

Sea level rise and the social cost of flood insurance subsidies

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Abstract

We consider the interaction between sea level rise due to climate change and imperfect flood insurance subsidies in an analytically tractable model with forward-looking investment decisions. Using the vulnerable Tampa Bay area to calibrate the model, we demonstrate that subsidised flood insurance leads to large social costs, driven by excessive development in flood-prone coastal areas. We then consider optimal adaptation in terms of forward-looking urban planning. With actuarially fair insurance, sea level rise leads to cities gradually retreating uphill from the coast, with areas proximate to the encroaching shoreline progressively abandoned.

1 Introduction

Coastal communities across the world are threatened by inexorable sea level rise, driven by anthropogenic climate change. The IPCC Fifth Assessment Report predicts the global mean sea level to increase by up to one metre by 2100 (Stocker et al., 2013); but there have been suggestions that ice-sheet break-up might lead to even more rapid increases (Hansen et al., 2016).¹ Without adaptation measures, floods could affect some 100-200m people every year by 2100.

Indeed, past coastward migration implies that flood risks are already substantial today, even before any further increases in the sea level. Some \$2tn of assets, and 270m people, are currently exposed to sea level extremes recurring once every 100 years (Field et al., 2014). The movement towards the coast is often exacerbated by poor incentives. Improper design of flood insurance schemes has been suggested to have driven excessive development in areas exposed to flood hazards (Ben-Shahar and Logue, 2016).

This paper makes two key contributions to the economic analysis of insurance subsidies and sea level rise. First, we assess the social cost (or deadweight loss) of existing insurance schemes. Using a tractable theoretical model calibrated to the

¹The topic is hotly debated; for a less alarming perspective, see Ritz et al. (2015).

key institutional features on the US National Flood Insurance Program (NFIP), we assess the incentives of households to purchase insurance.² Using the vulnerable region around Tampa Bay as our leading example, we demonstrate that the NFIP offers substantially underpriced insurance for residential building located at low elevations. The present value of the deadweight loss can reach 50% of the construction cost of the property. There are thousands of properties near Tampa Bay which receive such implicit subsidies, and the cost of these subsidies, once it materialises in the form of major hurricane-induced flooding, is likely to be large and, ultimately, to fall on the taxpayer.

Second, we look into the future and consider how coastal development can adapt to future sea level rise. We find that cities should start gradually migrating up the shore, with the region at the greatest risk of flooding gradually abandoned (and at most only partially rebuilt in response to flood events). Underpriced insurance will exacerbate the cost of sea level rise, as long-lived capital stocks are developed in areas which will be vulnerable to progressively more severe storm surges and eventual inundation by the rising sea.

We construct a tractable theoretical model to study these questions. The model allows for the analysis of different types of insurance schemes and could be readily extended to consider related issues, including distributional questions related to coastal insurance and optimal investments in sea wall defense.

The model also offers a set of empirical implications. The data required to test many of these implications do exist at a highly disaggregate form. We calibrate the model carefully to the Tampa Bay region, using data at a fine spatial resolution, to assess the aggregate costs of underpriced flood insurance in this particular case.³

2 Model

2.1 The city

Consider the city depicted in figure 1. It stretches from the ocean on the left and continues indefinitely to the right. The city is on a linear gradient and rises in distance from the sea at angle $\theta > 0$. Sea level rises at constant rate $\tilde{\kappa} \geq 0$.

While the city exists in two dimensions, it is convenient to study an equivalent setting in one dimension. The one-dimensional city stretches from the left-hand interval at the ocean, and parcels of land are distinguished by their distance x from the coast. Since the sea rises at constant rate $\tilde{\kappa}$, it is equivalent to having the

²In particular, we focus on the imprecisely differentiated insurance premia. There are many other features, such as the explicit subsidies offered to old properties constructed before the local flood insurance maps we first drawn up, which we leave for further work. See Michel-Kerjan (2010) for more details.

³More detailed empirical testing is work in progress.

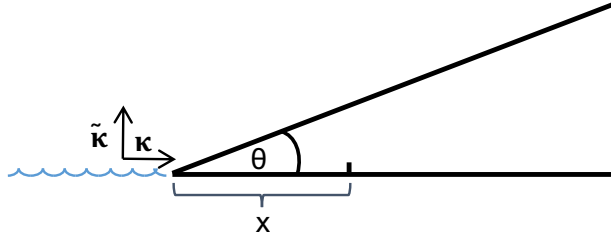


Figure 1: θ is the gradient of the hill. $\tilde{\kappa}$ is the rate of sea level rise, and κ is the rate at which the coastline moves horizontally inwards.

left-hand interval move inwards to the right at constant rate

$$\kappa = \frac{\tilde{\kappa}}{\tan(\theta)}.$$

The coastal distance of a fixed parcel of land decreases over time until it reaches zero, at which point the property is enveloped by the sea.

