependent on the pollution dynamics, which impacts directly human capital accumulation. Therefore, migration and its interactions with pollution are a key feature of the explanation of the productive stocks. Indeed, on the one hand, migration structure – the gain from migration and the scale of emigration – changes strongly the parents' choices in terms of education and savings, knowing that intergenerational transfers are included in the budget constraint. On the other hand, it also changes the production in the domestic area and then the pollution stock. This last can hamper widely the accumulation of human capital in the first periods and then leads to a reduction of the steady-state value of human capital. Therefore, even if the possibility to receive transfers – and especially remittances from migrants – creates a strong incentive to invest in education for the children, it could results in a reduction of human capital. In that case, while remittances are identified as a lever for economic growth in the literature, this model show that they can have a strong negative impact on development, and that a *brain gain* is not possible in most of the cases.

In that context, the numerical analysis of this theoretical model gives clear insights on the importance of the pollution structure if there is an environmental externality on human capital. Indeed, in a context where pollution intensity is low, a tax on pollution accompanied by a maintenance effort, improve the economic results while it is not possible if the emissions are too high. This result is robust to different specifications of the environmental features or the migration parameters. Moreover, the model reveals that the steady-state level of human capital efficiency was not impacted by the steady-state value of pollution, but by its dynamics during the transitional period. Therefore, it appears that the capital stock and the human capital generated in the early stages of the economic development are key features to trigger a strong economic growth in the next periods. In that context the existence of remittances or intergenerational transfers can hamper the accumulation of human capital, because they boost quickly the emissions of pollution.

Finally in this model, the interaction between the migration rate and the pollution stock is not simple. Indeed, on the one hand migration can be a source of economic dynamism and thus a source of pollution. But on the other hand more departures leads to a decrease in the population size and then to a decrease in the pollution stock. Therefore, in some cases an economic gain and an environmental gain can be obtained at the same time thanks to migration. For the Caribbean economies represented in this theoretical model, it seems that an increase of the emigration can be positive for the economic development, but only if migration is already high. Therefore, an auto-regulation of pollution is introduced thanks to the population dynamics and the reduction of the population size if migration is high. However, utility is most of the time decreased if the gains from migration are higher. To overcome this effect, other types of public policies could be tested, for example a tax on remittances or a policy on education to compensate the environmental damage on human capital accumulation.

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Appendix A. Proof of proposition 1

The size of the adult generation in the steady-state is directly given by the dynamics of the model:

$$N^* = \frac{\beta\gamma(1-\gamma)(1-\alpha)(1-\mu)(1-\rho+\rho\varepsilon)(1-\rho)}{\sigma(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} N^{*1-\delta} \quad (\text{A.1})$$

Therefore a relation between the steady-state values of the capital stock K^* and the human capital level h^* is obtained through the equation of K_{t+1} in the system (24).

$$K^* = \left[\frac{\alpha\beta A(1-\tau)(1-\alpha)(1-\gamma)(1-\rho)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \right]^{\frac{1}{1-\alpha}} N^* h^* \quad (\text{A.2})$$

Replacing the precedent equation and N^* in the human capital dynamics leads to the steady-state value of $\theta(Z_t) \equiv \chi$:

$$\chi = [\gamma\mu(1-\rho+\rho\varepsilon)]^{-\mu} \left[\frac{\beta A \alpha^\alpha (1-\tau)(1-\rho)(1-\gamma)(1-\alpha)(2-\alpha)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \right]^{-\frac{\mu}{1-\alpha}} \quad (\text{A.3})$$

Therefore, in the steady-state the stock of pollution can be defined as the value of the inverse function of $\theta(Z_t)$ ⁷ written $\theta^{-1}(\cdot)$, thus Z^* is defined as:

$$Z^* = \theta^{-1}(\chi) \quad (\text{A.4})$$

With the dynamics of the pollution stock (in the sytem (24)), another equation of the human capital level can be defined:

$$h^* = \left[\frac{aZ^*}{(\Omega-\tau)A} K^{*\alpha-1} N^{*\alpha-1} \right]^{\frac{1}{1-\alpha}} \quad (\text{A.5})$$

