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Transport policies in a two-sided market

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Abstract

We use a two-sided market to analyze a potential route of decarbonizing the transport sector, which relies on the adoption of electric vehicles (EVs). In our model EVs and gasoline vehicles (GVs) are demanded on one side of the market whereas demand for investment in electric vehicles charging stations (EVCSs) represents the other side. We assume that consumers can either buy EVs or GVs, which are substitutes, and their relative demand is affected by the availability of EVCSs. On the other side of the market, investors decide how many EVCSs to buy, depending on the number of EVs. A monopolistic platform producing both types of vehicles and EVCSs intermediates between the two sides of the market. The model allows us to capture the network effect operating between the number of EVCSs and the one of EVs; we assume this effect to be of a different intensity on each side. We first analyze a static setup of the model and we study the impact of one-sided and of two-sided policies. We find that the results depend on the relative values of the network effects on the two sides, but also on the degree of sostitutability between EVs and GVs and the slopes of the inverse demand functions. Then, we move to a dynamic setup of the model where we assume that demand is unknown as the markets for EVCSs and EVs are relatively new and also the impact that they will have on the traditional market for dirty cars is unclear. In this context, the monopolist uses a rule of the thumb (the gradient rule) to set prices and quantities that maximize profits.

Key words: transport, two-sided markets, network effects, unknown demand, electric vehicles.

JEL codes: C61, L91, R40.

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1 Introduction

Greenhouse gas (GHG) emissions caused by human activities are a major threat for our planet [41]. Transportation activities have a substantial role in contributing to GHG emissions; in 2014, the transportation sector accounted for 23% of the global carbon dioxide emissions, making it the second largest contributor after the electricity and heat generation sector. Moreover, road traffic alone accounted for three-quarters of transport emissions [2]. Consequently, reducing carbon emissions in the transport sector is a crucial action in facing climate change. One potential route to decarbonize the transport sector is through affecting driving behavior and vehicle choice. Recent years have seen increasing interest in electric vehicles (EVs) as a key technology for achieving efficient transportation while lowering emissions, although it has been stressed that the life cycle environmental implications of EVs depends on the fuel mix of electricity generation [72]. EVs include battery EV (BEV) and plug-in hybrid EV (PHEV). BEVs use electricity as the sole power source while PHEVs have the flexibility of using both electricity and liquid fuels. Governments are using a wide array of incentives, (income-tax credit or deduction for purchase of EVs, reduction or exemption from purchase or registration tax, free battery charging, free parking, support fo the deployment of charging infrastructure, grants for private installation of a charger and so on), to expand the proportion of electric vehicles (EVs) on the roads and switching from fossil fuels to EVs is a target of many policy programs. Many countries have set goals for EV adoption. For example, the U.S. plans to have more than 1.8 million plug-in hybrid electric vehicles (PHEVs); and China hopes to put 5 million hybrid and electric vehicles on the road by 2020 [69]. Also, the 2050 Swiss energy strategy envisages 30-75% introduction of EVs by 2050. Similarly, the California Air Resource Board requires a growing percentage of automakers' overall sales to be zero emissions. These policies, together with the increasing awareness of the environmental impact of GVs and performance of EVs, have led to an increase in the number of EVs: the EV market has expanded since its inception and is projected by industry analysts to grow much more in the coming decades. However, the diffusion of EVs is still limited in many countries and the actual effectiveness of the current collection of policies in supporting EV adoption is debated. The reasons behind the slow adoption of EVs are multiple, but as EVs were introduced to the broader consumers market only in 2010 and since they represent less than 1% of the total vehicle stock, there is little research that uses empirical data to analyze factors that affects adoption rates; thus much of our knowledge comes from stated preference studies [70]. In the literature ([71], [77], [39], [59]), we can identify at least three reason explaining the slow growth of the EV market; first, the purchase costs are still very high with respect to GVs prices: for comparable vehicles, such as Nissan Leaf and a Nissan Sentra, the difference in price is about $10,000; however, it is projected that, by 2020, EVs will become cheaper than GVs even without government subsidies; this optimistic view is partly due to the fact that the cost of electric vehicle batteries, which is the major cost of electric vehicles, has been reduced by more than 65% since 2010 [57]. Second, the driving distances that can be covered are limited; by way of example a Nissan Leaf can cover about 107 miles with one full charge, whereas GVs can achieve 400 miles per tank; this aspect restricts adoption significantly. The concern of running out of electricity (range anxiety) makes the EV charging infrastructure an important element when consumers choose vehicles. This leads us to the third reason for a slow adoption, which is also the most important for us: the lack of adequate infrastructures. EVs cannot be charged in normal gasoline stations and as long as the complementary EVs charging stations (EVCSs) are not available, consumers will be reluctant to buy EVs, regardless of their price or of the subsidies provided to them.

EVCSs would improve the possibility for consumers to perceive EVs and GVs are substitutes. On the contrary, low availability of charging infrastructures could hinder EV adoption, which could in turn reduce incentives to invest in charging infrastructures development. It has also to be taken into account that the charging time of EVs is significantly long (up to an hour) and this implies that they should be strategically located in a way that consumers do not perceive charging as a waste of time. Investment in EVCSs (whose costs range from 1000 to 4000 Swiss franc excluding installation, which is about double of the price) are undertaken by governments but also by private actors such as the EV manufacturers (e.g. Tesla and BMW) in order to favor the adoptions of their products. Additional actors investing in EVCSs can be identified in shopping malls and hotels, willing to strengthen their sustainability credentials, and in hospitals and schools for healthy reasons [49]; the latter group of actors will be the focus of this work. Measures have been adopted in many countries to increase the availability of EVCSs hoping that this would stimulate the EV market; by way of example in 2009 and 2010, the American Recovery and Reinvestment Act allotted \$100 million to build charging stations. However, the number of EVCSs is still limited and this is mostly due to the small share of EVs circulating. This means that the relationship between EVs and EVCSs can be depicted as a 'chicken-egg' problem [16], as common for goods that are characterized by complementarity and network externalities: as the number of EVCSs increases,the value of EVs is enhanced. This leads to more electric vehicles sales, which increase demand for charging stations and hence their profitability. However, we cannot tell which of the two events come first. Despite the lack of information on causality, there is empirical evidence that the growth trend of EVs and charging stations has strong temporal and geographical couplings, which are a manifestation of a cross network effect [76].

In this paper, we model the issue at hand within the framework of a two-sided market with indirect network externalities [4], in which a platform provides EVs and GVs to consumers (first side of the market) and EVCSs to investors (second side). We use this setup to analyze the potential route of decarbonizing the transport sector through the adoption of EVs. Two-sided markets are particularly suited to capture this problem because they allow us to model explicitly the indirect network externalities across the two different sides of the market (consumers and investors in EVCSs). EV owners value the existing charging station network, and charging providers value the circulating base of EVs.

Two sided markets are characterized by three elements [63]. First, two-sided markets are characterized by the presence of a platform providing distinct services to two or more distinct groups of consumers, which rely on the platform to intermediate transaction between them. Members of one side can rely on one platform only (single-homing), or on multiple platforms at the same time (multi-homing). Second, indirect network externalities exists across or within groups of consumers: this refers to the idea that, other things being equal, it is better to be connected to a bigger network; in other words, one side's utility from participation depends not only on the value of the good itself (membership fee), but also on the number of users on the other side that the platform attracts (interaction value). Network externalities might emerge in a large variety of contexts and may be positive or negative depending on the circumstances. Positive networks effect can be found in the choice of location by firms, as argued in the New Economic Geography literature [8]. Road congestion and traffic jams are examples of negative networks effects. The interaction between the different sides gives rise to strong complementarities as emphasized by network economics. A network externality is different from the notion of complementary goods; when goods are complementary the consumers internalize the purchase decision of the complement good e.g. razor and blades; on the other hand, when network externalities are present, the externality of the purchase decision is not internalized. The platform can internalize the network effect as it recognizes that a larger network raises the users' willingness to pay and therefore its revenues. Third, two-sided markets are characterized by a price structure that is non neutral and platforms must design it so as to bring both sides on board. The allocation of the total price between the two sectors (the price structure) affects economic outcome additionally to the total price charged by the platform to the two sides of the market. Pricing to one side of the market depends not only on the demand and costs that those consumers bring but also on how their participation affects participation on the other side and the profit that is extracted from that participation. Prices on both sides of the market depend on the joint set of demand elasticities and marginal costs on each side. The platform can affect the volume of transactions by charging more to one side of the market and reducing the price paid by the other in an equal amount. Platforms can end up making little money on one side (or making losses as well) and recouping the costs on the other side [63]; indeed, in two-sided market, we can observe prices below marginal cost or even negative (e.g the selling for newspapers for free covering the losses with the money from advertisement). An essential condition for the existence of a non neutral price structure is that bargaining between both sides of a platform is impossible and therefore there is no possibility to pass through the price allocation through side payments between end users [64]. Indeed, a necessary (but not sufficient) condition for a market to be two-sided is that the Coase theorem [1] does not apply to the relation between the two sides of the market, that is, the price structure is not neutral.

Classical examples of two-sided markets are the newspaper market ([66], [28]), where a reader and an advertiser interact through a newspaper, representing a platform; software platforms, where a user buys an application developed by a developer on the platform [22]; credit or debit cards markets, where a cardholder settles a transaction with a seller through the payment card provider platform [5]; shopping malls, which represent a platform where stores and shoppers interact; video-games, where gamers buy a game developed by a seller and plays it using the console build buy the platform [18]; the compact disc (CD) player and CD title markets ([30],[62]).

In the present paper, we use the framework described above to investigate the following issues: (1) what is the role of network effects on the introduction of EVs considering a two-sided market framework; (2) which policies can foster the adoption of EVs in a two-sided market with network externalities; (3) whether the results we obtain change in a dynamic setting.

The baseline model presents the analysis for a monopolistic platform with constant marginal cost selling EVs and GVs to consumers and EVCSs to investors. The assumption of a monopolistic market structure [4] is made in order to study the two-sided market in the easiest market structure and to focus on the demand side. As future research, we will focus on more complex market structures. We specify quasilinear utility functions for consumers and investors and we derive the demand functions for EVs, GVs and EVCSs, which clearly show the role played by the network effect (between EVs and EVCSs) and of the substitution effect (between EVs and GVs). Network effects are only across (intergroup externalities) and not within (intragroup externalities) the two sides. By solving the maximization problem of the platform we find the optimal quantities and prices for each good, which depend on the parameters of own demands, but also of demands from the other sectors. We use this setup to study

[1] The Coase theorem states that if property rights are clearly established and tradeable, and if there are no transaction costs nor asymmetric information, the outcome of the negotiation between two (or several) parties will be Pareto efficient, even in the presence of externalities [65].

the impact of policy measures: we first focus on one-sided policies affecting demand or production of EVs, GVs and EVCSs respectively. We then analyze two-sided policies addressing GVs and EVCSs, assuming lump sum transfers and a balanced government budget. It emerges that it is important to account for the two-sided character of the EVs market and for the presence of network effects in the EV industry, when designing incentives to foster the adoption of EVs. Indeed, for a given level of government spending which side is being subsidized, the buyers or the investors, has an impact on economic outcomes and the efficiency of the policy depends on key vehicles and EVCSs demands structural parameters. This result is a consequence of the failure of price neutrality characterizing the framework, that carries over to the application of subsidies. Moreover, we find that it is no equivalent to subsidize demand or production of the goods on each side of the market.

We then study the model in a dynamic framework, assuming that the platform has incomplete knowledge of the demand functions it faces. The bounded rationality assumption follows from the fact that the market for EVs and EVCSs are relatively new and that the effect that they will have on the established market for GVs is also unknown ([33], [59]). Moreover, given the presence of network effects, the amount of information the platform needs to collect is not a trivial requirement to satisfy: not only it needs the usual information related to the elasticity of demand, but also additional information about the strength of the network effect linking demands on the different sides. As the platform does not know the demand function, it uses a rule of the thumb, the gradient rule, to choose the prices and quantities that maximize its profits. The monopolist looks at the change in the profits and decides quantities accordingly: if the change in profits is positive, it moves the quantities in the same direction; if negative, it moves the quantities in the opposite direction; if the change is null, the platform will not change the quantities. This strategy represents an attempt to learn the willingness to pay of consumers and their elasticity to the network effect. We obtain the standard result that the critical points of the static model are the steady states of the dynamic model but not viceversa, and we prove the stability of the system for some values of the parameters. We then study how moving key parameters of the model affects the stability of the system and we find that very high network effects as well as high values of the subsidy to EVCSs or taxes to GVs destabilize the system.

1.1 Related literature

The paper lies at the intersection between the literature on the effect of environmental policies in the automobile market and technical change, the literature on two-sided and network effect, also with the introduction of environmental policies and the literature on unknown demand.

There is a rich body of research studying the effect of environmental policies in the automobile market. Many studies focus on the effectiveness of fuel taxes and fuel standards as a response to environmental issues related to the transportation sector ([42], [36], [21]). [34] assess California's Zero Emission Vehicles (ZEV) program that requires auto manufacturers to produce a certain percentage of zero emissions vehicles. Another policy approach is to establish eco-friendly rules like the Corporate Fuel Economy (CAFE) standard that led to a 50% reduction of fuel consumption per passenger car mile [35]. Other recent studies investigate policies targeting hybrid vehicles and the response of consumers to subsides to adopt EVs or to install charging stations ([70], [50], [47], [20], [77], [39], [59]) Also, policies targeting other alternative fuel vehicles are investigated ([52], [58]). The impacts of U.S. government programs both at the federal and state level in promoting the adoption of hybrid vehicles are examined in recent studies including [29], [68] and [67]. [40] show that there is substantial geographic variation in the environmental benefits of EV adoption and argue for spatially differentiated incentives. Apart from monetary incentives such as tax credits or direct subsidies, there exist other measures that could boost EVs sales or remove some obstacles to their purchases. These includes improvement of the charging station infrastructure [51], road tax exemptions, free use of bus line and parking areas. [44] analyses the effects of a wider range of policies on EV market share in the U.S. Their analysis examines the impact of policies that include free parking, carpool vehicle lane access, public chargers, subsidized home chargers, license fee reduction, on addition to subsidies. They find that the States in which EVs represent the largest share of the new vehicle market are not those with the highest incentives, and they rely significantly on policies other than direct subsidies, such as access to carpool lane and free parking. Moreover, sales are highly correlated with the percentage of people with income above US$ 100.000. [49] studies the effect of compatibility in the U.S. EV market in which three incompatible standards for charging stations are present. The paper shows that the impact of mandating compatibility standards has ambiguous effect on market outcome and welfare. [33] analyzes the market evolution for EVs in Germany until 2020, revealing a great uncertainty in the development due to external factors such as advancement of battery technologies, availability of charging infrastructures and on users' willingness to pay. This implies that policies to foster market diffusion of EVs should be dynamically adaptable to react to changing framework conditions. [15] estimates a nested multinomial logit model for the clean-fuel vehicles demand. Similar discrete decision model of vehicle is used in [14], [31] and [38] to study the preference of hybrid electric vehicle. It is pointed out that charging convenience is a major concern when consumers make the purchase. The characteristics of EVs adoption connects the issue to two broader strand of the literature, that are the one on externalities in the new technology market ([43], [23], [6], [13], [54]) and the one on directed technical change ([1], [3]). [43] examines the energy efficient technologies in buildings. [23] points out that the network externality is often observed such that the value of a unit of good increases with the number of units sold. [6] examines the outcomes of two competing new technologies under various conditions and shows that the new technology market gradually lock itself into an outcome that is not entirely predictable in advance. [13] characterizes the equilibrium and strategic behavior within each segment of the computer industry. More closely related to our work is [54] which is the first articles to explore the indirect network effects between alternative fuel vehicles and refueling stations. [1] develops an endogenous growth model to show that the optimal climate policy path includes both carbon taxes and research subsidies; [3] shows that firms in th automobile industry innovate more in alternative fuel technologies (electric, hybrid, hydrogen), when tax-inclusive fuel prices are higher.