Apart from property loss arising from deterministic sea level rise, the city is also threatened by stochastically occurring storm surges. When a storm surge occurs, it reaches a stochastic height h . We assume the city has a constant slope θ , so that the elevation of a property at x is given by $y \equiv \kappa x$. It is to be understood in what follows that location x and elevation y have a one-to-one relationship, so that $y = y(x)$; we do not explicitly indicate this relation in what follows. A storm surge causes damage according to a function $\gamma D(y, h)$, in which γ is the construction cost and $D(\cdot)$ gives the proportional damage. Denote the maximal possible damage fraction by $\bar{D}(y) \equiv \max_h D(y, h)$.

Storm surges arrive at Poisson rate λ per year. We assume that storm surge heights are exponentially distributed, which is consistent with the evidence of Lin and Emanuel (2016), obtained from hurricane models embedded within larger climate models. In other words, the distribution of h is given by

$$\Pr[h' \leq h] = F(h) = 1 - \exp(-h/\sigma), \text{ for } h \geq 0, \quad (1)$$

with the pdf given by $f(h) = \sigma^{-1} \exp(-h/\sigma)$. In particular, this means that, for any $y \geq 0$, $h \geq y$, and defining the depth to which a property is submersed as $d \equiv h - y$,

$$\Pr[h' \geq h] = \Pr[h' \geq y] \Pr[d' \geq d] = \exp(-y/\sigma) \exp(-d/\sigma),$$

so that the conditional distribution is location-independent.

2.2 Agents

An unlimited number of agents have the option to purchase a unit of capital at exogenous cost $\gamma \geq 0$. They can place capital anywhere they want, though once placed, it cannot be moved. Capital depreciates at rate $\delta > 0$.

A resident with a unit of capital at location x obtains additive flow utility from consumption of amenities $u(x)$; disutility from congestion $\beta k(x)$, in which $k(x)$ denotes the capital installed at location x ; and a linear benefit from consumption of money. The utility flow is thus given by

$$u(x) - \beta k(x) - p(i, y)$$

in which $p(\cdot)$ denotes the premium flow paid for flood insurance, which is a function of the chosen level of coverage i and the elevation y . Individuals prefer living close to the shore, so that $u' \leq 0$. Agents are small and take the aggregate quantity of capital as given. They discount future benefits at constant rate $\rho > 0$.⁴

2.3 Insurance

We will consider two cases for insurance. The first case is one in which the insurance contract follows the features of the NFIP. In particular, we assume the premia follow a two-part tariff:

$$p(i, y) = \begin{cases} ip^1(y) & \text{for } i \leq \tilde{i}; \\ \tilde{i}p^1(y) + (i - \tilde{i})p^2(y) & \text{for } i > \tilde{i}, \end{cases} \quad (2)$$

where the schedules $p^1(y)$, $p^2(y) > p^1(y)$, are calibrated later to the NFIP premium structure. In other words, one (high) rate applies to insurance coverage up to a given threshold, and for coverage in excess of this threshold a different (lower) rate applies.

If a resident has insurance, she can obtain a payment when a storm surge occurs, given by $C(y, h, i)$. These are given by

$$C(y, h, i) = \min\{i, \gamma D(y, h), \bar{C}\}.$$

In words, the claim covers all damage up to the amount of coverage subscribed to, but with an exogenous cap on total claims.

The second case is with actuarially fair insurance. This yields the benchmark for our welfare comparisons.⁵

⁴We assume consumers are fully rational and have perfect information regarding flood risk. There is an extensive literature on how incomplete information or behavioral issues (such as salience or optimism) may affect the demand for flood insurance (Browne and Hoyt, 2000; Hallstrom and Smith, 2005; Gallagher, 2014; Bakkensen and Barrage, 2017). On the other hand, private information and adverse selection may imply that insurance is selectively taken up by those who are most at risk (although Botzen and Van Den Bergh (2012) find that this does not seem to be an issue in the Netherlands. We leave such considerations for further extensions.

⁵Note that this is not the efficient case. There is a congestion externality in the model, so that optimal policy would tax capital, the more highly so, the more people live in a given location. We choose to use the actuarially fair case as the policy-relevant benchmark.