Combining equations (A.5), (A.2) and (A.4) gives the values of h^* and K^* :

$$\begin{aligned} h^* &= \frac{a\theta^{-1}(\chi)}{\Omega-\tau} [\alpha A(1-\tau)]^{-\frac{\alpha}{1-\alpha}} \left[\frac{\sigma}{\gamma(1-\mu)(1-\rho+\rho\varepsilon)} \right]^{\frac{1}{\delta}} \left[\frac{\beta(1-\alpha)(1-\gamma)(1-\rho)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \right]^{-\frac{1-\alpha(1-\delta)}{\delta(1-\alpha)}} \\ K^* &= \frac{\beta\alpha(1-\alpha)(1-\gamma)(1-\rho)(1-\tau)}{(1+\beta)[\alpha(1-\rho)+\gamma(1-\alpha)(1-\rho+\rho\varepsilon)]} \frac{a\theta^{-1}(\chi)}{\Omega-\tau} \end{aligned}$$

⁷The properties of the function $\theta(\cdot)$ will be studied in the numerical analysis

Appendix B. Stability of the Steady-State

A numerical analysis has been conducted to evaluate the stability of the steady-state. It is clear that these results hold for the specification tested in this study, however the parameters tested here are plausible for Caribbean economies and the interval tested are large enough to insure good generality of the results. The only parameters that have an impact on the steady-state are the environmental features. Indeed the migration parameters or the initial values have no impacts on the steady-state stability. Therefore, in this analysis, the focus is on the natural absorption rate, the pollution intensity and the damage function.

First, in the figure B.11 the production (Y), the capital stock (K), the pollution stock (Z) and the human capital level (h) are represented according to different values of absorption rate – *i.e.* $a = \{0.2, 0.5, 0.8\}$ – and of pollution intensity – *i.e.* $\Omega = \{0.2, 0.5, 0.8\}$. It shows that the environmental parameters have an impact on the necessary time to reach the steady-state, but not on the stability of the steady-state. Moreover, the steady-state value of the stock of pollution is not impacted by the environmental features. This is due to the expression of χ – which is totally independant of these parameters. Nevertheless, even if Z^* is not changed by the environmental features, the dynamics of the pollution stock is very different according to these parameters. Indeed, if a is small or Ω is high, the pollution stock is higher in the first periods and thus leads to a decrease in the steady-state values Y^* , K^* and h^* .