Seminal papers in the literature on two-sided markets are [16], [24], [64], [65] and [4]. [63] firstly proposed a restrictive definition to distinguish between one-sided and two-sided markets in the context of charge per usage. [25] argues that a multi-sided platform has two or more group of consumers who need each other, but cannot capture the value of their interaction and relies on a catalyst to facilitate it. Similarly, [66] states that a multi-sided market exists when there is some kind of interdependence or externality between groups of agents that are served by an intermediary. [16], [65], [4], and [5] focus on pricing and the coordination issues typical in two-sided markets. Subsequent work, such as [73] and [74], generalized the modeling framework to examine different market structures and types of platforms. The theory of two-sided markets is related to the theories of network externalities and compatibility and of (monopoly or competitive) multi-product pricing. From the former, it borrows that there are non-internalized externalities among end-users; from the second, the focus on the price structure [64]. Positive network externalities arise when a user has more utility from a good when other users have the same good. [45] look at a case in which consumers value a product more highly when it is compatible with other consumer products. [26] also analyze a general model with network externalities. Firms choose whether to switch from an old technology to a new technology. The decisions of the firms are modeled as a multi-stage game in which one firm starts and the other firms follow sequentially. In another paper, [27] develop their ideas further, and introduce an installed base of the old technology. Due to the installed base, users of the old technology will adopt the new technology at a slow pace, depending on how fast the installed base depreciates. Early adopters of the new technology must then bear the cost of a small network while waiting for more consumers to adopt the new technology. [17] shows that the provision decision by software firms determines the value and hence the market share of competing hardware technologies. They show that in equilibrium either more hardware technologies are supported or there is standardization, with all consumers on the same network. As well the empirical literature on network externalities is growing, see for example [48], [32], and [12].

Following the increased awareness about environmental issues, environmental policies have started to be introduced in two-sided markets, taking into account the positive feedback loop between EV purchases and charging stations ([76], [71]). Our goal is to contribute to this strand of the literature. [76] considers a two-sided model with indirect network effects for EVs and EVCSs and studies how policies affect plug-in electric vehicle adoption. Differently from us, however, they consider a sequential games, assume perfect information and do not include dynamic analysis. A work close to our is [71], which studies the impact of network externality and subsidy structure on the diffusion of EVs in a two-sided market. However, differently from us, the paper considers a simultaneous move game based on complete knowledge of market demand and does not include the possibility of substitution with GVs. Both [76] and [71] conclude that each side of th market respond positively to each other and that subsidizing charging station entry is more cost effective in increasing EV sales. Our work adds to this literature by studying the growing industry of EVs in a two-sided market setting, introducing a wide array of policy measures, the dynamic decision of the platform and unknown demand.

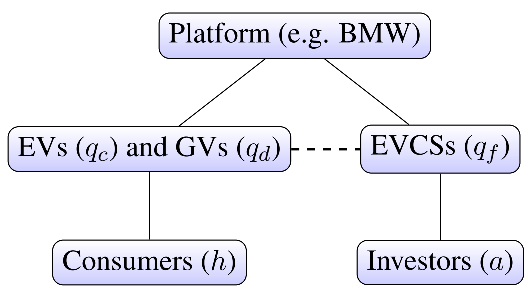
The paper borrows from the literature on firms' behavior when demand is unknown. [19] studies the learning processes and delayed responses when the monopolist has no information on the demand function. When firms have incomplete knowledge, they might try to extrapolate information using rules of the thumb [7]. In particular, they propose a rule of the thumb called gradient rule, which states that if by changing the quantity produced in one direction, firms observe an increase in the profits, they will change the quantity in the same direction in the next period. On the contrary, if the change in profits is negative, the firm will adjust the quantity in the opposite direction. If profits do not change, the monopolist will not change the quantity. This adjustment process has been applied to monopoly ([61], [56], [55]) and oligopoly ([60], [11], [10]) settings to show that the adjustments depends on the reactivity of firms to changes in profits. We follow this approach and we focus on the easiest way to show how quantities evolve, the one which requires the smaller information set, being aware that this is just an approximation of reality. To the best of our knowledge, rules of the thumb have not been applied to two-sided market with the only exception of [28], which studies a monopolist facing demand for advertising and readership in a two-sided market framework. We build on the model introducing an additional group of agents and we include the impact of sustitability in addition to the network effects.

EVs are seen by many as a way to decarbonize the transport sector. However, to the best of our knowledge, there exists, to date, little research which explores the ways in which both sides of the EVs market interact with each other and on which governmental efforts work best to advance electric vehicle sales. This paper tries to make progress in this area by explicitly modeling the equilibrium relationships between EVs adoption and EVCSs availability in a two-sided market framework. In the context of the EV industry, the platform can be thought of as the vehicles and charging stations manufacturer, while the two sides consist of buyers of EVs and GVs and electric charging station providers, which invest in the new technology. We focus on private cars as these dominate transport demand and fuel consumption in most countries, but the same structure applies to all forms of road transport and types of road vehicles. Our setup is close to [71], however our paper differs because we also include the production of dirty cars, we consider imperfect information and a dynamic setting. Similarly to [71], in our model the importance of the price structure emerges in the policy analysis: for a given level of a tax (subsidy), it does matter which side is charged for (benefit from) it; however, our richer setup allows us to consider a wider array of possible policies measures and the model allows us to compare policies in a static and in a dynamic framework. The subsidy to EVCSs increases the quantity of EVs and EVCSs but from the dynamic setup it emerges that when its value is very high it destabilizes the system. Similarly, very high values of the network destabilize the dynamic system from the static analysis we only obtained that the quantities of EVs and EVCSs are increasing with the subsidy. Differently from standard models on two-sided markets ([64], [4]), we introduce unknown demand functions for the firms. The monopolist uses the gradient rule to choose the quantities maximizing profits. The paper closer to our is [28], which studies a monopoly with unknown demands for advertising and readership. However, we add an additional demand function and we consider the presence of substitute other than complement goods. The main contribution of this work is to study indirect network effects in the EV market both in a static and dynamic setting including the substitution possibility with GVs and incomplete knowledge of the demand functions by the platform. Our analysis provides useful insights about the effects operating in the EVs market and their implications for policy making.

The paper is organized as follows. Section 2 develops the baseline static model with the platform setting quantities on both sides. We analyze the policy instruments available to favor the transition towards EVs in Section 3 and we compute the welfare maximizing policies. Section 4 studies the dynamic framework introducing bounded rationality and the rule of the thumb. Section 5 concludes and proposes some lines of future research.

2 A platform setting quantities on both sides

Our economy is populated by three sets of agents: consumers, denoted by h, investors, denoted by a, and a monopolistic platform. Consumers are agents willing to buy a car and we assume that all the consumers will buy one. Investors can be thought to as shopping malls, companies or hotels willing to boost their sustainability credentials, but also hospitals and schools for healthy reasons. Clearly, investors can also be the government or EV manufacturers, but this is not the focus of our work. The monopoly assumption is due to the fact that at the launch stage of EVCSs market investments are mainly conducted by few actors. Moreover, we want to study the easiest supply setup in order to focus on the demand side of the market. In a future work, we want to relax this assumption and study the impact of introducing other platform structures, such as an oligopoly. We consider a two-sided market. On the consumers' side, two competing technologies exist: EVs, c, and GVs, d. The two types of vehicles are substitutes and consumers can choose between them. As there is empirical evidence of the network effect on both the station side and the consumer side (Springel, 2016), we assume that consumers' decision is affected by the availability of EVCSs (we assume that the number of gasoline stations does not affect the decision as their supply is already sufficient to meet demand). Notice that EVs and EVCSs are not perfect complements - because owners of EVs could charge their vehicles at home - but they have some degree of complimentary. p_c and p_d are the purchase price for an EVs and a GVs. Notice that this price represent the lifetime cost of the cars. We do not consider the fuel cost because it can be included in the lifetime cost and adding it does not affect the results. On the investors' side, agents invest in EVCSs, f, and derive utility from the purchase of the infrastructure as they can sell the service afterwards. Investors demand charging stations and their decision is affected by the number of EVs as a higher number will increase their profits (Springel, 2016). There is a continuum of potential users on each side with mass normalized to 1. p_f is the price investors pay to the platform producing EVCSs. This price does not only represent the cost of buying a charging station, but also the maintenance and operating costs. As for the fuel cost, we do not consider this additional cost because it does not affect the results.



Given the framework described above, we have two utility functions. First, the utility function of consumers, which depends on EVs, GVs and on the quantity of EVCSs; the latter enters the utility function of consumers because, even though they do not optimize for it, it affects the quantity of EVs chosen. We assume that all the consumers buy a car of one of the two types. Second, the utility function for investors, which depends on charging stations, but also of the number of EVs, which they do not optimize for. Following [37] and [53], we assume quasi-linear utility functions; however, we assume different coefficients for the interaction terms instead of having a unique one.

The utility function of consumers is

$$U_h(q_c, q_d, q_f) = q_0 + \sum_i \alpha_i q_i - \frac{1}{2}\left[\sum_i \beta_i q_i^2 + 2(\gamma_1 q_c q_d + \gamma_2 q_c q_f + \gamma_3 q_d q_f)\right] \tag{1}$$

where $q_0 > 0$ is the individual consumption level of the numeraire good. $q_i \geq 0$ with $i = \{c, d, f\}$ is the individual consumption level of EVs, GVs and EVCSs respectively. Each agent on one side derives an inherent fixed benefit α_i, which represents the size of market i, independently from the number of agents on the other side. The market size together with the utility derived from the consumption of a good by itself represent a stand alone benefit, which come from the immediate use of the good. β_i represents the slope of the inverse demand function for each good, $\beta_d > 0$. Also we assume the the slope of demand is the same for EVs and GVs, so that $\beta_c = \beta_d$. $\gamma_1 \in (0, 1)$ is a parameter capturing the substitution effect between EVs and GVs whereas $\gamma_2 \in (-1, 0)$ represents the complementarity between EVs and EVCSs; each agent buying an EV enjoys a net transaction benefit (γ_2) for every agent that joins the platform on the other side (each investor in EVCSs). This term represents the network benefit. (1) shows that an increase in the quantity of EVCSs increases the utility of consumers even if consumers do not choose the quantity of EVCSs; hence, there is an indirect network effect between these goods. We also assume that there is no direct interaction between q_f and q_d ($\gamma_3 = 0$): the impact of EVCSs on the quantity of GVs only realizes through the quantity of EVs. The budget constraint of consumers is

$$q_0 + p_c q_c + p_d q_d = m_c$$

meaning that, given total income on the consumers' side (m_c) a share of it is allocated to the purchase of the numeraire good, a share to the purchase of EVs and a share to the purchase of GVs. The price of the numeraire good is normalized to 1. The quasi-linear specification implies that there are no income effects so that the shadow value of income is 1.

The utility function of investors is

$$U_a(q_c, q_f) = q_0 + \sum_j \alpha_j q_j - \frac{1}{2}\left[\sum_j \beta_j q_j^2 + 2\gamma_4 q_c q_f\right] \tag{2}$$

where q_0 is the consumption level of the numeraire good. q_j with $j = \{c, f\}$ is the consumption level of EVs and EVCSs. Since EVs are complementary to EVCSs, we include the quantity of EVs in the utility function of investors. However, we assume that for investors the degree of complementary between EVs and charging stations (γ_4) is different than for consumers (γ_2). We also assume that $\gamma_2 > \gamma_4$ meaning that the network effect is stronger for investors than for consumers. This assumption follows from an asymmetric information argument: investors have higher ability to analyze the market before making their decisions; they have perfect information and they can determine the total number of EVs. Moreover, it does not make sense for them to invest in EVCSs if there are no EVs as they would make losses. On the other hand, consumers have less information and they mostly care of the number of EVCSs in their neighborhood. Also, they have the option to charge their EVs at home, so the network effect is weaker for them. The different intensity of the network effects plays a crucial role in the analysis, particularly with regard to the effect of the policies.
The budget constraint of investors is

$$q_0 + p_f q_f = m_a$$

meaning that, given total income on the investors' side (m_c) a share of it is allocated to the purchase of the numeraire good and a share to the purchase of EVCSs.

Solving the consumers' and investors' problem, the FOCs are

$$
\begin{aligned}
q_0 &: \quad \lambda - 1 = 0 \\
q_c &: \quad \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f - \lambda p_c = 0 \\
q_d &: \quad \alpha_d - \beta_d q_d - \gamma_1 q_c - \lambda p_d = 0 \\
q_f &: \quad \alpha_f - \beta_f q_f - \gamma_4 q_c - \lambda p_f = 0
\end{aligned} \tag{3}
$$

Hence we find the demand functions for EVs, GVs and EVCSs

$$
\begin{aligned}
q_c &= \frac{\alpha_c}{\beta_c} - \frac{\gamma_1}{\beta_c} q_d - \frac{\gamma_2}{\beta_c} q_f - \frac{p_c}{\beta_c} \\
q_d &= \frac{\alpha_d}{\beta_d} - \frac{\gamma_1}{\beta_d} q_c - \frac{p_d}{\beta_d} \\
q_f &= \frac{\alpha_f}{\beta_f} - \frac{\gamma_4}{\beta_f} q_c - \frac{p_f}{\beta_f}
\end{aligned} \tag{4}
$$

Demand for EVs is linear in the quantity of GVs and EVCSs and in its own price. The linear specification follows from the choice of a quasi-linear utility function. From (4) we can see that the quantity of GVs has a negative impact on the demand for EVs as the two types of good are substitutes. On the opposite, the quantity of EVCSs increase the demand for EVs because of the network effect (captured by γ_2) and similarly the quantity of EVs increases the demand for EVCSs (captured by γ_4). The positive network externality arises in the context of EVs due to complementarities: if the number of stations increases for some exogenous reason, then demand for EVs increases. This leads to a further increase in the number of charging points, and so on. The positive feedback loop between new EV sales and charging station suggests that a small change on either side can lead to a large change in both EVs purchases and investments in EVCSs. From the demand functions, inverse demands are

$$
\begin{aligned}
p_c &= \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f \\
p_d &= \alpha_d - \beta_d q_d - \gamma_1 q_c \\
p_f &= \alpha_f - \beta_f q_f - \gamma_4 q_c
\end{aligned} \tag{5}
$$

which show that the βs represent the slopes of the inverse demand functions and, again, the role of the network effects γ_2 and γ_4.