2.4 Equilibrium

When investing a unit of capital, the agent anticipates how sea level rise will cause the distance of a given parcel from the coast to decrease in time. To model this, it is useful to view the city at each date in “relative” terms defined by distance to the current (though ever moving) coastline. In effect, we allow the origin to move in time as the sea rises. Let $x(\tau; x', t)$ denote the distance from the coast that parcel x' in period t will be in future period τ , i.e.

$$x(\tau; x', t) = x' - \kappa(\tau - t).$$

From now on, we will always describe space relative to the shoreline, that is, in terms of x .

To simplify notation, define the distribution of capital in period τ by

$$K^{(\tau)} \equiv [k^{(\tau)}(x)]_{x \geq 0}.$$

Given a time path for the capital profile $[K^{(\tau)}]_{\tau \geq t}$, the investment rate at time $t \geq 0$ and location x is defined as

$$I(x, t; [K^{(\tau)}]_{\tau \geq t}) \equiv \lim_{dt \downarrow 0} \frac{k^{(t+dt)}(x - \kappa dt) - k^{(t)}(x)}{dt} + \delta k^{(t)}(x)$$

In other words, the rate of change of capital on a given parcel is given by investment less depreciation. Investment is irreversible, implying $I(x, t) \geq 0$. The investment rate can go to infinity if there is a discrete jump in the capital distribution.

A steady state distribution of capital is constant: $k^{(t)} = k^S$ for all t . In terms of absolute space, the distribution of capital moves like a blob up the hill, in tandem with the shoreline: never stopping and never changing shape. As $dx/d\tau = -\kappa$, we can express time in terms of space. With a slight abuse of notation, we write the steady state investment rate, as a rate of investment per unit of space, as $I^S \equiv I(x, t; k^S(x))/\kappa$. The steady state investment rate satisfies

$$I^S \equiv \begin{cases} \frac{\delta}{\kappa} k(x) - k'(x) & \text{if } \lim_{z \uparrow x} k(z) = \lim_{z \downarrow x} k(z); \\ \infty & \text{otherwise.} \end{cases}$$

Thus, wherever the steady state capital distribution is continuous, the investment rate is given by the difference between the depreciation rate and the rate of change of the steady state capital profile away from the shore. With zero investment, the capital stock decreases exponentially towards the shore, at rate δ/κ . The steady state profile must be left-continuous; i.e. upward jumps, moving away from the shore, cannot exist.

At time t , an agent takes as given the expected equilibrium distribution of capital at future dates, and seeks to maximise the net benefit of a unit of capital invested at location x' . Consider a short interval of time dt and the value of a unit of capital

at location x :

$$\begin{aligned}
V(x; [K^{(\tau)}]_{\tau \geq t}) &= \max_i (u(x) - \beta k(x) - p(i, y)) dt \\
&\quad + e^{-\rho dt} \mathbb{E} [V(x + dx; [K^{(\tau)}]_{\tau \geq t+dt}) e^{-\delta dt} + C(y, h, i)] \\
&= \max_i (u(x) - \beta k(x) + p(i, y)) dt \\
&\quad + e^{-(\rho+\delta)dt} ((1 - \lambda dt) V(x + dx; [K^{(\tau)}]_{\tau \geq t+dt}) \\
&\quad + \lambda dt \mathbb{E}_h [V(x + dx; [K^{(\tau)}]_{\tau \geq t+dt}) - \gamma D(y, h) + C(y, h, i)]).
\end{aligned}$$

in which the last expectation is taken with respect to h , given that a storm surge occurs. In words, the value of the unit of capital accrues from the flow utility over dt , plus the expected, discounted continuation value. The latter incorporates the depreciation of the unit. If there is no storm surge in the interval dt , the continuation value just reflects the fact that the unit is slightly closer to shore because of gradual sea-level rise. If there is a storm surge, the capital is damaged and must be repaired at the cost of $\gamma D(y, h)$, for which an insurance claim of $C(y, h, i)$ is received.

From now on, we will also omit the notation indicating the dependence of capital value on future capital distributions. Standard manipulations confirm that the unit value of capital satisfies

$$\begin{aligned}
(\rho + \delta)V(x) &= \max_i u(x) - \beta k(x) - p(i, y) \\
&\quad - \kappa V'(x) + \lambda \mathbb{E}_h (-\gamma D(y, h) + C(y, h, i)).
\end{aligned} \tag{3}$$

Note that this is degenerate Hamilton-Jacobi-Bellman (HJB) equation, with the investor having no dynamic control variables. Thus, the value of capital is discounted at the rate $\rho + \delta$. Value evolves due to the flow payoff, the change in the value of capital due to gradual sea-level rise, plus the expected damage due to storm surges net of insurance claims.