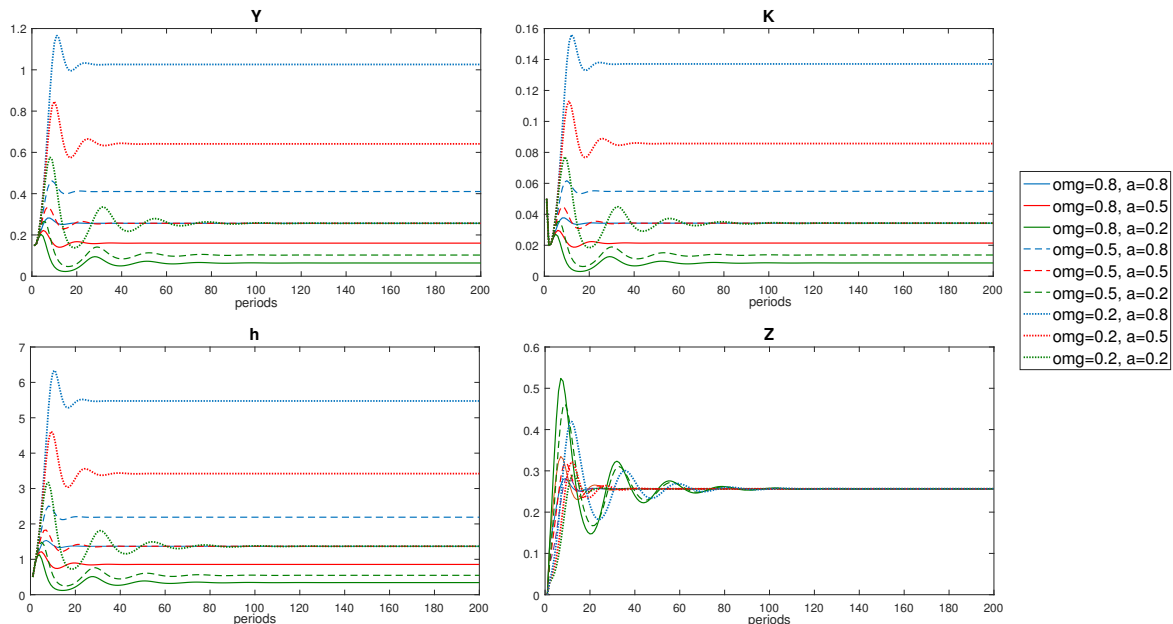


Figure B.11: The effect of Ω and a on the convergence

Second it seems that the steady-state stability is strongly linked to the form of the function $\theta(Z_t)$. Indeed, if this function is concave, the equilibrium is unstable and the stock values of the economy oscillate around the steady-state values. As an illustration, the production (Y), the capital stock (K), the pollution stock (Z) and the human capital level (h) of the benchmark economy are represented in figure ???. The plain line represents the benchmark economy with $\theta_1(Z_t)$, the dashed line is related to the situation with $\theta_2(Z_t)$ and finally the dotted line represents the economy with a highly concave function (*i.e.* $\theta_3(Z_t)$), where the function are given by:

$$\theta_1(Z_t) = \frac{\chi}{1 + Z_t} \quad (\text{B.1})$$

$$\theta_2(Z_t) = \frac{\chi}{(1 + Z_t^2)} \quad (\text{B.2})$$

$$\theta_3(Z_t) = \frac{\chi}{1 + Z_t^4} \quad (\text{B.3})$$

$$(\text{B.4})$$

In the figure B.13, the second derivatives of the functions $\theta_1(Z_t), \theta_2(Z_t), \theta_3(Z_t)$ are represented in order to define whether these functions are convexe or concave. The signs of these second derivative are not always the same in the interval represented here. However, according to the parameters chosen for the benchmark economy, values of the pollution stock stay under a threshold for which the sign of the second derivative are more or less stable. Therefore here it appears that convergence is possible if the interval where $\theta(Z_t)$ is convexe is large enough. Otherwise, there are oscillations around the steady-state values.

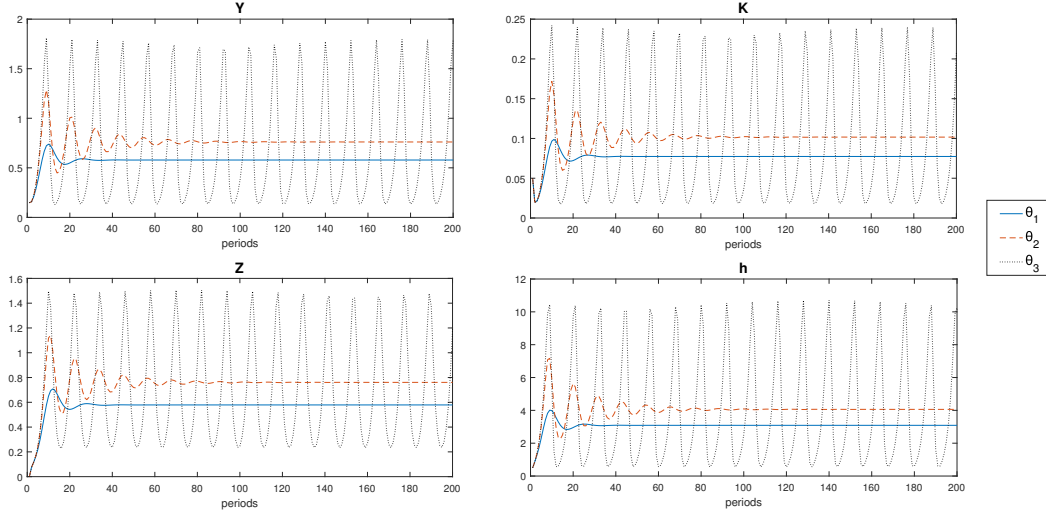


Figure B.12: The effect of $\theta(Z_t)$ on the steady-state stability

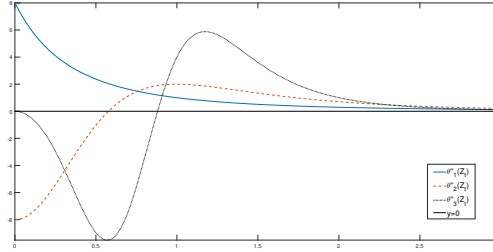


Figure B.13: Second derivatives of the functions $\theta_1(Z_t), \theta_2(Z_t), \theta_3(Z_t)$

Appendix C. Comparative static analysis

Proposition 2. *On the SS, the population size N^* is negatively correlated to the cost for rearing children $\sigma = \frac{1}{\lambda} \left(\frac{1}{T}\right)^\delta$, therefore it is decreasing in δ – the importance of land in the reproduction function – and it is positively correlated to T – the land size – as well as λ – the reproduction function efficiency. Moreover, on the SS, N^* is positively impacted by:*