2.1 Static profit maximization

In two-sided markets, the platform chooses the profit maximizing quantities and prices given the interrelated demands of two or more groups of customers. In our model, decisions of car manufacturer are not explicitly modeled as we want to focus on the demand side.

We assume that the platform incurs a constant marginal cost c_c and c_d on the consumer side (marginal cost of car manufacturing), while the marginal cost on the investor side is c_f. Hence, total costs of production are $TC = c_c q_c + c_d q_d + c_f q_f$.

Profits are given by

$$\pi = (p_c - c_c)q_c + (p_d - c_d)q_d + (p_f - c_f)q_f \tag{6}$$

where the first two terms represent profit from buyers/consumers and the third term is profit from investors in EVCSs. The FOCs of the maximization problem are a set a simultaneous equations which take into account impact of the demand from the other groups. The simultaneity is a direct result of the dependence of demand by members of one group on the demand by members of another group.

$$\begin{aligned} \frac{\partial \pi}{\partial q_c} &: \alpha_c - 2\beta_c q_c - 2\gamma_1 q_d - (\gamma_2 + \gamma_4) q_f - c_c = 0 \\ \frac{\partial \pi}{\partial q_d} &: \alpha_d - 2\beta_d q_d - 2\gamma_1 q_c - c_d = 0 \\ \frac{\partial \pi}{\partial q_f} &: \alpha_f - 2\beta_f q_f - (\gamma_2 + \gamma_4) q_c - c_f = 0 \end{aligned} \tag{7}$$

By solving the system, we find the profit maximizing quantities

$$\begin{aligned} q_c^* &= \frac{1}{X}\left[2\beta_d\beta_f(\alpha_c - c_c) - 2\beta_f\gamma_1(\alpha_d - c_d) - \beta_d(\gamma_2+\gamma_4)(\alpha_f - c_f)\right] \\ q_d^* &= \frac{1}{2X}\left[-4\beta_f\gamma_1(\alpha_c - c_c) + [4\beta_c\beta_f - (\gamma_2+\gamma_4)^2](\alpha_d - c_d) + 2\gamma_1(\gamma_2+\gamma_4)(\alpha_f - c_f)\right] \\ q_f^* &= \frac{1}{X}\left[-\beta_d(\gamma_2+\gamma_4)(\alpha_c - c_c) + \gamma_1(\gamma_2+\gamma_4)(\alpha_d - c_d) + 2(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right] \end{aligned} \tag{8}$$

where $X = 4\beta_c\beta_d\beta_f - 4\beta_f\gamma_1^2 - \beta_d(\gamma_2+\gamma_4)^2$. We assume that the conditions $q_c^* \geq 0$, $q_d^* \geq 0$ and $q_f^* \geq 0$ are always satisfied, which implies $X > 0$. As we can see from the equations above, an increase in the market size of EVs, α_c, increases the optimal quantity of EVs and of EVCSs, but reduces the optimal quantity of GVs as they are substitutes. An increase in the market size of EVCSs, α_c, has the same effect, whereas an increase in the market size of GVs has the opposite effect. Notice that the impact on a specific quantity of the parameters of the other demands is always mediated by one of the interaction terms γ_1, γ_2 and γ_4, but for the impact of the market size and cost of EVs on its own quantity. The network effects have a positive impact on the quantity of EVs and a negative impact on the quantity of GVs. However, the impact on EVCSs is positive only if $\beta_d > \gamma_1$. The substitution effect, or its combination with the network effects, has a negative impact on all the quantities.

Given the optimal quantities in (8), we can find the price maximizing profits as

$$\begin{aligned} p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c) \\ &\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)] \\ p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\ p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\ &\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_2^2\alpha_f + \gamma_4^2 c_f)] \end{aligned} \tag{9}$$

Because of the network effects, the prices of EVs and EVCSs depend on the parameters of demands from the other side of the market. Moreover, prices of EVs and EVCSs also depend on the parameters of demand for GVs. The price of GVs, differently from its quantity - which also depends on the parameters of the demands for the other goods - only depends on the parameters of its own demand. Notice that if $\gamma_2 = \gamma_4$ demands for EVs and EVCSs would as well depend on the parameters of their own demand only.

3 Policies

In this section, we analyze different policy measures that can be adopted to foster the energy transition towards EVs. We adopt the number of EVs as the signal of welfare and hence the model can be used to analyze the consumer welfare impact of the EV incentives. This is in line with the current policy strategy of setting specific targets for EV sales in a given time period. We investigate the impact that subsidies or taxes to different sides of the market have on the diffusion of EVs; in a two-sided market with network externalities subsidies are non-neutral and we can determine which side of the market is more efficient to subsidize depending on key vehicle demand and charging station supply parameters. We conclude that policies should take into account the presence of network effects when designing the energy transition. As we said before, we assume that $\gamma_2 > \gamma_4$, i.e. that the intensity of the network effect is stronger for investors than for consumers; this implies that the investors care more about the availability of EVs than what consumers do about the number of EVCSs.

First, we consider one-sided policies addressing demand or production (i.e consumers or the platform); then, we focus on two-sided policies addressing GVs and EVCSs, assuming lump sum transfers and balanced government budget.

In the considerations below, we do not take into account the denominator X, affecting the impacts of all the policies. Since all the effects are normalized by this term we consider it as a standardization factor.

3.1 Subsidy to production of EVs only

If a revenue subsidy is provided to the production of EVs, the profit of the platform from selling EVs are increased, as we include the value of the credit as an additive term in the static profit function. The optimal quantities become

$$\begin{aligned}
q_c^* &= \frac{1}{X}\left[2\beta_d\beta_f(\alpha_c + s_c - c_c) - 2\beta_f\gamma_1(\alpha_d - c_d) - \beta_d(\gamma_2+\gamma_4)(\alpha_f - c_f)\right] \qquad (10)\\
q_d^* &= \frac{1}{2X}\left[-4\beta_f\gamma_1(\alpha_c + s_c - c_c) + [4\beta_c\beta_f - (\gamma_2+\gamma_4)^2](\alpha_d - c_d) + 2\gamma_1(\gamma_2+\gamma_4)(\alpha_f - c_f)\right]\\
q_f^* &= \frac{1}{X}\left[-\beta_d(\gamma_2+\gamma_4)(\alpha_c + s_c - c_c) + \gamma_1(\gamma_2+\gamma_4)(\alpha_d - c_d) + 2(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right]
\end{aligned}$$

The effect on the quantity of EVs of a subsidy to production of EVs is not mediated by the network effects but only by the slope of the inverse demand for GVs and EVCSs. On the other hand, the effect on the quantity of GVs is affected by the substitution parameter (γ_1) and by the slope of the inverse demand for EVCSs; as for the impact on the number of EVCSs, the parameters capturing the network effects (γ_2 and γ_4) shape the impact of the subsidy together with the slope of the inverse demand function for GVs. Specifically, the effects are

$$\begin{aligned} \frac{\partial q_c^*}{\partial s_c} &= \frac{2\beta_d\beta_f}{X} > 0 \\ \frac{\partial q_d^*}{\partial s_c} &= -\frac{4\beta_f\gamma_1}{2X} < 0 \\ \frac{\partial q_f^*}{\partial s_c} &= -\frac{\beta_d(\gamma_2+\gamma_4)}{X} > 0 \end{aligned} \tag{11}$$

showing that the subsidy increases both the quantities of EVs and of EVCSs, whereas it decreases the quantity of GVs. Notice that larger slopes of the inverse demand functions for GVs and EVCSs imply that the quantity of EVs sales is more reactive to the subsidy. Similarly, the quantity of EVCSs is more reactive if the slope of the inverse demand function for EVCSs is larger and if the network effects are stronger.

The optimal prices when the subsidy is in place are

$$\begin{aligned} p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c - s_c + c_c) - 2\beta_d[\gamma_4^2\alpha_c + \gamma_2^2(c_c - s_c)]] \\ &\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)] \\ p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\ p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c + s_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\ &\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_2^2\alpha_f + \gamma_4^2 c_f)] \end{aligned} \tag{12}$$

Notice that the effect of the subsidy to production of EVs on the price of EVs is affected by the slopes of the three demand functions and of the substitution and network parameters. The price of GVs only depends on of its own market size and marginal cost and it is not affected by the subsidy. The effect of the subsidy on the price of EVCSs depends on the network intensities and on the slope of the inverse demand for GVs and EVCSs.

The impacts of the policy on the optimal prices are given by

$$\begin{aligned} \frac{\partial p_c^*}{\partial s_c} &= -\frac{2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4 - 2\beta_d\gamma_2^2}{X} < 0 \\ \frac{\partial p_d^*}{\partial s_c} &= 0 \\ \frac{\partial p_f^*}{\partial s_c} &= \frac{\beta_d\beta_f(\gamma_2 - \gamma_4)}{X} > 0 \end{aligned} \tag{13}$$

showing that the price of EVs is decreased by the subsidy, whereas the one of EVCSs is increased. The price of GVs is not affected by the subsidy. Whether the reduction in the price of EVs is smaller or larger than the increase in the price of EVCSs depends on the values of the parameters.

3.2 Subsidy to demand of EVs

A price subsidy directly affects the buyers' vehicle purchasing decision by making the high purchase cost of EVs comparable to (or even lower than) the purchase cost of a GVs. If a subsidy is provided to the demand for clean cars only, the optimal quantities are as above, whereas the optimal prices

$$\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + s_c + c_c) - 2\beta_d[\gamma_4^2(\alpha_c + s_c) + \gamma_2^2 c_c]] \\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)] \\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c + s_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_2^2\alpha_f + \gamma_4^2 c_f)]
\end{aligned} \tag{14}$$

The coefficient capturing the impact of the subsidy on the price of EVs differs from the previous case because of the relative relevance of the network effect (now the network effect on the investors' side plays a more important role) and of the opposite sign. The impact on the price of GVs is null as before, whereas the on the price of EVCSs is identical to the previous case.

The impacts of the policy on the optimal prices are given by

$$\begin{aligned}
\frac{\partial p_c^*}{\partial s_c} &= \frac{2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4 - 2\beta_d\gamma_4^2}{X} > 0 \\
\frac{\partial p_d^*}{\partial s_c} &= 0 \\
\frac{\partial p_f^*}{\partial s_c} &= \frac{\beta_d\beta_f(\gamma_2 - \gamma_4)}{X} > 0
\end{aligned} \tag{15}$$

showing that both the price of EVs and of EVCSs are increased by the subsidy to demand of EVs. Whether the increase in p_c is larger or smaller than the increase in p_f depends on the value of the parameters; however, we observe that this policy is less effective than a subsidy to production of EVs because it increases the price of p_c other than increasing the price of p_f by the same amount as before. Moreover, as we assumed $\gamma_2 > \gamma_4$,the increase in p_c caused by a subsidy to demand of EVs is larger than the decrease in p_c caused by a subsidy to production of EVs.

3.3 Taxes on production of GVs

If a tax is imposed on the production of dirty cars, the profit of the platform from selling GVs are reduced. The optimal quantities become

$$
\begin{aligned}
q_c^* &= \frac{1}{X}\left[2\beta_d\beta_f(\alpha_c - c_c) - 2\beta_f\gamma_1(\alpha_d - s_d - c_d) - \beta_d(\gamma_2+\gamma_4)(\alpha_f - c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-4\beta_f\gamma_1(\alpha_c - c_c) + [4\beta_c\beta_f - (\gamma_2+\gamma_4)^2](\alpha_d - s_d - c_d) + 2\gamma_1(\gamma_2+\gamma_4)(\alpha_f - c_f)\right] \\
q_f^* &= \frac{1}{X}\left[-\beta_d(\gamma_2+\gamma_4)(\alpha_c - c_c) + \gamma_1(\gamma_2+\gamma_4)(\alpha_d - s_d - c_d) + 2(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right]
\end{aligned} \tag{16}
$$

which show that the impact of the tax on production of GVs on the quantity of EVs is affected by the degree of substitution between EVs and GVs and by the slope of the inverse demand function for EVCSs. As for the quantity of GVs, the relevant parameters influencing the impact of the tax are the network effects and the slope of the inverse demand for EVs and EVCSs. Considering the impact on the quantity of EVCSs, the impact of the tax depends on the parameters capturing the network and substitution effects.

Specifically, the impacts of the policy on the optimal quantities are given by

$$
\begin{aligned}
\frac{\partial q_c^*}{\partial s_d} &= \frac{2\beta_f\gamma_1}{X} > 0 \\
\frac{\partial q_d^*}{\partial s_d} &= \frac{4\beta_c\beta_f - (\gamma_2+\gamma_4)^2}{2X} < 0 \\
\frac{\partial q_f^*}{\partial s_d} &= -\frac{\gamma_1(\gamma_2+\gamma_4)}{X} > 0
\end{aligned} \tag{17}
$$

which shows that the tax on production of GVs increases the quantity of EVs and EVCSs, whereas it decreases the quantity of GVs. The impact on the quantity of EVs is higher the stronger the substitution effect and the slope of the inverse demand function for EVCSs. Moreover, for $\beta_d > \gamma_1$, Δq_c is larger when the subsidy is provided to EVs rather than when a tax is imposed to GVs. As for q_d the impact is larger the larger the slope of the inverse demand for EVs and EVCSs and the difference between the two network effects. Moreover, for $\beta_c > \gamma_1$, Δq_f (reduction) is lower when the subsidy is provided to EVs rather than when a tax is imposed to GVs. The impact on the quantity of EVCSs is larger the larger the difference in the intensity between the two network effects and the stronger the substitution effect. Moreover, for $\beta_d > \gamma_1$, Δq_f is larger when the subsidy is provided to EVs rather than when a tax is imposed to GVs.

The optimal prices are

$$
\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)] \\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - s_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)] \\
p_d^* &= \frac{1}{2}(\alpha_d + s_d + c_d) \\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - s_d - c_d) \\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_2^2\alpha_f + \gamma_4^2 c_f)]
\end{aligned} \tag{18}
$$

Considering the price of EVs, the impact of the tax is mediated by the network and substitution parameters. The price of GVs is affected directly by the tax, with no role played by any parameter. Finally, the impact on the price of EVCSs depends on the slope of its inverse demand curve and on the substitution and network parameters.