Given an unlimited number of agents (free entry) and a fixed external cost of capital, utility maximization implies that all opportunities for strictly positive gain will be exploited. We use this requirement to define an equilibrium.

Definition 1 *Given an initial distribution of capital, $K^{(0)}$, an equilibrium of the city model is a continuum of capital distributions, $[K^{(t)}]_{t \geq 0}$, such that at all dates t*

$$\begin{aligned}
V(x) &\leq \gamma, \quad I(x) \geq 0, \quad (V(x) - \gamma)I(x) = 0, \\
i(y) &= i^*(y) \equiv \arg \max_i -p(i, y) + \lambda [C(y, h, i)].
\end{aligned}$$

3 Impact of insurance subsidies

We first consider the impact of flood insurance subsidies in the absence of sea level rise, setting $\kappa = 0$. In this special case, investment is clearly given by $I(x) = \delta k(x)$. We first consider the NFIP flood insurance scheme (2).

3.1 NFIP insurance

The first-order condition for an interior solution to the insurance coverage problem is given by

$$p_i(i, y) = \lambda \frac{\partial}{\partial i} \mathbb{E}_h [C(y, h, i)].$$

In other words, if the desired quantity of insurance falls into the interior of the insurance premium schedule, then the cost of a marginal unit of coverage must equal the expected marginal claim. From (2), it is obvious that the marginal cost is given by $p^1(y)$ for $i < \tilde{i}$ and $p^2(y)$ for $i > \tilde{i}$. It is also possible that the optimum is a corner solution: thus, we must compare any interior maxima also with the corner candidates $i = 0$, $i = \tilde{i}$, $i = \min\{\gamma\bar{D}, \bar{C}\}$.

The steady state equilibrium under NFIP insurance premia must now satisfy

$$k^N(x) = \beta^{-1} (u(x) - p(i^*, y) - \gamma(\rho + \delta) + \lambda \mathbb{E}_h [-\gamma D(y, h) + C(y, h, i^*)]). \quad (4)$$

3.2 Actuarially fair insurance

It is well-known that actuarially fair insurance involves charging agents the expected value of the damages as premium, and fully compensating for any realised damages good. The agents will be exactly indifferent between purchasing insurance or not.⁶ In other words, the maximised HJB equation is given by

$$(\rho + \delta)V(x) = u(x) - \beta k(x) - \lambda \mathbb{E}_h \gamma D(y, h).$$

This is easily solved for the steady-state capital distribution

$$k^F(x) = \beta^{-1} (u(x) - \gamma(\rho + \delta) - \lambda \mathbb{E}_h \gamma D(y, h)). \quad (5)$$

3.3 Calibration to the Tampa Bay area

We now want to calibrate the steady state model to existing insurance schemes for NFIP, and to the characteristics of the Tampa Bay area. Tampa Bay is a bay on the western coast of Florida, surrounded by a large conurbation of the cities of Tampa, St. Petersburg and Clearwater, with a total population of over 4 million. The area is vulnerable to storm surges due to hurricane activity in the Mexican Gulf. The region was last severely hit in 1921, by a powerful hurricane which caused a storm surge exceeding 10 ft (3 m) in height. However, the region has not been severely impacted since and there has been extensive coastal development, including on areas designated as floodplain by the Federal Emergency Management Agency (FEMA).

⁶Because we work with risk-neutral agents. Risk-averse agents would insure themselves fully, passing all risk onto risk-neutral reinsurance companies, which would be making zero profits in equilibrium.

We take the insurance premia from FEMA (2017), approximating the discrete premium structure piecewise linear functions:

$$\begin{aligned} p^1(\tilde{y}) &= \max\{.02 * ((\tilde{y}^{\text{FZ}} - \tilde{y}) + 1), .14\}, \\ p^2(\tilde{y}) &= .00365 * (\tilde{y}^{\text{FZ}} - y) + .002. \end{aligned} \tag{6}$$

Here, tildes indicate elevations measured in feet; we convert back to meters for the model. The constant \tilde{y}^{FZ} is the elevation designated as the boundary of flood zone vulnerable to the ‘100-year flood’, that is, a flood with an annual probability of .01. In the Tampa Bay area, $\tilde{y}^{\text{FZ}} = 10$ ft. Figure 2 displays the premium schedules together with the actual (discrete) rates, as a function of elevation relative to the flood zone boundary. Thus, the first \$100 of coverage costs \$14 at elevations of 4 ft from the sea level or less (i.e. more than 6 ft below the flood zone boundary), but only \$2 at the flood zone boundary. Above the threshold of $\tilde{i} = \$60,000$, an additional \$100 of coverage costs \$3.5 at 1 ft above the shoreline, and \$.2 at the flood zone boundary.