- The preference for the future β
- The net gain from migration ε
- The level of intergenerational transfers γ under a condition:

$$\frac{\partial N^*}{\partial \gamma} > 0 \iff \gamma < \frac{2\sqrt{\alpha(1-\rho)}[\sqrt{\alpha(1-\rho)} + \sqrt{[1-\rho + \rho\varepsilon(1-\alpha)]]}{(1-\alpha)(1-\rho + \rho\varepsilon)}$$

- The probability of migration ρ under a condition:

$$\frac{\partial N^*}{\partial \rho} > 0 \iff \frac{1-\rho}{1-\rho + \rho\varepsilon} > \left[\frac{\gamma(1-\alpha)}{\alpha(\varepsilon-1)} \right]^{1/2}$$

In this economy, the size of the population will be controlled, on one side by the natality and on the other side by migration. Natality increases the number of children, therefore, in the steady-state, the size of the population depends negatively on the cost of raising children. This last is correlated to the congestion effect and thus to the population size, the available land size as well as the reproduction function efficiency. The impact of migration and thus the parameters that control its effect is less clear. Indeed, the probability of migration ρ boosts the natality and thus increases the number of children. Nevertheless, at the same time it decreases the number of adults who will stay in the territory at the next period. These two effects are competing, and then in some cases a higher emigration rate could lead to a growth (reduction) of the population size, if the increase in the number of children is larger (lower) than the decrease in the number of adults through migration. The effects of γ and ε on population size depend on conditions (given in **Proposition 2**) which are linked to an interaction at least three effects. On the one hand an increase in these parameters has positive effects on population size thanks to a rise of production – and then wealth – or the substitution effect between savings and natality. On the other hand they result in a negative effect from a higher pollution stock (cf. **Proposition 4**) that induces a decrease in human capital and thus of the household's income.

To study the variations of the pollution stock, the human capital level or the capital stock according to the parameters, it is necessary to define the effect of the different parameters on $\chi \equiv \theta(Z^*)$.

Proposition 3. *On the SS, χ is positively correlated to:*

- The environmental tax rate τ :
- The net gain from migration ε under a condition:

$$\frac{\partial \chi^*}{\partial \varepsilon} < 0 \iff \varepsilon < \frac{(1-\rho)(1-\gamma)}{\gamma p}$$

- The level of intergenerational transfers γ under a condition:

$$\frac{\partial \chi^*}{\partial \gamma} > 0 \iff \gamma > \frac{\sqrt{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]^2 + 4\alpha(1-\rho)(1-\alpha)^3(1-\rho+\rho\varepsilon)}}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)} - \frac{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)}$$

- The probability of migration ρ under a condition:

$$\frac{\partial \chi^*}{\partial \rho} > 0 \iff \frac{1-\rho}{1-\rho+\rho\varepsilon} < \frac{\gamma\varepsilon}{(\varepsilon-1)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}$$

And χ is negatively correlated to the preference for the future β and the economic efficiency A .

Proposition 4. *On the SS, the variations of Z^* depend directly on the variations of χ , knowing that $\theta(Z_t)$ and its inverse are decreasing and monotonic functions. The stock of pollution is positively correlated to:*

- The preference for the future β
- The net gain from migration ε under a condition:

$$\frac{\partial Z^*}{\partial \varepsilon} > 0 \iff \varepsilon > \frac{(1-\rho)(1-\gamma)}{\gamma p}$$

- The level of intergenerational transfers γ under a condition:

$$\frac{\partial Z^*}{\partial \gamma} > 0 \iff \gamma < \frac{\sqrt{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]^2 + 4\alpha(1-\rho)(1-\alpha)^3(1-\rho+\rho\varepsilon)}}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)} - \frac{[\alpha(1-\alpha)(1-\rho+\rho\varepsilon) + \alpha(1-\rho)(2-\alpha)]}{2(1-\alpha)^2(1-\rho+\rho\varepsilon)}$$

- The probability of migration ρ under a condition:

$$\frac{\partial Z^*}{\partial \rho} > 0 \iff \frac{1-\rho}{1-\rho+\rho\varepsilon} > \frac{\gamma\varepsilon}{(\varepsilon-1)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho+\rho\varepsilon)]}$$

And Z^* is negatively correlated to the tax level τ and the natural absorption rate, a .