Specifically, the impacts of the policy on the optimal prices are given by

$$\begin{aligned}
\frac{\partial p_c^*}{\partial s_d} &= \frac{\gamma_1(\gamma_2^2 - \gamma_4^2)}{2X} < 0 \\
\frac{\partial p_d^*}{\partial s_d} &= \frac{1}{2} > 0 \\
\frac{\partial p_f^*}{\partial s_d} &= \frac{\beta_f \gamma_1(\gamma_2 - \gamma_4)}{X} > 0
\end{aligned} \tag{19}$$

showing that the price of EVs is decreased whereas the prices of GVs and EVCSs are increased. The negative effect on the price of EVs is stronger the larger the difference between the intensities of the two network effects and the larger the sostitutability parameter. However, we are not able to draw conclusions about the change in the price of EVs in this case as compared to a subsidy to production of EVs. The tax on production increases the price of dirty cars but only by one half of its value as the monopolist is able to pass half on the increase in the price on to consumers. The impact on the price of GVs is larger when a tax is applied to production of GVs rather than when a subsidy is provided to EVs, as before the impact was null. The positive effect on the price of EVCSs is stronger the larger the difference between the intensities of the two network effects and the larger the sostitutability parameter and the slope of the inverse demand function for EVCSs. Moreover, for $\beta_d > \gamma_1$, Δp_f is larger when the subsidy is provided to EVs rather than when a tax is imposed to GVs.

3.4 Taxes on demand of dirty cars

If a tax is imposed on the demand for dirty cars only, the optimal quantities are as before, whereas the optimal prices

$$\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)] \\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - s_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)] \\
p_d^* &= \frac{1}{2}(\alpha_d - s_d + c_d) \\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - s_d - c_d) \\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_2^2\alpha_f + \gamma_4^2 c_f)]
\end{aligned} \tag{20}$$

When the tax is imposed on demand instead that on production, the impact on the price of EVs and of EVCSs is as before. The same hold for the price of GVs although the sign is reversed.

Specifically, the impacts of the policy on the optimal prices are given by

$$\begin{aligned}
\frac{\partial p_c^*}{\partial s_d} &= \frac{\gamma_1(\gamma_2^2 - \gamma_4^2)}{2X} < 0 \\
\frac{\partial p_d^*}{\partial s_d} &= -\frac{1}{2} < 0 \\
\frac{\partial p_f^*}{\partial s_d} &= \frac{\beta_f \gamma_1(\gamma_2 - \gamma_4)}{X} > 0
\end{aligned} \tag{21}$$

which shows that the monopolist has to reduce the price of GVs because of the reduction in demand due to the tax, even though only by one half of the tax. The impact of the tax on the prices of EVs and EVCSs is the same regardless on whether it applies to production or demand. Hence, the same comments of the previous paragraph applies, but for the price of GVs: the impact on the price of GVs is reduced more when a tax is applied to demand of GVs rather than when a subsidy is provided to EVs, as before the impact was null.

3.5 Subsidy to production of charging stations

Subsidies to EVCSs can eliminate the problem of electricity availability through the development of the charging infrastructure. Removing this crucial barrier to the EV industry can indirectly increase buyers demand for EVs. If a subsidy is provided to production of charging station, the optimal quantities are

$$\begin{aligned}
q_c^* &= \frac{1}{X}\left[2\beta_d\beta_f(\alpha_c - c_c) - 2\beta_f\gamma_1(\alpha_d - c_d) - \beta_d(\gamma_2+\gamma_4)(\alpha_f + s_f - c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-4\beta_f\gamma_1(\alpha_c - c_c) + [4\beta_c\beta_f - (\gamma_2+\gamma_4)^2](\alpha_d - c_d) + 2\gamma_1(\gamma_2+\gamma_4)(\alpha_f + s_f - c_f)\right] \\
q_f^* &= \frac{1}{X}\left[-\beta_d(\gamma_2+\gamma_4)(\alpha_c - c_c) + \gamma_1(\gamma_2+\gamma_4)(\alpha_d - c_d) + 2(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)\right]
\end{aligned} \tag{22}$$

which show that the impact of the subsidy to production of EVCSs on the quantity of EVs depend on the slope of the inverse demand for GVs and on the network effects. The impact of q_d, instead, depends on the parameters capturing the substitution and the network effects. As for q_f the impact is mediated by the slopes of the inverse demand functions for EVs and GVs and by the substitution parameter. Specifically, the impacts of the policy on the optimal quantities are given by

$$\begin{aligned}
\frac{\partial q_c^*}{\partial s_f} &= -\frac{\beta_d(\gamma_2+\gamma_4)}{X} > 0 \\
\frac{\partial q_d^*}{\partial s_f} &= \frac{\gamma_1(\gamma_2+\gamma_4)}{X} < 0 \\
\frac{\partial q_f^*}{\partial s_f} &= \frac{2(\beta_c\beta_d - \gamma_1)}{X} > 0
\end{aligned} \tag{23}$$

which show that both EVs purchases and charging stations investments are positively related to subsidies to production of EVCSs, whereas this policy reduces the quantity of GVs. As in Springel

(2016), we obtain that positive feedback loops between EVCSs and EVs sales amplify the impact of subsidies on both sides of the market in terms of quantities. Notice that the magnitude and the sign of the impact of a subsidy to EVCSs on the quantity of EVs are the same of the impact of a subsidy to EVs on the quantity of EVCSs. Similarly, the magnitude and impact of a subsidy to EVCSs on the quantity of GVs are the same of the impact of a tax to GVs on the quantity of EVCSs.

Optimal prices are

$$\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)] \\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)] \\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f - s_f + c_f) - \beta_d[\gamma_2^2\alpha_f + \gamma_4^2(c_f - s_f)]]
\end{aligned} \tag{24}$$

The effect on the price of EVs is affected by the sostitutability and network effects but also by the slope of the inverse demand for EVs and GVs. There is no impact of the subsidy on the price of GVs, whereas the impact on the price of EVCSs depends on all the parameters of the model.

Specifically, the impacts of the policy on the optimal prices are given by

$$\begin{aligned}
\frac{\partial p_c^*}{\partial s_f} &= \frac{(\beta_c\beta_d - \gamma_1^2)(\gamma_2 - \gamma_4)}{X} > 0 \\
\frac{\partial p_d^*}{\partial s_f} &= 0 \\
\frac{\partial p_f^*}{\partial s_f} &= \frac{2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4 - \beta_d\gamma_4^2}{X} > 0
\end{aligned} \tag{25}$$

As for the impact on the price of EVs a larger difference between the intensities of the two network effects generates a higher increase in the price. Larger slopes of the inverse demand functions for EVs and GVs generate the same impact, whereas an in crease in the sostitutability effect reduces the increase in the price. As for the price of EVCSs, we observe that the impact of a subsidy to production of EVCSs on p_f is the same as the impact of a subsidy to demand of EVs on the price of EVs.

3.6 Subsidy to demand of charging stations

The government can subsidize the provision of charging stations by investors in order to generate a positive externality on EVs consumption (through the network effect). Subsidies can be provided to charging station investors for purchasing and installing charging equipment. When a subsidy is provided to the demand for EVCSs by investors, the optimal quantities are as in the case of subsidizing production; however, the optimal prices become

$$
\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)] \\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)] \\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + s_f + c_f) - \beta_d[\gamma_2^2(\alpha_f + s_f) + \gamma_4^2 c_f]]
\end{aligned} \tag{26}
$$

When demand of charging stations is subsidized instead of production, the impact on the price of EVs is as before and there is no impact on the price of GVs. However, the parameter that now plays a major role in shaping the impact of the subsidy on the price of EVCSs is the network effect on the consumer side and not the investors side as when production of EVCSs was subsidized.

Specifically, the impacts of the policy on the optimal prices are given by

$$
\begin{aligned}
\frac{\partial p_c^*}{\partial s_f} &= \frac{(\beta_c\beta_d - \gamma_1^2)(\gamma_2 - \gamma_4)}{X} > 0 \\
\frac{\partial p_d^*}{\partial s_f} &= 0 \\
\frac{\partial p_f^*}{\partial s_f} &= -\frac{2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4 - \beta_d\gamma_2^2}{X} < 0
\end{aligned} \tag{27}
$$

which shows that the impact of the subsidy on the price of clean cars is the same regardless of whether demand or production of charging stations is subsidized. However, the impact on the price of charging stations differs because the network effect that now plays a major role in shaping the impact is the one on the consumers side. Specifically, the effect of a subsidy to demand is negative and smaller than the positive effect that a subsidy to production has on p_f, as we assumed $\gamma_2 > \gamma_4$. Notice that the impact that a subsidy to demand of EVCSs has on the price of EVCSs is the same that a subsidy to production of EVs has on the price of EVs.

It appears that, in line with the literature on two-sided markets, subsidies are non-neutral, in the sense that results are different if one side (consumers) or the other (investors) is subsidized. In general, the effectiveness of policy on one side of the market is closely tied to the importance that agents on the other side place on the operating network effect. We can also see that the slopes of the inverse demand functions (as captured by the βs) play a major role in the determination of the impact of the policies. A larger slope means that demand less elastic and larger values of β amplifies the effects of the other parameters.

The results above can be summarized in the following table

One-sided policies have clear-cut effects on quantities and the direction of the impact is the expected one. As for prices, the results deserve some additional comments.

First, the price of EVs is reduced by a subsidy to EVs production, whereas the impact of a subsidy to demand is positive. This follows from the fact that if production is subsidized, the monopolist is able to benefit from the reduction in costs and to charge a lower price. On the contrary, if demand is

Table 1: One-sided policies

		EVs		GVs		EVCSs	
		Δq_c	Δp_c	Δq_d	Δp_d	Δq_f	Δp_f
EVs sub.	prod.	$+$	$-$	$-$	0	$+$	$+$
	dem.	$+$	$+$	$-$	0	$+$	$+$
GVs tax	prod.	$+$	$-$	$-$	$+$	$+$	$+$
	dem.	$+$	$-$	$-$	$-$	$+$	$+$
EVCSs sub.	prod.	$+$	$+$	$-$	0	$+$	$+$
	dem.	$+$	$+$	$-$	0	$+$	$-$

subsidized, it increases, and the monopolists can charge a higher price. Taxes on both production and demand for GVs decrease the price of EVs. On the contrary, a subsidy to production or demand for EVCSs increases the price of EVs.

The price of GVs is only affected by policies addressing the GV sector directly, even though the quantity of GVs is affected by the parameters of the demand from the other two sectors because of substitution with EVs and, indirectly, because of the network effect between EVs and EVCSs. Notice that when production is taxed, the price increases because of the increased costs; however, we can see that the price is increased only by one half of the value of the tax, because the monopolist is able to pass half on the increased costs on to consumers. On the contrary, when demand is taxed, the quantity that the monopolist is able to sell is lower; hence it has to reduce the price.

Finally, the price of EVCSs: assuming $\gamma_2 > \gamma_4$, p_f increases with the subsidies to both production and demand of EVs. Similarly, a tax on GVs production and demand increases the price of EVCSs. Hence, the sector that benefits the most from policies aiming at reducing the number of GVs is the EVs sector as not only the quantity is increased - as for EVCSs - but also the price is reduced. The effect of a subsidy to production of EVCSs itself is to increase the price. On the contrary, a subsidy to demand of EVCSs decreases the price as the demand faced by the monopolist is larger.

Notice that, by way of example, a tax on production of dirty cars has an opposite effect of the prices of EVs and EVCSs depending on the strength of the network effect. If $\gamma_2 > \gamma_4$, meaning that investors give more importance to the availability of EVs than consumers to the number of EVCSs, then the price of EVs will decrease and the price of EVCSs will increase. The opposite would happen if consumers give more importance to the network effect than investors. This follows from the fact that the monopolist can choose on which side of the market to change the price and it will do that on the side which gives more importance to the complementary good.

The three most effective one-sided policies are a subsidy to production of EVs, a tax on production of GVs and a subsidy to demand of EVCSs. However, which of the three policies dominates depends on the parameters of the model.

3.7 Tax on dirty cars production and subsidy to demand for charging stations

The government can also choose to combine different policy instruments in order to increase the share of EVs in the economy. One possibility is to have a tax on the production of dirty cars and a subsidy to the demand for EVCSs. In this case we assume that the two incentives are government revenue equivalent or lump sum, that is,

$$s_d q_d = s_f q_f \tag{28}$$

meaning that the government's budget is always balanced.

Demand functions when investment in EVCSs are subsidize are

$$\begin{aligned} q_c &= \frac{\alpha_c}{\beta_c} - \frac{\gamma_1}{\beta_c} q_d - \frac{\gamma_2}{\beta_c} q_f - \frac{p_c}{\beta_c} \\ q_d &= \frac{\alpha_d}{\beta_d} - \frac{\gamma_1}{\beta_d} q_c - \frac{p_d}{\beta_d} \\ q_f &= \frac{\alpha_f}{\beta_f} - \frac{\gamma_4}{\beta_f} q_c - \frac{p_f}{\beta_f} + \frac{s_f}{\beta_f} \end{aligned} \tag{29}$$

which shows that the demand for charging stations is increased by the subsidy s_f. Inverse demand functions are

$$\begin{aligned} p_c &= \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f \\ p_d &= \alpha_d - \beta_d q_d - \gamma_1 q_c \\ p_f &= \alpha_f - \beta_f q_f - \gamma_4 q_c + s_f \end{aligned} \tag{30}$$

3.7.1 Static profit maximization

The subsidy provided to purchase EVs is obtained trough a tax levied on the profits from GVs production. Hence, profits of the monopolist are given by

$$\pi = (p_c - c_c)q_c + (p_d - s_d - c_d)q_d + (p_f - c_f)q_f \tag{31}$$

The FOCs for the maximization problem are

$$\begin{aligned} \frac{\partial \pi}{\partial q_c} &: \alpha_c - 2\beta_c q_c - 2\gamma_1 q_d - (\gamma_2 + \gamma_4) q_f - c_c = 0 \\ \frac{\partial \pi}{\partial q_d} &: \alpha_d - 2\beta_d q_d - 2\gamma_1 q_c - s_d - c_d = 0 \\ \frac{\partial \pi}{\partial q_f} &: \alpha_f - 2\beta_f q_f - (\gamma_2 + \gamma_4) q_c + s_f - c_f = 0 \end{aligned} \tag{32}$$

So that the profit maximizing quantities are

$$
\begin{aligned}
q_c^* &= \frac{1}{X}\left[2\beta_d\beta_f(\alpha_c - c_c) - 2\beta_f\gamma_1(\alpha_d - s_d - c_d) - \beta_d(\gamma_2+\gamma_4)(\alpha_f + s_f - c_f)\right] \qquad (33)\\
q_d^* &= \frac{1}{2X}\left[-4\beta_f\gamma_1(\alpha_c - c_c) + [4\beta_c\beta_f - (\gamma_2+\gamma_4)^2](\alpha_d - s_d - c_d) + 2\gamma_1(\gamma_2+\gamma_4)(\alpha_f + s_f - c_f)\right]\\
q_f^* &= \frac{1}{X}\left[-\beta_d(\gamma_2+\gamma_4)(\alpha_c - c_c) + \gamma_1(\gamma_2+\gamma_4)(\alpha_d - s_d - c_d) + 2(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)\right]
\end{aligned}
$$

where $X = 4\beta_c\beta_d\beta_f - 4\beta_f\gamma_1^2 - \beta_d(\gamma_2+\gamma_4)^2$. An increase in tax on profits on the production of GVs, s_d, increases the quantity of clean cars and charging stations, whereas it decrease the quantity of dirty cars. Similarly, a subsidy to charging stations increases the quantity of clean cars and charging stations, whereas it decrease the quantity of dirty cars.