We calibrate damages using USACOE (2006), who report expert estimates on marginal flooding damages to one-story residential buildings as a function of water depth, given in terms of a fraction of the replacement cost. We smooth the marginal damages to take an exponential form:

$$D'(y, h) = \alpha_1 \exp -\alpha_2(h - y),$$

with the level parameter $\alpha_1 = .9$ and the slope parameter $\alpha_2 = 1.1$. Note that the damages depend only on the depth of submersion d . Setting $D(y, y) = 0$, i.e. that damages are negligible until the flood reaches the elevation of the floor of the building, we can integrate to obtain the damage function

$$D(d) = \frac{\alpha_1}{\alpha_2} (1 - \exp(-\alpha_2(d)))$$

where we have slightly abused notation. Figure 3 displays the approximation we use. For construction cost, we use the average Florida residential construction cost of $\gamma = \$270,000$.

We now turn to the storm surge distribution. Lin and Emanuel (2016) report modelling storm surge heights as a Generalized Pareto distribution. We take their reported heights of a 100-year (3.2 m), 1000-year (4.6 m), and 10000-year (5.9 m) flood and fit these approximately to our exponential distribution (1) (a special case of the Generalized Pareto distribution), so that $\sigma \approx .659$. The return period is thus subsumed into the storm surge distribution, and we set $\lambda = 1$.

The calibration is convenient enough to characterise the equilibrium analytically. First, the insurance claim never hits the cap, so that

$$C(y, h, i) = \begin{cases} \gamma D(h - y) & \text{if } h < D^{-1}(i/\gamma); \\ i & \text{otherwise.} \end{cases}$$

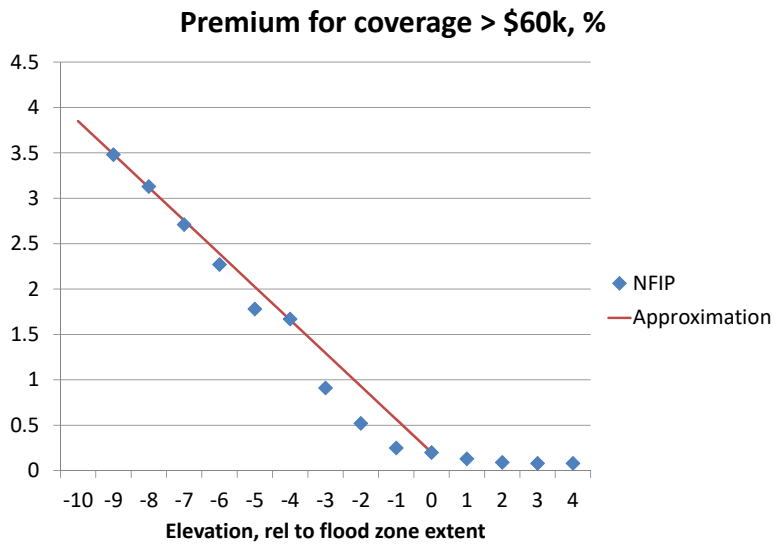
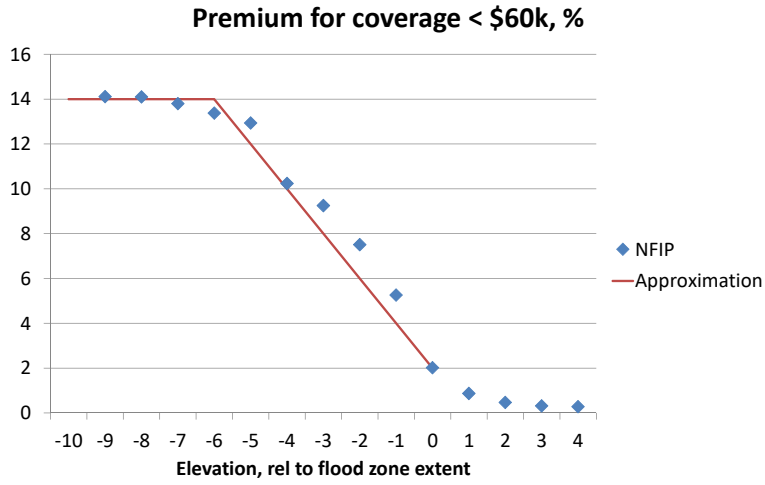


Figure 2: Calibration of insurance premia.