Proposition 5. *On the SS, the capital stock K^* is positively impacted by:*

- The technology factor A
- The preference for the future β
- The natural absorption of pollution a
- The net gain from migration ε under a condition:

$$\frac{\partial K^*}{\partial \varepsilon} > 0 \iff \zeta_\varepsilon > \frac{(1-\alpha)(1-\rho)(1-\gamma)}{\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)}$$

$$\text{where } \zeta_\varepsilon = \frac{\partial \theta^{-1}(x)}{\partial \varepsilon} \frac{\varepsilon}{\theta^{-1}(x)}$$

- The level of intergenerational transfers γ under a condition:

$$\frac{\partial K^*}{\partial \gamma} > 0 \iff \zeta_\gamma > \frac{\gamma}{1-\gamma} \left[\frac{1-\rho + \rho\varepsilon(1-\alpha)}{\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)} \right]$$

$$\text{where } \zeta_\gamma = \frac{\partial \theta^{-1}(x)}{\partial \gamma} \frac{\gamma}{\theta^{-1}(x)}$$

- The probability of migration ρ under a condition:

$$\frac{\partial K^*}{\partial \rho} > 0 \iff \zeta_\rho > \frac{\rho}{1-\rho} \left[\frac{\gamma\varepsilon(1-\alpha)}{\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)} \right]$$

$$\text{where } \zeta_\rho = \frac{\partial \theta^{-1}(x)}{\partial \rho} \frac{\rho}{\theta^{-1}(x)}$$

- The environmental tax rate τ under a condition:

$$\frac{\partial K^*}{\partial \tau} > 0 \iff \zeta_\tau > \frac{\tau}{1-\tau} \left[\frac{1-\Omega}{\Omega-\tau} \right]$$

$$\text{where } \zeta_\tau = \frac{\partial \theta^{-1}(x)}{\partial \tau} \frac{\tau}{\theta^{-1}(x)}$$

And the capital stock is negatively correlated to the emissions of pollutions Ω .

Proposition 6. *On the SS, the human capital level h^* is positively impacted by:*

- The natural absorption of pollution a
- The technology factor A under a condition:

$$\frac{\partial h^*}{\partial A} > 0 \iff \zeta_A > \frac{1}{1-\alpha}$$

$$\text{where } \zeta_A = \frac{\partial \theta^{-1}(x)}{\partial A} \frac{A}{\theta^{-1}(x)}$$

- The preference for the future β under a condition:

$$\frac{\partial h^*}{\partial \beta} > 0 \iff \zeta_\beta > \frac{1-\alpha(1-\delta)}{\delta(1-\alpha)(1+\beta)}$$

$$\text{where } \zeta_\beta = \frac{\partial \theta^{-1}(x)}{\partial \beta} \frac{\beta}{\theta^{-1}(x)}$$

- The net gain from migration ε under a condition:

$$\frac{\partial h^*}{\partial \varepsilon} > 0 \iff \zeta_\varepsilon > \frac{\varepsilon \alpha p}{\delta} \left[\frac{\gamma \delta \varepsilon p - (1-\rho)(1-\gamma \delta)}{[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)]} \right]$$

- The level of intergenerational transfers γ under a condition:

$$\frac{\partial h^*}{\partial \gamma} > 0 \iff \zeta_\gamma < \frac{\gamma}{1-\gamma} \left[\frac{\alpha(1-\rho)[\gamma(1-\alpha(1-\delta)) - (1-\alpha)(1-\gamma)] + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)[\alpha(1-\rho) + p]}{\delta(1-\alpha)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)]} \right]$$

- The probability of migration ρ under a condition:

$$\frac{\partial h^*}{\partial \rho} > 0 \iff \zeta_\rho < \frac{p}{1-\rho} \left[\frac{(1-\rho)(\varepsilon-1)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)] - \gamma\varepsilon[1-\alpha(1-\delta)](1-\rho + \rho\varepsilon)}{(1-\rho + \rho\varepsilon)[\alpha(1-\rho) + \gamma(1-\alpha)(1-\rho + \rho\varepsilon)]} \right]$$

- The environmental tax rate τ under a condition:

$$\frac{\partial h^*}{\partial \tau} > 0 \iff \zeta_\tau > \frac{\tau}{1-\tau} \left[\frac{1-\tau - \alpha(1-\Omega)}{(\Omega-\tau)(1-\alpha)} \right]$$

And the human capital level is negatively correlated to the emissions of pollutions Ω .