Given the optimal quantities in (33), we can find the prices maximizing profits as

$$
\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)]\\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - s_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)]\\
p_d^* &= \frac{1}{2}(\alpha_d + s_d + c_d) \qquad (34)\\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - s_d - c_d)\\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + s_f + c_f) - \beta_d[\gamma_2^2(\alpha_f + s_f) + \gamma_4^2 c_f]]
\end{aligned}
$$

3.8 Tax on demand for dirty cars and subsidy to investment in charging stations

Another possible combination of policy instruments is to tax the demand for dirty cars and subsidize the investment in charging stations. The optimal quantities are as in the previous case, whereas optimal prices are

$$
\begin{aligned}
p_c^* &= \frac{1}{2X}[2(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_c + c_c) - 2\beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_c)]\\
&\quad -\gamma_1(\gamma_2^2 - \gamma_4^2)(\alpha_d - s_d - c_d) - 2(\gamma_2 - \gamma_4)(\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)]\\
p_d^* &= \frac{1}{2}(\alpha_d - s_d + c_d) \qquad (35)\\
p_f^* &= \frac{1}{X}[\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) - \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - s_d - c_d)\\
&\quad +(2\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f - s_f + c_f) - \beta_d[\gamma_2^2\alpha_f + \gamma_4^2(c_f - s_f)]]
\end{aligned}
$$

Assuming $\gamma_2 > \gamma_4$, that means that the intensity of the network effect is stronger for investors than for consumers, the results above can be summarized as in Table 2.

Table 2: Policy mix

	EVs		GVs		EVCSs	
	Δq_c	Δp_c	Δq_d	Δp_d	Δq_f	Δp_f
Tax GVs prod. + sub. EVCSs dem.	$+$	$-^*$	$-$	$+$	$+$	$\hat{+}$
Tax GVs dem. + sub. EVCSs prod.	$+$	$-^*$	$-$	$-$	$+$	$+$

As in the case of one-sided policies, the effects on the quantities of the goods are the same regardless on whether the tax/subsidy applies to production or demand. However, the impact on prices differs according to the policies adopted. Considering the price of EVs, both policies decrease the price assuming that $\gamma_1(\gamma_2 + \gamma_4) > \beta_c\beta_d(\gamma_2 - \gamma_4)$. The price of GVs is increased by a tax on the production of GVs and decreased by a tax on demand. As for p_f, the price is always increased when demand of GVs is taxed and production of EVCSs is subsidized. However, if production of GVs is taxed and demand for EVCSs is subsidized, the price increases only if $\beta_d(\gamma_2 + \gamma_4) > 2\beta_f\gamma_1$.

The combination of these different policies marks a difference from [71] as she cannot include them in the analysis as she does not include the GVs sector.

Beyond the tax subsidy policy, the proposed model can be directly used to study other policies, such as regulations. The government may require a minimal share of EVs in the vehicle fleet [75]. This could stimulate the EV market and attract more investment on the EV charging station via the feedback loop between the EV and charging station market. Similarly, the government may mandate the minimal number of EV charging stations in public parking lots.

4 Dynamics when the monopolist sets quantities

In this section, we move to the analysis of the dynamic of the model. We assume that the monopolist has incomplete knowledge of the demand functions coming from the cars consumer and charging stations investors. Hence, he uses an updating rule, the gradient rule, in order to maximize profits: he changes quantities in the same direction as long as profits are positive; he changes quantities in the opposite direction if the profits are negative. If profits are constant, the monopolist will not change the quantities. Formally,

$$
\begin{aligned}
q_{c,t+1} &= q_{c,t} + \chi(q_{c,t})\frac{\partial \pi}{\partial q_{c,t}} \\
q_{d,t+1} &= q_{d,t} + \chi(q_{d,t})\frac{\partial \pi}{\partial q_{d,t}} \\
q_{f,t+1} &= q_{f,t} + \chi(q_{f,t})\frac{\partial \pi}{\partial q_{f,t}}
\end{aligned} \tag{36}
$$

where $\chi(q_{i,t})$ is a positive function which gives the extent of production variation of the platform following a given profit signal. With this kind of local adjustment mechanism, the platform only needs to infer how the market will respond to small production changes by an estimate of the marginal profit. We notice that, in this case, the platform does not decide their future production quantities by solving an optimization problem, but the selection occurs just following the direction of increasing profits. This adjustment mechanism has been proposed by many authors (see e.g. [7], [56], [55], [11]). In the following, we also assume linear functions $\chi(q_{i,t}) = \chi q_{i,t}$, since this assumption captures the fact that relative production variations are proportional to marginal profits, i.e.

$$
\frac{q_{i,t+1} - q_{i,t}}{q_{i,t}} = \chi \frac{\partial \pi}{\partial q_i} \tag{37}
$$

where χ is a positive speed of adjustment, which represents the platform reaction to profit signals per unitary production. Notice that the second term on the right hand side of the equations above is 0 when the quantity is the one that maximizes profits, that is the quantities we found in (8).

Substituting (7) in (36), we get the following quadratic three dimensional dynamic nonlinear system in discrete time

$$
\begin{aligned}
q_{c,t+1} &= q_{c,t} + \chi q_{c,t}\left(\alpha_c - 2\beta_c q_{c,t} - 2\gamma_1 q_{d,t} - (\gamma_2 + \gamma_4) q_{f,t} - c_c\right) \\
q_{d,t+1} &= q_{d,t} + \chi q_{d,t}\left(\alpha_d - 2\beta_d q_{d,t} - 2\gamma_1 q_{c,t} - c_d\right) \\
q_{f,t+1} &= q_{f,t} + \chi q_{f,t}(\alpha_f - 2\beta_f q_{f,t} - (\gamma_2 + \gamma_4) q_{c,t} - c_f)
\end{aligned} \tag{38}
$$

Proposition 1. All the critical points of the static system are stationary point in the dynamic system and are maxima.

Proof. Let $q_{t+1} = q_t = q^*$ be the condition to have a steady state. Hence, substituting this condition in (38), the possible steady states satisfy

$$
\begin{aligned}
q_c^* &= q_c^* + \chi q_c^*\left(\alpha_c - 2\beta_c q_{c,t} - 2\gamma_1 q_{d,t} - (\gamma_2 + \gamma_4) q_{f,t} - c_c\right) \\
q_d^* &= q_d^* + \chi q_d^*\left(\alpha_d - 2\beta_d q_{d,t} - 2\gamma_1 q_{c,t} - c_d\right) \\
q_f^* &= q_f^* + \chi q_f^*(\alpha_f - 2\beta_f q_{f,t} - (\gamma_2 + \gamma_4) q_{c,t} - c_f)
\end{aligned} \tag{39}
$$

There is a unique steady state which is given by the system of (8). Exactly the quantities that maximize profits. This concludes the proof.

We can represent system (38) as

$$q_{t+1} = F(q_t) \tag{40}$$

where

$$F(q) = F(q_c, q_d, q_f) = \begin{pmatrix} q_c \\ q_d \\ q_f \end{pmatrix} + \chi \begin{pmatrix} q_c \dfrac{\partial \pi}{\partial q_c} \\ q_d \dfrac{\partial \pi}{\partial q_d} \\ q_f \dfrac{\partial \pi}{\partial q_f} \end{pmatrix} \tag{41}$$

Notice that F can be represented as

$$F = I + \chi \begin{pmatrix} q_c & 0 & 0 \\ 0 & q_d & 0 \\ 0 & 0 & q_f \end{pmatrix} \begin{pmatrix} \dfrac{\partial \pi}{\partial q_c} \\ \dfrac{\partial \pi}{\partial q_d} \\ \dfrac{\partial \pi}{\partial q_f} \end{pmatrix} \tag{42}$$

where I is the identity matrix. Equilibrium points of (42) are critical point of the static maximization problem.

In order to study the stability of the equilibrium points, we compute the Jacobian of the map F wrt the quantities

$$J_F = \begin{pmatrix} 1 + \chi\pi'_{q_c} + \chi q_c \pi''_{q_c,q_c} & \chi\pi'_{q_c} + \chi q_c \pi''_{q_c,q_d} & \chi\pi'_{q_c} + \chi q_c \pi''_{q_c,q_f} \\ \chi\pi'_{q_d} + \chi q_d \pi''_{q_d,q_c} & 1 + \chi\pi'_{q_d} + \chi q_d \pi''_{q_d,q_d} & \chi\pi'_{q_d} \\ \chi\pi'_{q_f} + \chi q_f \pi''_{q_f,q_c} & \chi\pi'_{q_f} & 1 + \chi\pi'_{q_f} + \chi q_f \pi''_{q_f,q_f} \end{pmatrix} \tag{43}$$

Given that the Jacobian is evaluated at an interior equilibrium point (q_c, q_d, q_f), the first order partial derivatives $\pi'_{q_c}, \pi'_{q_d}, \pi'_{q_f}$ vanish. Hence, obtain

$$\hat{J}_F = \begin{pmatrix} 1 + \chi q_c \pi''_{q_c,q_c} & \chi q_c \pi''_{q_c,q_d} & \chi q_c \pi''_{q_c,q_f} \\ \chi q_d \pi''_{q_d,q_c} & 1 + \chi q_d \pi''_{q_d,q_d} & 0 \\ \chi q_f \pi''_{q_f,q_c} & 0 & 1 + \chi q_f \pi''_{q_f,q_f} \end{pmatrix} \tag{44}$$

where q_c, q_d and q_f are given by (8), (8) and (9). From (44) we can compute the characteristic polynomial $P(Z)$ that we need to check for the stability of the dynamic system

$$Det(ZI - \hat{J}_F) = P(Z) \tag{45}$$

The characteristic polynomial we obtain is of the form

$$P(Z) = a_0 Z^3 + a_1 Z^2 + a_2 Z + a_3 \tag{46}$$

where

$$\begin{aligned}
a_0 &= 1 \\
a_1 &= 2\chi(\beta_c q_c + \beta_d q_d + \beta_f q_f) - 3 \\
a_2 &= 3 - 4\chi[\beta_c q_c + \beta_d q_d + \beta_f q_f] + 4\chi^2(\beta_d\beta_f q_d q_f + \beta_c\beta_f q_c q_f + \beta_c\beta_d q_c q_d) \\
&- \chi^2 q_c[4\gamma_1^2 q_d + (\gamma_2+\gamma_4)^2 q_f] \\
a_3 &= -1 - 2\chi(\beta_c q_c + \beta_d q_d + \beta_f q_f) - 4\chi^2(\beta_d\beta_f q_d q_f + \beta_c\beta_f q_c q_f + \beta_c\beta_d q_c q_d) \\
&+ \chi^2 q_c[4\gamma_1^2 q_d + (\gamma_2+\gamma_4)^2 q_f] + \chi^3 q_c q_d q_f[4\beta_c\beta_d\beta_f - 4\beta_f\gamma_1^2 - \beta_d(\gamma_2+\gamma_4)^2]
\end{aligned} \tag{47}$$

4.1 Proof of the stability

We apply the Jury stability test to the linearized system. The necessary and sufficient conditions for $\hat{J}_F$ to have eigenvalues in the unit circle states

1. $|a_3| < a_0$, that is $a_0 > a_3$ and $a_0 < -a_3$.
2. $P(1) = a_0 + a_1 + a_2 + a_3 > 0$
3. $P(-1) = -a_0 + a_1 - a_2 + a_3 < 0$
4. Jury Table

$$\begin{matrix} a_3 & a_2 & a_1 & a_0 \\ a_0 & a_1 & a_2 & a_3 \end{matrix}$$

where

$$b_2 = \begin{vmatrix} a_3 & a_0 \\ a_0 & a_3 \end{vmatrix}$$

$$b_1 = \begin{vmatrix} a_3 & a_1 \\ a_0 & a_2 \end{vmatrix}$$

$$b_0 = \begin{vmatrix} a_3 & a_2 \\ a_0 & a_1 \end{vmatrix}$$

the condition that need to be satisfied is $|Det(b_2)| > |Det(b_0)|$, that is,

$$|a_3^2 - a_0^2| > |a_3 a_1 - a_2 a_0| \tag{48}$$

that is, $a_3^4 - 2a_3^2 + 1 - a_3^2 a_1^2 + 2a_1 a_2 a_3 - a_2^2 > 0$.

4.2 Numerical Simulation

In this section, we investigate the impact of relevant parameters on the stability of the dynamic system and we compare the outcomes of the model when the system is stable and when it is unstable.

4.2.1 Stability

Because EVs were introduced to the broader consumers market only in 2010 and since they represent less than 1% of the total vehicle stock, only few studies try to estimate the parameters of demand functions or the network effects. However, based on the empirical studies available ([49], [46]), we assume the parameters values presented in Table 3.

Table 3: Parameters value for the dynamic system simulation

EVs quantity	q_c	1
GVs quantity	q_d	1
EVCSs quantity	q_f	1
Substitution effect	γ_1	0.5
Network effect consumers	γ_2	-0.4
Network effect investors	γ_4	-0.5
Subsidy to EVs	s_c	0
Tax on GVs	s_c	0
Subsidy to EVCSs	s_f	0
Speed of adjustment	χ	0.05
Market size EVs	α_c	20
Market size GVs	α_d	30
Market size EVCSs	α_f	3
Slope demand EVs	β_c	1.1
Slope demand GVs	β_d	1.1
Slope demand EVCSs	β_f	0.5
Marginal costs	c_i	0

Except when differently indicated these parameters values will be used in the analysis. As stated above the substitution effect lies in the interval [0,1] and the network effects are between -1 and 0 [37]. Moreover, we have assumed that the network effect is stronger for investors, hence $\gamma_2 > \gamma_4$. The policy parameters are initially assumed to be null and we will then look how they affect the outcome of the model when changing. Important parameters are the market sizes of the three goods. Indeed, although the quantities of EVs and EVCSs are not directly comparable, we can use the market size i.e. the quantity of the good when the price is zero, α, to account for the different order of magnitude. Assuming $\alpha_f << \alpha_c$ we capture this effect. The parameters capturing the slopes of the inverse demand functions are chosen on the basis of the estimates for the elasticities that we could find in the literature. In [49], the estimated elasticity for EVs with respect to price is above one in absolute value which is line with the prior literature on the automobile industry ([9], [46]). Hence, even though we are aware of the fact that it is just an approximation, we assume that the slope of the inverse demand function is above one. Moreover, we assume the same slope for EVs and GVs, i.e. $\beta_c = \beta_d$. Following [49] we assume a smaller value for the charging station elasticity, as the latter is assumed to be larger, but we are considering inverse demand.