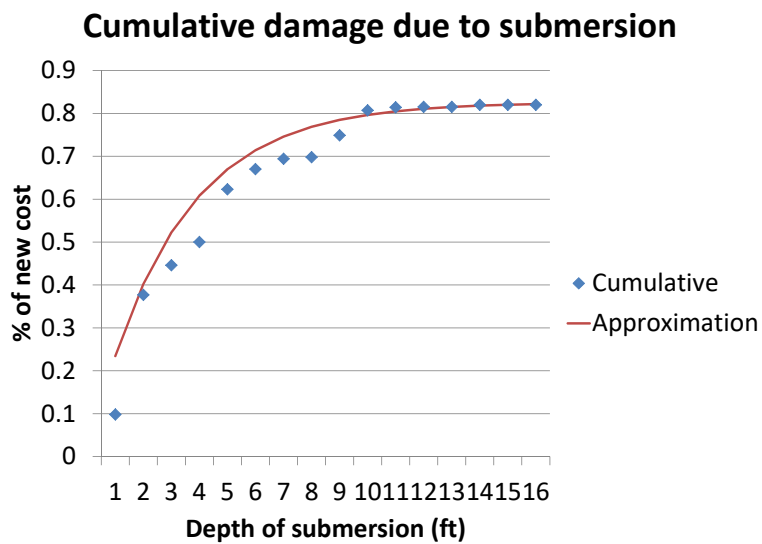
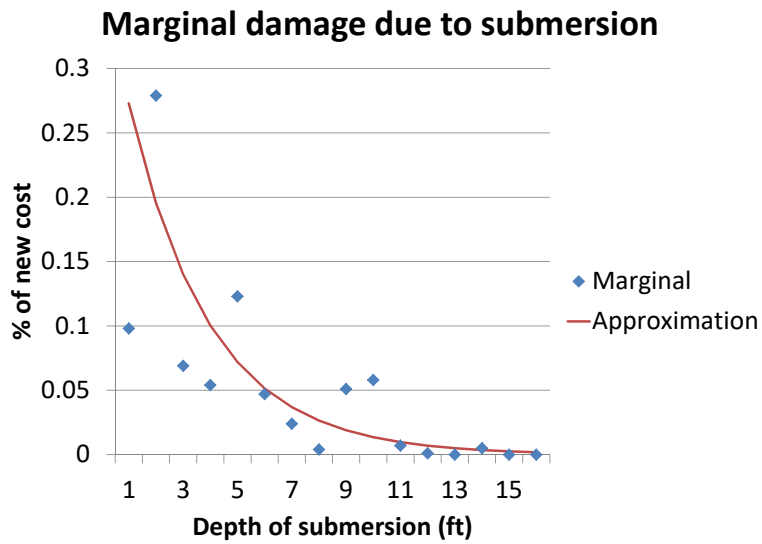


Figure 3: Calibration of flooding damages.

Thus, the expected value of the claim is

$$\mathbb{E}_h C(y, h, i) = \exp\left(-\frac{y}{\sigma}\right) \left[\int_0^{D^{-1}(\frac{i}{\gamma})} \gamma D(d) f(d) dd + \int_{D^{-1}(\frac{i}{\gamma})}^{\infty} i f(d) dd \right]$$

so that, using the Leibniz integral rule,

$$\frac{\partial}{\partial i} \mathbb{E}_h C(y, h, i) = (1 - F(y + D^{-1}(i/\gamma))) = \exp\left(-\frac{y + D^{-1}(i/\gamma)}{\sigma}\right).$$

In other words, taking more flood insurance yields one-to-one benefits, but only if the flood event is sufficiently severe to cause damages in excess of the current coverage (i.e. $h - y > D^{-1}(i/\gamma)$). This marginal benefit is clearly lower, the higher is the elevation of the property y . However, the marginal cost of insurance, given by (6), is also decreasing (stepwise) in y . It is still straightforward to obtain the insurance coverage taken at any interior optimum, or

$$i^*(y, p_j(y)) = \gamma \frac{\alpha_1}{\alpha_2} (1 - \exp(\alpha_2 y) p_j(y)^{\alpha_2 \sigma / \lambda}).$$

This interior candidate is only valid if i^* is indeed consistent with the constraints on the insurance premium schedule.

The optimum is given by comparing the surplus due to insurance between any valid interior candidates and the corner solutions. This surplus is given, for any i , by

$$\mathbb{E}_h C(y, h, i) = \exp\left(-\frac{y}{\sigma}\right) \gamma \frac{\sigma \alpha_1}{1 + \sigma \alpha_2} \left[1 - \left(1 - \frac{\alpha_2 i}{\alpha_1 \gamma}\right)^{\frac{1}{\sigma \alpha_2} + 1} \right]$$

We can now obtain an analytical characterisation of the cost of flood insurance subsidies in the Tampa Bay region:

Proposition 2 *The deadweight loss caused by the NFIP insurance scheme, compared to actuarially fair insurance, is given by*

$$DWL = (\rho + \delta)^{-1} \int_0^{y^{DWL}} k^N(x) [p(i^*(y), y) - \mathbb{E}_h C(y, h, i^*(y))] dy, \quad (7)$$

where y^{DWL} is given by

$$p_2(y^{DWL}) i^*(y^{DWL}, p_2(y^{DWL})) = \mathbb{E}_h C(y^{DWL}, h, i^*(y^{DWL}, p_2(y^{DWL}))).$$

Proof. The deadweight loss is just $V^N - V^F$. However, under actuarially fair insurance, each unit yields an expected benefit exactly equal to the opportunity cost, so that $V^F = 0$. Note that the insurance premia and claims are just transfers. We can write

$$\begin{aligned} V^N &= (\rho + \delta)^{-1} \int_0^{\infty} k^N(x) (u(x) - \beta k^N(x) - \gamma(\rho + \delta) - \mathbb{E}_h \gamma D(y, h)) dx \\ &= (\rho + \delta)^{-1} \int_0^{\infty} k^N(x) (p(i^*, y) - \mathbb{E}_h C(y, h, i^*)) dx, \end{aligned}$$

where the first equality is just the definition of social surplus, and the second equality follows from (3), setting $V(x) = \gamma$.

The last condition yields the point above which households prefer not to purchase insurance. It is easy to verify that the optimal insurance coverage is never an interior optimum below, nor a corner optimum at, the threshold \tilde{i} . Instead, an interior optimum above the threshold obtains for $y < y^{\text{DWL}}$, and the corner optimum with zero coverage elsewhere. ■

In words, the deadweight loss due to subsidised insurance arises because the extra insurance encourages development close to the shoreline. With the NFIP structure, insurance is only worth buying in $y \in [0, y^{\text{DWL}}]$; beyond this location, insurance is overly expensive and the risk-neutral individuals behave as they would in the case of actuarially fair insurance. The deadweight loss is calculated as the sum of the deviations between the NFIP insurance premia and the expected claim across all units of capital; in the absence of insurance, an installed unit of capital provides no benefit due to the opportunity cost equalling the constant marginal (and thus average) benefit. Finally, it is worth noting that the actuarially fair benchmark involves a congestion externality, and the insurance subsidies amplify this pre-existing distortion.

The above analysis allows us to obtain values for the deadweight loss associated with homes close to the shoreline. In the Tampa Bay area, very few homes are situated at the elevation of 1 ft. However, there are many homes at 2 ft and 3 ft, and the capitalised deadweight loss of having an average residential building in these locations is \$302,000 and \$140,000, or 112% and 52% of the average construction cost, respectively. A large number of homes in northwestern St. Petersburg are located at an elevation of 4 ft, implying a lifetime deadweight loss of \$41,000, or 15% of the construction cost, per building. The premium rates on low-lying buildings are capped at about 14% for coverage up to \$60,000, and are below 4% for coverage exceeding this. While the resulting premia are high, at above \$12,000 per year, they are still cheap in the sense that the expected claims easily exceed the premium payments.

Thus it is clear that the NFIP subsidised insurance may carry substantial costs for buildings close to the shoreline. From (7), it is clear that these costs are borne ultimately by the taxpayer, as they represent a net outflow from the accumulated premia. These losses will not be realised until a large storm surge materialises and leads to substantial material damages, forcing the NFIP to be bailed out.

From the above analysis, it is clear that we can obtain an estimate of the aggregate deadweight losses in the Tampa Bay area, using more fine-grained GIS data. It is to this that we turn next.