For these values of the parameters the system is stable and then we let the parameters of interest vary to see how this affects the stability of the system. In order to perform this analysis we use bifurcation diagrams. Bifurcation diagrams are such that when a single steady state is reached in the dynamic system it corresponds to the equilibrium of the static case: even if the platform does not know the demand, for some values of the parameters, there is a unique internal solution, which coincides with the critical point to of the static system. However, the dynamic system looses stability for some values of the parameters and more complex dynamics emerges; for these values the platform is not able to achieve the optimal solutions of the static case: we want to analyze which parameters destabilize the system when assuming different values.

We start by considering how the stability of the system is affected when χ changes (Figure 1). χ captures the percentage variation of the quantity given a change in marginal profits, that is, it captures the reactivity of the platform to a change in marginal profits. For low values of the parameter, the system is stable and it converges to the maximum of the static system. For $\chi = 0.061$, a double period cycle emerges, so that the system converges to a stable orbit; in this case, we may have an equilibrium with low sales of electric cars or an equilibrium with high sales of EVs depending on the initial conditions. Notice that at the bifurcation point the system looses stability because for values just below it the system converges to an equilibrium whereas for values just above it the system converges to the stable orbit. For $\chi = 0.081$ the system does not converge and it gets into chaos. The model in this case move from stability through a sequence of period bifurcation to chaos. Similar figures can be obtained considering q_d and q_f. As an experiment, we also try to use a different updating rule in which the platform does not only consider the quantity produced in the last period, but an average of the quantities produced in the last n periods. However, this does not affect the initial bifurcation point but only the dynamic after the two period cycle; hence, we did not report the results. We also try to reduce the impact of χ by scaling it down; this means that we are assuming a more conservative firm. In this case, the bifurcation takes place later because the reactivity of the platform is lower and hence it is more likely that she will be able to achieve the optimal equilibrium.

Another parameter of interest to us is the network effect and we investigate its impact on the stability of the system. Figure 2, 3, 4 show that high values of the network effect destabilize the system. As in the static case, we obtain that for high values of the network effect the quantity of EVs and EVCSs is higher and the quantity of GVs is lower. Similar results are obtained varying γ_4 instead of γ_4. Similarly, high values of the the tax on GVs destabilize the system (see Figures 5, 6, 7). Also in this case the policy measures generate the expected effects, such that high values of the tax reduce the quantity of GVs and increase the quantities of EVs and EVCSs. Finally, we study the impact of a subsidy to demand for EVCSs: the subsidy on demand of EVCSs increases the quantity of EVs up to a value, but then the system becomes unstable and a two period cycle emerges such that the initial steady state (the critical point of the static problem) is not stable anymore. Since the platform does not know the demand function he chooses quantities wrongly moving from the upper to the lower branch each period, for the same value of the subsidy. However, this is not a systematic mistake as the platform doe not know the demand.

The results we obtain in the dynamic system confirm the ones of the static model; however, they also highlight some aspect that did not appear in the static analysis. In particular they allow to identify the range of parameters such that the policy measures will have the expected impacts and the values for which the impact of the policies is uncertain.

Figure 1: Bifurcation diagram of q_c as a function of χ; χ varies from 0.001 to 0.1.

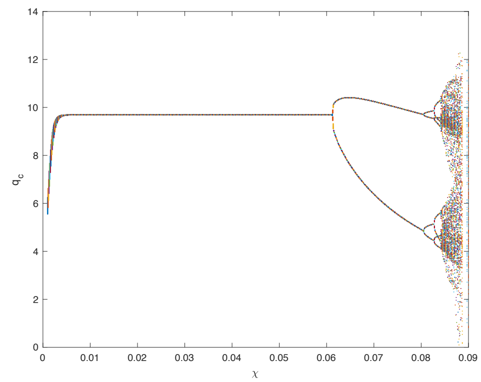


Figure 2: Bifurcation diagram of q_c as a function of γ_2; γ_2 varying from -1 to 0.

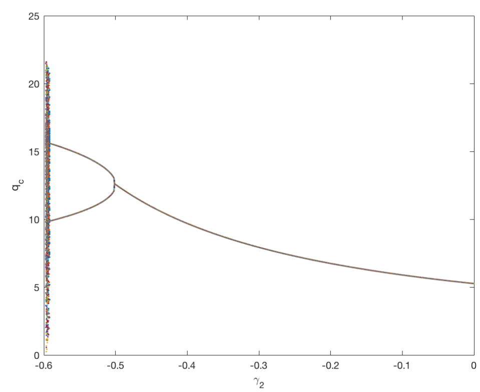


Figure 3: $qd(\gamma_2)$, γ_2 varying from -1 to 0.

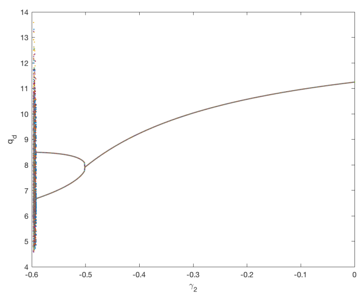


Figure 4: $qf(\gamma_2)$, γ_2 varying from -1 to 0.

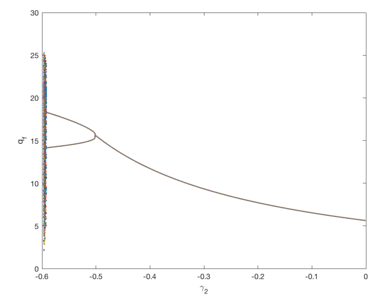


Figure 5: Bifurcation diagram of q_c as a function ofs_d

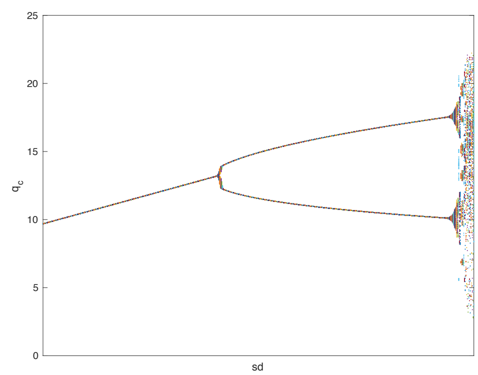


Figure 6: $qd(s_d)$

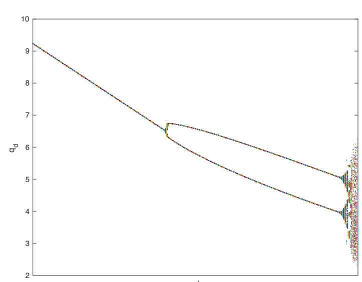


Figure 7: $qf(s_d)$

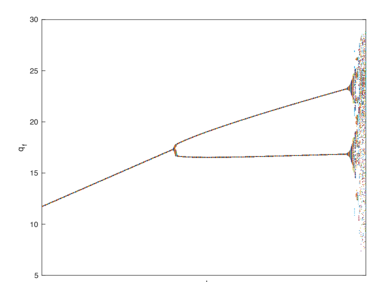


Figure 8: Bifurcation diagram of q_c as a function ofs_f

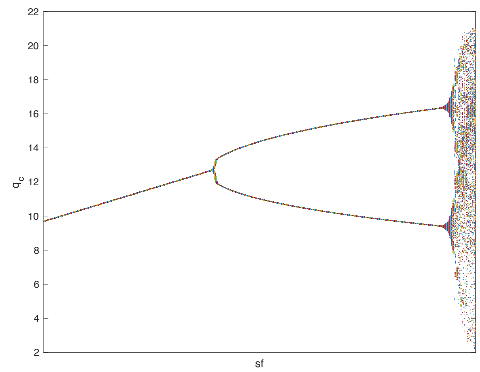


Figure 9: $qd(s_f)$

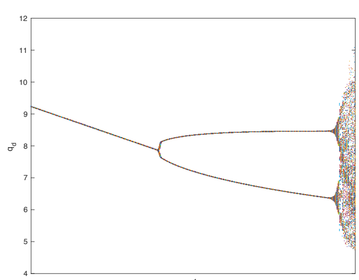
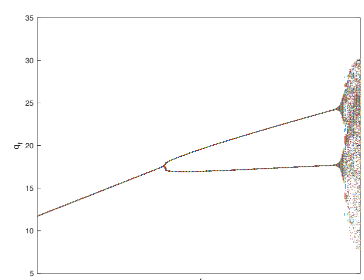


Figure 10: $qf(s_f)$



4.2.2 Economic outcomes under stability and instability of the system

In order to compare economic outcomes when the system is stable and unstable, we start by computing profits and cumulated profits in the two cases. We find that the profits when the system is stable ($\chi = 0.05$) are always above the profits when the system is unstable ($\chi = 0.08$) (see Figure 11); hence, cumulated profits are higher when the system is stable (see Figure 12). This implies that the platform realizes higher profits when it is able to reach the optimal solution of the static case, that is, when it behaves as if it knew the demand function.

Figure 11: Profits when the system is stable, pi_s, compared to profits when the system is unstable, pi_u.

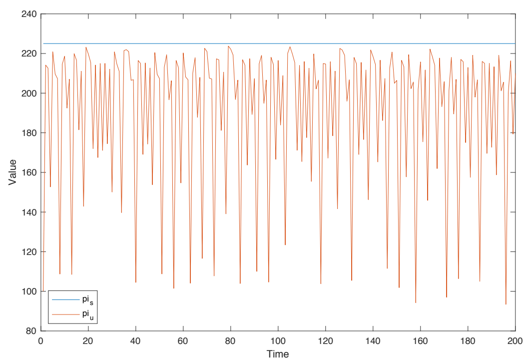
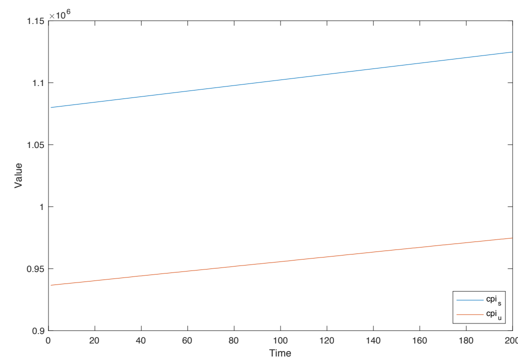


Figure 12: Cumulated profits when the system is stable, cpi_s, compared to cumulated profits when the system is unstable, cpi_u.



We also compute welfare and cumulated welfare when the system is stable and when the system is unstable. We find that welfare when the system is stable is not always higher than welfare when the system is unstable (see Figure 13); however, cumulated welfare is higher when the system is stable (see Figure 14).

Figure 13: Welfare when the system is stable, w_s, compared to welfare when the system is unstable, w_u.

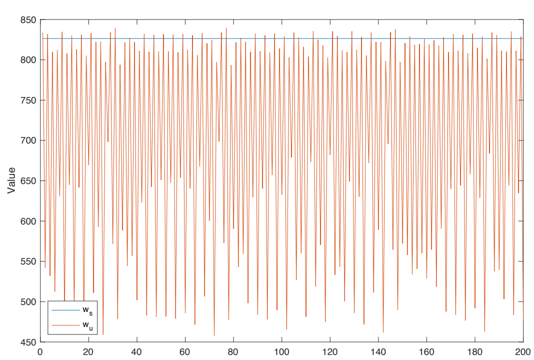
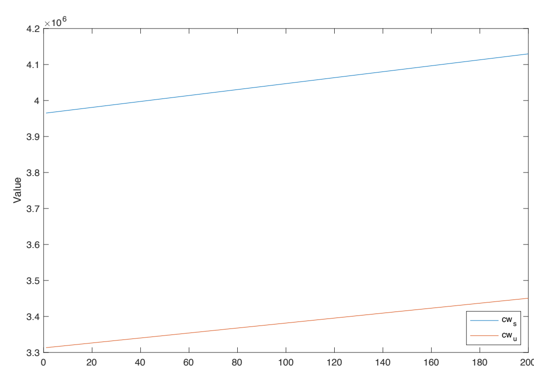


Figure 14: Cumulated welfare when the system is stable, cw_s, compared to cumulated welfare when the system is unstable, cw_u.



5 Conclusions

There are a variety of opportunities to reduce greenhouse gas emissions from the transportation sector, such as improving fuel efficiency, reducing travel demand, improving driving practices, and switching to alternative fuel. In many countries, EVs have started to be considered an increasingly important option in achieving lower emissions related to transportation. However, there is no general consensus on the design of the supporting policies that work best to encourage EV adoption and it is not clear what the dynamic of the penetration of EVs might be.

We present a two-side market model characterized by a platform serving consumers of EVs and GVs on one side of the market and investors in EVCSs on the other side. We develop a static modeling framework in which consumers make their car purchasing decisions by maximizing their utility (affected by the number of EVCSs) across EVs and GVs and investors maximize their utility (affected by the number of EVs) by choosing the quantity of EVCSs. The model allows us to capture the network effects operating in the EV market and to study one-sided and two-sided policy measures. The work highlights the importance of accounting for the network externalities present in the EV market due to its two-sided nature when designing EV promoting policies.

We then move to a dynamic setting to compare the results of this model with the ones of the static model. Since the market for EVs and EVCSs are relatively new and their impact on the market for GVs in unclear, we assume that the platform has incomplete information about the demand functions and it uses a rule of the thumb to choose the profit maximizing prices and quantities. We then look at the impact of the most relevant parameters for our analysis. The results we obtain in the dynamic system confirm the ones of the static model; however, they also highlight some aspect that did not appear in the static analysis. In particular, they allow to identify the range of parameters such that the policy measures will have the expected impact and the values for which the impact of the policies is uncertain, as high values of network effects, subsidies and taxes destabilize the system. We also find that the platform realizes higher profits when it is able to reach the optimal solution of the static case, that is, when it behaves as if it knew the demand function. Similarly, welfare is higher in when the system is stable.

Future research will focus on modifying the market structure introducing an oligopolistic platform. Another assumption we want to relax is that we have modeled that all the consumers in the market buy a new car simultaneously. In a more realistic setting only a fraction of the installed base of traditional cars is replaced in each period. We also want to extend our research by introducing a spatial dimension of charging stations location; this aspect is particularly relevant because charging an EVs requires time; hence, charging stations cannot be located, for instance, on highways, as it is unpleasant location for consumers to wait. In a future (empirical) work, we want to investigate how much the location of charging stations affects their profitability. Finally, we would like to include region-specific factors, such as larger driving distances or colder winters, as they are likely to affect consumers' vehicle purchasing and operating decisions to a larger degree.

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Appendix A - Corner Solutions

Starting from the optimal quantities in (8), we consider, for illustration, four limit cases:

1. $\gamma_1 = \gamma_2 = \gamma_4 = 0$.

   The equilibrium quantities become

$$\begin{aligned} q_c &= \frac{1}{2\beta_c}(\alpha_c - c_c) \\ q_d &= \frac{1}{2\beta_d}(\alpha_d - c_d) \\ q_f &= \frac{1}{2\beta_f}(\alpha_f - c_f) \end{aligned} \tag{49}$$

   (49) shows that the optimal quantity of each good is only influenced by the parameters related to its own demand.

   The equilibrium prices reduces to

$$\begin{aligned} p_c &= \frac{1}{2}(\alpha_c + c_c) \\ p_d &= \frac{1}{2}(\alpha_d + c_d) \\ p_f &= \frac{1}{2}(\alpha_f + c_f) \end{aligned} \tag{50}$$

   (50) shows that the market size of clean cars and their marginal cost of production have an impact on the quantity of dirty cars but not on their price. This means that market size and marginal cost move demand and supply of the same amount. If you imagine a graph with q on the x axis and p on the y axis, and you draw demand and supply given that the price remain the same it means that supply and demand change by the same amount.

2. $\gamma_1 = 1, \gamma_2 = \gamma_4 = 0$.

   The equilibrium quantities become

$$\begin{aligned} q_c &= \frac{1}{2(\beta_c\beta_d - 1)}[\beta_d(\alpha_c - c_c) - (\alpha_d - c_d)] \\ q_d &= \frac{1}{2(\beta_c\beta_d - 1)}[-(\alpha_c - c_c) + \beta_c(\alpha_d - c_d)] \\ q_f &= \frac{1}{2\beta_f}(\alpha_f - c_f) \end{aligned} \tag{51}$$

   (51) shows that the quantity of charging stations does not have any impact on clean cars now, whereas the substitution effect with dirty cars is still there. The same is true for the quantity of GVs. The optimal quantity of charging stations only depends on parameters of its own demand. Prices are as in (50).

3. $\gamma_1 = 0, \gamma_2 = \gamma_4 = -1$.

   The equilibrium quantities become

$$
\begin{aligned}
q_c &= \frac{1}{2(\beta_c\beta_f - 1)}[\beta_f(\alpha_c - c_c) + (\alpha_f - c_f)] \\
q_d &= \frac{1}{2\beta_d}(\alpha_d - c_d) \\
q_f &= \frac{1}{2(\beta_c\beta_f - 1)}[(\alpha_c - c_c) + \beta_c(\alpha_f - c_f)]
\end{aligned} \tag{52}
$$

   (52) shows that the quantity of dirty cars does not have any impact on clean cars now, whereas the complementarity effect with charging station is still present. The optimal quantity of dirty cars only depends on parameters of its own demand. Notice that $\gamma_2 = \gamma_4 = -1$ implies that clean cars and charging stations are perfect complements; indeed, we can see that the impact of the parameters of the demand for charging stations and clean cars respectively is not mediated by any parameter. The ratio of the two inputs in the utility is 1 to one. This result holds because we turn off the substitution effect. Prices are as in (50).

4. $\gamma_1 = 1, \gamma_2 = \gamma_4 = -1$.

   The equilibrium quantities become

$$
\begin{aligned}
q_c &= \frac{1}{2X_4}[\beta_d\beta_f(\alpha_c - c_c) - \beta_f(\alpha_d - c_d) + \beta_d(\alpha_f - c_f)] \\
q_d &= \frac{1}{2\beta_d X_4}[-\beta_d\beta_f(\alpha_c - c_c) - \beta_d(\alpha_f - c_f) + (\beta_f + X_4)(\alpha_d - c_d)] \\
q_f &= \frac{1}{2\beta_f X_4}[\beta_d\beta_f(\alpha_c - c_c) - \beta_f(\alpha_d - c_d) + (\beta_d + X_4)(\alpha_f - c_f)]
\end{aligned} \tag{53}
$$

   where $X_4 = \beta_c\beta_d\beta_f - \beta_f - \beta_d$. (53) shows that the quantity of dirty cars and of charging stations have an impact of opposite sign on the quantity of clean cars. q_d is negatively affected by the parameters of the demand for clean cars and charging stations and positively affected by the parameters of its own demand. The optimal quantity of charging stations if positively affected by the demand for clean cars and by its own demand, but negatively affected by the demand for dirty cars. Even if $\gamma_2 = \gamma_4 = -1$ implies that clean cars and charging stations are perfect complements, we are also assuming that clean and dirty cars are perfect substitutes and this explains why in (53) the impact of the demand for charging stations and of its own demand on the quantity of clean cars is affected by the parameter related to dirty cars. Similarly, the impact of demand for clean cars and own demand on the optimal quantity of charging stations is mediated by β_d. Prices are as in (50).

5. γ_1 free, $\gamma = \gamma_2 = \gamma_4$.

The equilibrium quantities become

$$\begin{aligned} q_c^* &= \frac{1}{2Y}\left[\beta_d\beta_f(\alpha_c - c_c) - \gamma_1\beta_f(\alpha_d - c_d) - \gamma\beta_d(\alpha_f - c_f)\right] \\ q_d^* &= \frac{1}{4Y}\left[-2\beta_f\gamma_1(\alpha_c - c_c) + 2(\beta_c\beta_f - \gamma^2)(\alpha_d - c_d) + \gamma_1\gamma(\alpha_f - c_f)\right] \\ q_f^* &= \frac{1}{2Y}\left[-\gamma\beta_d(\alpha_c - c_c) + \gamma_1\gamma(\alpha_d - c_d) + (\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right] \end{aligned} \tag{54}$$

where $Y = \beta_c\beta_d\beta_f - \beta_f\gamma_1^2 - \beta_d\gamma^2$. (54) shows that the quantity of dirty cars does not have any impact on clean cars now, whereas the complementarity effect with charging station is still present. The optimal quantity of dirty cars only depends on parameters of its own demand. Notice that $\gamma_2 = \gamma_4 = -1$ implies that clean cars and charging stations are perfect complements; indeed, we can see that the impact of the parameters of the demand for charging stations and clean cars respectively is not mediated by any parameter. The ratio of the two inputs in the utility is 1 to one. This result holds because we turn off the substitution effect. Prices are as in (50).

Appendix B - Equal network effect

In this Appendix we assume that the intensity of the network effect is the same for consumers and investors, that is, $\gamma_2 = \gamma_4$.

The utility function and budget constraint for consumers is as before; the utility function for investors is then

$$U_a(q_c, q_f) = q_0 + \sum_i \alpha_i q_i - \frac{1}{2}\left[\sum_i \beta_i q_i^2 + 2\gamma_2 q_c q_f\right] \tag{55}$$

where q_0 is the consumption level of the numeraire good. q_i where $i = \{c, f\}$ is the consumption level of EVs and charging stations. Since EVs are complementary to EVCSs, we include the quantity of EVs in the utility function of investors, but now the degree of complementarity is the same of consumers γ_2. The budget constraint for investors is not affected.

Solving the consumers' and investors' problem, the FOCs are

$$\begin{aligned} q_0 &: \lambda - 1 = 0 \\ q_c &: \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f - \lambda p_c = 0 \\ q_d &: \alpha_d - \beta_d q_d - \gamma_1 q_c - \lambda p_d = 0 \\ q_f &: \alpha_f - \beta_f q_f - \gamma_2 q_c - \lambda p_f = 0 \end{aligned} \tag{56}$$

Hence we find the demand for EVs, GVs and charging stations

$$\begin{aligned} q_c &= \frac{\alpha_c}{\beta_c} - \frac{\gamma_1}{\beta_c} q_d - \frac{\gamma_2}{\beta_c} q_f - \frac{p_c}{\beta_c} \\ q_d &= \frac{\alpha_d}{\beta_d} - \frac{\gamma_1}{\beta_d} q_c - \frac{p_d}{\beta_d} \\ q_f &= \frac{\alpha_f}{\beta_f} - \frac{\gamma_2}{\beta_f} q_c - \frac{p_f}{\beta_f} \end{aligned} \tag{57}$$

where the only difference from the model in the paper is that the quantity of EVCSs depends on the quantity of dirty cars according to the parameter γ_2.

From the demand functions, inverse demands are

$$\begin{aligned} p_c &= \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f \\ p_d &= \alpha_d - \beta_d q_d - \gamma_1 q_c \\ p_f &= \alpha_f - \beta_f q_f - \gamma_2 q_c \end{aligned} \tag{58}$$

5.1 Static profit maximization

Profits are given by

$$\pi = (p_c - c_c)q_c + (p_d - c_d)q_d + (p_f - c_f)q_f \tag{59}$$

where the first two terms represent profit from buyers/consumers and the third term is profit from investors in charging stations. The FOCs of the maximization problem are

$$
\begin{aligned}
\frac{\partial \pi}{\partial q_c} &: \quad \alpha_c - 2\beta_c q_c - 2\gamma_1 q_d - 2\gamma_2 q_f - c_c = 0 \\
\frac{\partial \pi}{\partial q_d} &: \quad \alpha_d - 2\beta_d q_d - 2\gamma_1 q_c - c_d = 0 \\
\frac{\partial \pi}{\partial q_f} &: \quad \alpha_f - 2\beta_f q_f - 2\gamma_2 q_c - c_f = 0
\end{aligned} \tag{60}
$$

The profit maximizing quantities are

$$
\begin{aligned}
q_c^* &= \frac{1}{2X}\left[\beta_d\beta_f(\alpha_c - c_c) - \gamma_1\beta_f(\alpha_d - c_d) - \gamma_2\beta_d(\alpha_f - c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-\gamma_1\beta_f(\alpha_c - c_c) + (\beta_c\beta_f - \gamma_2^2)(\alpha_d - c_d) + \gamma_1\gamma_2(\alpha_f - c_f)\right] \\
q_f^* &= \frac{1}{2X}\left[-\gamma_2\beta_d(\alpha_c - c_c) + \gamma_1\gamma_2(\alpha_d - c_d) + (\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right]
\end{aligned} \tag{61}
$$

where $X = \beta_c\beta_d\beta_f - \beta_f\gamma_1^2 - \beta_d\gamma_2^2$.
Given the optimal quantities in (61), we can find the price maximizing profits as

$$
\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f + c_f)
\end{aligned} \tag{62}
$$

(62) shows that the price of each good only depends on the parameters of its own demand. This means that market size and marginal cost move demand and supply of the same amount.

5.2 Policies

In this section, we analyze different policy measures that can be adopted to foster the energy transition towards EVs, assuming that the strength of the network effect is the same on the two sides of the market. We decided to perform this policy analysis because, differently from the model presented in the paper, this assumption allows us to derive clear cut results without the help of any numerical simulation. We compare the impact of direct purchasing price subsidies to the impact of charging station subsidies to understand whether it is preferable to subsidize consumers, by lowering the upfront costs associated with EV purchases, or to subsidize charging stations, by lowering their sunk investment costs.

5.2.1 Subsidy to production of charging stations

If a subsidy is provided to production of charging station, the optimal quantities are

$$\begin{aligned}
q_c^* &= \frac{1}{2X}\left[\beta_d\beta_f(\alpha_c - c_c) - \gamma_1\beta_f(\alpha_d - c_d) - \gamma_2\beta_d(\alpha_f + s_f - c_f)\right] && (63)\\
q_d^* &= \frac{1}{2X}\left[-\gamma_1\beta_f(\alpha_c - c_c) + (\beta_c\beta_f - \gamma_2^2)(\alpha_d - c_d) + \gamma_1\gamma_2(\alpha_f + s_f - c_f)\right] && (64)\\
q_f^* &= \frac{1}{2X}\left[-\gamma_2\beta_d(\alpha_c - c_c) + \gamma_1\gamma_2(\alpha_d - c_d) + (\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)\right] && (65)
\end{aligned}$$

which show that both EVs purchases and EVCS investments are positively related to subsidies to the production of EVCS. The effectiveness of a charging station subsidy on the number of EV purchases is closely tied to the importance that consumers place on the operating charging station network (as captured by γ_2), but the impact will be, in any case, positive. The result indicates that the charging network influences buyers' vehicle choice. On the contrary, the subsidy to production of charging stations reduces the quantity of dirty cars.

Optimal prices are

$$\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c) && (66)\\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) && (67)\\
p_f^* &= \frac{1}{2}(\alpha_f - s_f + c_f) && (68)
\end{aligned}$$

which shows that subsidies to production of charging stations reduces their price although only by half the amount of the subsidy because of the monopolistic power of the platform.

5.2.2 Subsidy to demand of charging stations

The government can subsidize the provision of charging stations by investors in order to generate a positive externality on clean cars consumption (through the network effect). Subsidies can be provided to charging station investors for purchasing and installing charging equipment. If a subsidy is provided to the demand for charging stations by investors, the optimal quantities are as in the case of subsidizing production, showing that both the quantities of EVs and charging stations increase, whereas the quantity of GVs decreases. However, the optimal prices

$$\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c)\\
p_d^* &= \frac{1}{2}(\alpha_d + c_d) && (69)\\
p_f^* &= \frac{1}{2}(\alpha_f + s_f + c_f)
\end{aligned}$$

which shows that the price of charging stations increases because demand is higher and the monopolist can adjust demand thanks to his market power.

5.2.3 Subsidy to production of EVs only

If a subsidy is provided to the production of EVs only, the optimal quantities are

$$
\begin{aligned}
q_c^* &= \frac{1}{2X}\left[\beta_d\beta_f(\alpha_c+s_c-c_c)-\gamma_1\beta_f(\alpha_d-c_d)-\gamma_2\beta_d(\alpha_f-c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-\gamma_1\beta_f(\alpha_c+s_c-c_c)+(\beta_c\beta_f-\gamma_2^2)(\alpha_d-c_d)+\gamma_1\gamma_2(\alpha_f-c_f)\right] \\
q_f^* &= \frac{1}{2X}\left[-\gamma_2\beta_d(\alpha_c+s_c-c_c)+\gamma_1\gamma_2(\alpha_d-c_d)+(\beta_c\beta_d-\gamma_1^2)(\alpha_f-c_f)\right]
\end{aligned} \tag{70}
$$

showing that the subsidy increases both the quantities of EVs and of charging stations, the effect mediated by the parameter capturing the network effect (γ_2). The quantity of GVs is decreased by the subsidy. The optimal prices when the subsidy is in place are

$$
\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c-s_c+c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d+c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f+c_f)
\end{aligned} \tag{71}
$$

which shows that a subsidy to production of EVs reduces the price of clean cars, but only by one half of its value because the monopolist is able to increase its profits.

5.2.4 Subsidy to demand of EVs

If a subsidy is provided to the demand for clean cars only, the optimal quantities are as above, whereas the optimal prices are

$$
\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c+s_c+c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d+c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f+c_f)
\end{aligned} \tag{72}
$$

which shows that when demand is subsidized the price of EVs increases because the monopolist is able to charge a higher price given that demand has been increased by the subsidy.

The above analysis shows that while the impact of subsidies to the production side reduces the price, this is not true for subsidies to the demand side, as prices increase because of the monopolistic power.

5.2.5 Taxes on production of GVs

If a tax is imposed on the production of dirty cars only, the optimal quantities are

$$\begin{aligned}
q_c^* &= \frac{1}{2X}\left[\beta_d\beta_f(\alpha_c - c_c) - \gamma_1\beta_f(\alpha_d - s_d - c_d) - \gamma_2\beta_d(\alpha_f - c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-\gamma_1\beta_f(\alpha_c - c_c) + (\beta_c\beta_f - \gamma_2^2)(\alpha_d - s_d - c_d) + \gamma_1\gamma_2(\alpha_f - c_f)\right] \\
q_f^* &= \frac{1}{2X}\left[-\gamma_2\beta_d(\alpha_c - c_c) + \gamma_1\gamma_2(\alpha_d - s_d - c_d) + (\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right]
\end{aligned} \tag{73}$$

which shows that the tax on production of GVs increases the quantity of EVs and charging stations, whereas it decreases the quantity of dirty cars. The optimal prices are

$$\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d + s_d + c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f + c_f)
\end{aligned} \tag{74}$$

Contrary to the cases analyzed above, the tax on production increases the price of dirty cars as the monopolist is able to pass half on the increase in the price on through consumers. The prices of the other goods are not affected.

5.2.6 Taxes on demand of dirty cars

If a tax is imposed on the demand for dirty cars only, the optimal quantities are as before, whereas the optimal prices

$$p_c^* = \frac{1}{2}(\alpha_c + c_c) \tag{75}$$

$$p_d^* = \frac{1}{2}(\alpha_d - s_d + c_d) \tag{76}$$

$$p_f^* = \frac{1}{2}(\alpha_f + c_f) \tag{77}$$

which shows that the monopolist has to reduce the price, given the reduction in demand due to the tax, even though only by one half of the tax.

Overall, considering taxes we get the opposite result as in the case of subsidies; the tax on production increases the price as the monopolist is able to pass the increased cost on through consumers, whereas the tax on demand reduces the price because the monopolist faces a lower demand.

5.2.7 Tax on dirty cars production and subsidy to demand for charging stations

The government can also choose to combine different policy instruments in order to increase the share of EVs in the economy. One possibility is to have a tax on the production of dirty cars and a subsidy on the demand for charging station. In this case we assume that the two incentives are government revenue equivalent, that is

$$s_d q_d = s_f q_f \tag{78}$$

Demand functions when investment in charging stations are subsidized are

$$\begin{aligned} q_c &= \frac{\alpha_c}{\beta_c} - \frac{\gamma_1}{\beta_c} q_d - \frac{\gamma_2}{\beta_c} q_f - \frac{p_c}{\beta_c} \\ q_d &= \frac{\alpha_d}{\beta_d} - \frac{\gamma_1}{\beta_d} q_c - \frac{p_d}{\beta_d} \\ q_f &= \frac{\alpha_f}{\beta_f} - \frac{\gamma_2}{\beta_f} q_c - \frac{p_f}{\beta_f} + \frac{s_f}{\beta_f} \end{aligned} \tag{79}$$

which shows that the demand for charging stations is increased by the subsidy s_f. Inverse demand functions are

$$\begin{aligned} p_c &= \alpha_c - \beta_c q_c - \gamma_1 q_d - \gamma_2 q_f \\ p_d &= \alpha_d - \beta_d q_d - \gamma_1 q_c \\ p_f &= \alpha_f - \beta_f q_f - \gamma_2 q_c + s_f \end{aligned} \tag{80}$$

5.2.8 Static profit maximization

The subsidy provided to buy EVs is obtained trough a tax levied on the profits from GVs production. Hence, profits of the monopolist are given by

$$\pi = (p_c - c_c) q_c + (p_d - s_d - c_d) q_d + (p_f - c_f) q_f \tag{81}$$

The FOCs for the maximization problem are

$$\begin{aligned} \frac{\partial \pi}{\partial q_c} &: \alpha_c - 2\beta_c q_c - 2\gamma_1 q_d - 2\gamma_2 q_f - c_c = 0 \\ \frac{\partial \pi}{\partial q_d} &: \alpha_d - 2\beta_d q_d - 2\gamma_1 q_c - s_d - c_d = 0 \\ \frac{\partial \pi}{\partial q_f} &: \alpha_f - 2\beta_f q_f - 2\gamma_2 q_c + s_f - c_f = 0 \end{aligned} \tag{82}$$

The profit maximizing quantities are

$$
\begin{aligned}
q_c^* &= \frac{1}{2X}\left[\beta_d\beta_f(\alpha_c - c_c) - \gamma_1\beta_f(\alpha_d - s_d - c_d) - \gamma_2\beta_d(\alpha_f + s_f - c_f)\right] \\
q_d^* &= \frac{1}{2X}\left[-\gamma_1\beta_f(\alpha_c - c_c) + (\beta_c\beta_f - \gamma_2^2)(\alpha_d - s_d - c_d) + \gamma_1\gamma_2(\alpha_f + s_f - c_f)\right] \\
q_f^* &= \frac{1}{2X}\left[-\gamma_2\beta_d(\alpha_c - c_c) + \gamma_1\gamma_2(\alpha_d - s_d - c_d) + (\beta_c\beta_d - \gamma_1^2)(\alpha_f + s_f - c_f)\right]
\end{aligned} \tag{83}
$$

where $X = \beta_c\beta_d\beta_f - \beta_f\gamma_1^2 - \beta_d\gamma_2^2$. An increase in tax on profits from production of EVs s_d increases the quantity of clean cars and charging stations, whereas it decrease the quantity of dirty cars. Similarly, a subsidy to charging stations increases the quantity of clean cars and charging stations, whereas it decrease the quantity of dirty cars.

Given the optimal quantities in (83), we can find the prices maximizing profits as

$$
\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d + s_d + c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f + s_f + c_f)
\end{aligned} \tag{84}
$$

Optimal prices show that a tax on production of dirty cars increases the price by one half because the monopolist is able to pass half of the cost on through consumers. Similarly, a subsidy to demand of charging stations increases the price charged by the monopolist by one half because demand is now higher and he can charge a higher price.

5.2.9 Tax on demand for dirty cars and subsidy to investment in charging stations

Another possible combination of policy instruments is to tax the demand for dirty cars and subsidize the investment in charging stations. The optimal quantities are as in the previous case, whereas optimal prices are

$$
\begin{aligned}
p_c^* &= \frac{1}{2}(\alpha_c + c_c) \\
p_d^* &= \frac{1}{2}(\alpha_d - s_d + c_d) \\
p_f^* &= \frac{1}{2}(\alpha_f - s_f + c_f)
\end{aligned} \tag{85}
$$

Optimal prices show that a tax on demand reduces the demand for GVs and hence the monopolist has to reduce the price of GVs in order to keep on selling them. However, it does not reduce the price by the same amount of the tax but only by half of it, because of its market power. Similarly, a subsidy to production only reduces the price of charging stations by half of the subsidy, because the monopoly is able to retain half of the subsidy thanks to its market power.

The present combination of tax on demand of GVs and subsidy to investment in charging stations seems superior to the previous combination as it leads to a reduction in price. However, it might also be, depending on the values of the subsidy and of the tax, the reduction in the price of dirty cars is such that demand for dirty cars increase more than the one for EVs. The more effective policy would be to have a low tax and a high subsidy, as this would imply a small reduction in the price of dirty cars and a large reduction in the cost of investing in charging station. This outcome would also be in line with the assumption of a balanced budget for the government. Since we assumed $s_d q_d = s_f q_f$ and give that it is plausible to assume that $q_d > q_f$ it could be possible to obtain the same revenue from both sides charging a smaller tax to demand of dirty cars as the revenues would be increased by the larger quantity. Also notice that this result depends on the choice of having a monopolistic platform.

Appendix C - Competitive production of dirty cars case

In this section we assume that the demand side is as before, with consumers on one side of the model and investors on the other side. However, we assume that the monopolistic platform only produces EVs and charging stations, whereas GVs are produced competitively.

Static profit maximization

Given that production of dirty cars is competitive, $p_d = c_d$, where c_d is the marginal cost of producing GVs.

We assume that total costs of production for the monopolist are $TC = c_c q_c + c_f q_f$. Hence, profits of the monopolist are given by

$$\pi = (p_c - c_c)q_c + (p_f - c_f)q_f$$

The FOCs for the maximization problem after substituting for the inverse demand for dirty cars, which is now given for the monopolist, are

$$\begin{aligned} \frac{\partial \pi}{\partial q_c} &: \quad \alpha_c - 2\beta_c q_c - (\gamma_2 + \gamma_4)q_f - \frac{\gamma_1}{\beta_d} q_d - c_c = 0 \\ \frac{\partial \pi}{\partial q_f} &: \quad \alpha_f - 2\beta_f q_f - (\gamma_2 + \gamma_4)q_c - c_f = 0 \end{aligned} \tag{86}$$

Moreover, we know

$$\alpha_d - \beta_d q_d - \gamma_1 q_c - c_d = 0 \tag{87}$$

Solving the system of equations, the profit maximizing quantities are

$$\begin{aligned} q_c^* &= \frac{1}{X_m}\left[2\beta_d\beta_f(\alpha_c - c_c) - 2\beta_f\gamma_1(\alpha_d - c_d) - \beta_d(\gamma_2 + \gamma_4)(\alpha_f - c_f)\right] \\ q_d^* &= \frac{1}{X_m}\left[-2\beta_f\gamma_1(\alpha_c - c_c) + [4\beta_c\beta_f - (\gamma_2 + \gamma_4)^2](\alpha_d - c_d) + \gamma_1(\gamma_2 + \gamma_4)(\alpha_f - c_f)\right] \\ q_f^* &= \frac{1}{X_m}\left[-\beta_d(\gamma_2 + \gamma_4)(\alpha_c - c_c) + \gamma_1(\gamma_2 + \gamma_4)(\alpha_d - c_d) + (2\beta_c\beta_d - \gamma_1^2)(\alpha_f - c_f)\right] \end{aligned} \tag{88}$$

where $X_m = 4\beta_c\beta_d\beta_f - 2\beta_f\gamma_1^2 - \beta_d(\gamma_2 + \gamma_4)^2$. The optimal prices are then

$$\begin{aligned} p_c^* &= \frac{1}{X_m}[\beta_d(2\beta_c\beta_f - \gamma_2\gamma_4)(\alpha_c - c_c) - \beta_d\gamma_4^2\alpha_c - (2\beta_f\gamma_1^2 + \beta_d\gamma_2^2)c_c \\ &\quad -\gamma_1[2\beta_c\beta_f + \gamma_4(\gamma_2 + \gamma_4)](\alpha_d - c_d)] - [\beta_c\beta_d(\gamma_2 - \gamma_4) - \gamma_1^2\gamma_4](\alpha_f - c_f) \\ p_d^* &= c_d \\ p_f^* &= \frac{1}{X_m}[-\beta_d\beta_f(\gamma_2 - \gamma_4)(\alpha_c - c_c) + \beta_f\gamma_1(\gamma_2 - \gamma_4)(\alpha_d - c_d) \\ &\quad +(2\beta_c\beta_d\beta_f - \beta_f\gamma_1^2 - \beta_d\gamma_2\gamma_4)(\alpha_f + c_f) - \beta_d(\gamma_4^2\alpha_c + \gamma_2^2 c_f)] \end{aligned} \tag{89}$$

Appendix E - Graphs

We use graphs to show what happens to the optimal quantities in (8) when moving crucial parameters as γ_2 and γ_4.

Figure 15: The optimal quantity of EVs is increasing with the network effects.

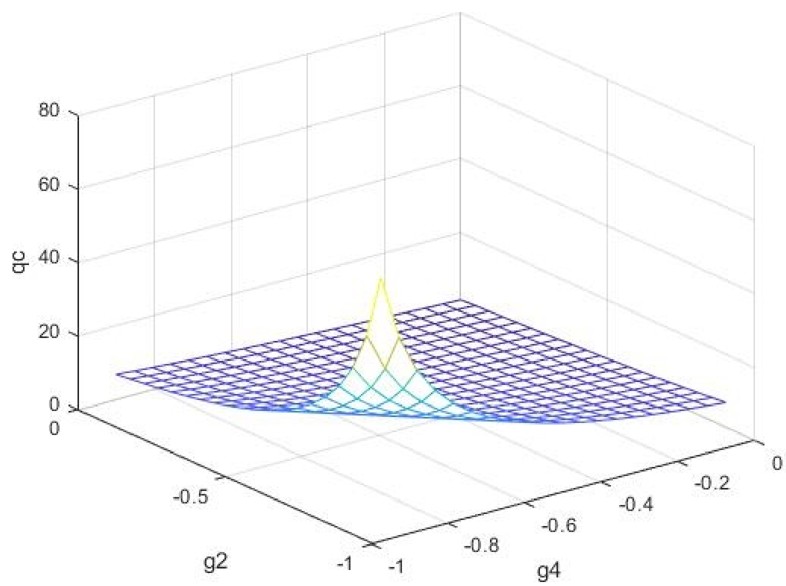

Figure 16: The optimal quantity of GVs is increasing with the network effects

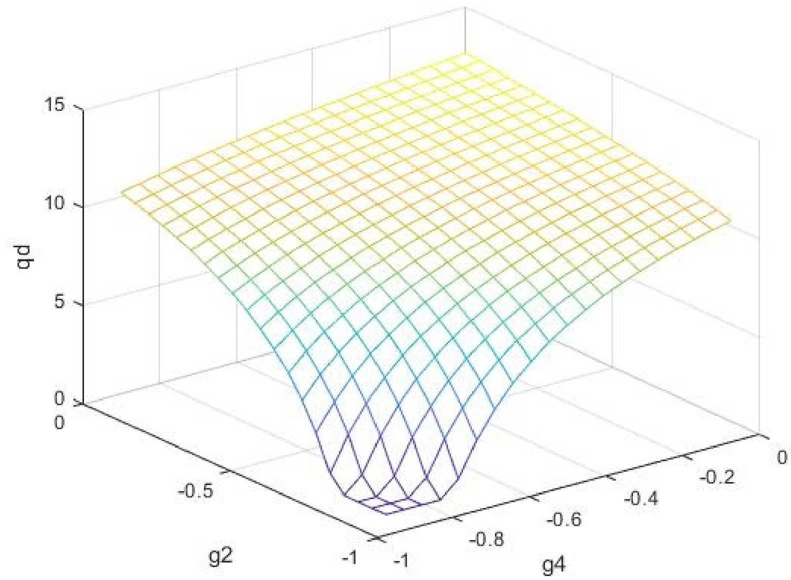

Figure 17: The optimal quantity of EVCSs is increasing with the network effects.