3.4 GIS analysis

TO BE WRITTEN.

4 Sea-level rise

We now consider how to model sea level rise, setting $\kappa > 0$. To obtain intuition, we set the insurance payments and claims to zero and utility from amenities to be linear. We also assume a simplified damage function. More precisely, we assume that when the storm surge arrives, it causes complete capital destruction up to some height \bar{h} , and no damage beyond that level. As capital is fully destroyed by a storm surge, the continuation value following such an event is zero; in particular, this value is independent of the height of the storm surge.⁷

The following proposition characterizes the unique steady state equilibrium under these restrictions:

Proposition 3 *Suppose $\Pr(h = \bar{h}) = 1$, $C(y, i, h) = 0$, and $u(x) = \bar{\alpha} - \alpha x$. Then, $\exists \bar{h}^*$ such that the unique steady state distribution is given by:*

1. *If $\bar{h} \leq \bar{h}^*$: The equilibrium capital stock is*

$$k^*(x) = \begin{cases} e^{\frac{\delta}{\kappa}(x-\underline{x}(\bar{h}))} \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x} - \gamma(\rho + \delta)], & \text{if } 0 \leq x < \underline{x}(\bar{h}), \\ \max \left\{ 0, \frac{1}{\beta} [\bar{\alpha} - \alpha x - \gamma(\rho + \delta)] \right\}, & x > \underline{x}(\bar{h}), \end{cases}$$

with $\underline{x} > \bar{h}$ defined by $k(\underline{x}) = \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x} - \gamma(\rho + \delta)]$ and

$$\gamma = \int_0^{\underline{x}} e^{\frac{\rho+\delta+\mu(x)}{\kappa}} \frac{1}{\kappa} \left(\bar{\alpha} - \alpha z - \beta k(\underline{x}) e^{\frac{\delta}{\kappa}(z-\underline{x})} \right) dz.$$

2. *If $\bar{h} > \bar{h}^*$: The equilibrium capital stock is*

$$k^*(x) = \begin{cases} e^{\frac{\delta}{\kappa}(x-\underline{x}_1)} \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x}_1 - \gamma(\rho + \delta + \lambda)], & \text{if } 0 \leq x < \underline{x}_1, \\ \frac{1}{\beta} [\bar{\alpha} - \alpha x - \gamma(\rho + \delta)], & \underline{x}_1 < x < \underline{x}_2(\bar{h}), \\ e^{\frac{\delta}{\kappa}(x-\underline{x}_3(\bar{h}))} \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x}_3 - \gamma(\rho + \delta)], & \text{if } \underline{x}_2(\bar{h}) \leq x < \underline{x}_3(\bar{h}), \\ \max \left\{ 0, \frac{1}{\beta} [\bar{\alpha} - \alpha x - \gamma(\rho + \delta)] \right\}, & x > \underline{x}_3(\bar{h}), \end{cases}$$

with $\underline{x}_1 < \bar{h}^*$, $\underline{x}_2 \in (\bar{h}^*, \bar{h})$, $\underline{x}_3 > \bar{h}^*$, satisfying $k(\underline{x}_1) = \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x}_1 - \gamma(\rho + \delta + \lambda)]$, $k(\underline{x}_2) = \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x}_2 - \gamma(\rho + \delta + \lambda)]$, $k(\underline{x}_3) = \frac{1}{\beta} [\bar{\alpha} - \alpha \underline{x}_3 - \gamma(\rho + \delta)]$, and

$$\begin{aligned} \gamma &= \int_0^{\underline{x}_1} e^{\frac{\rho+\delta+\lambda}{\kappa}} \frac{1}{\kappa} \left(\bar{\alpha} - \alpha z - \beta k(\underline{x}_1) e^{\frac{\delta}{\kappa}(z-\underline{x}_1)} \right) dz, \\ \gamma &= \int_{\underline{x}_2}^{\underline{x}_3} e^{\frac{\rho+\delta+\mu(x)}{\kappa}} \frac{1}{\kappa} \left(\bar{\alpha} - \alpha z - \beta k(\underline{x}_3) e^{\frac{\delta}{\kappa}(z-\underline{x})} \right) dz + \gamma e^{\int_{\underline{x}_2}^{\underline{x}_3} \frac{\rho+\delta+\mu(z)}{\kappa} (z-\underline{x}_3) dz}. \end{aligned}$$

⁷Partial destruction of capital would mean the continuation value would be related to the stock of capital remaining after the surge, complicating the model.

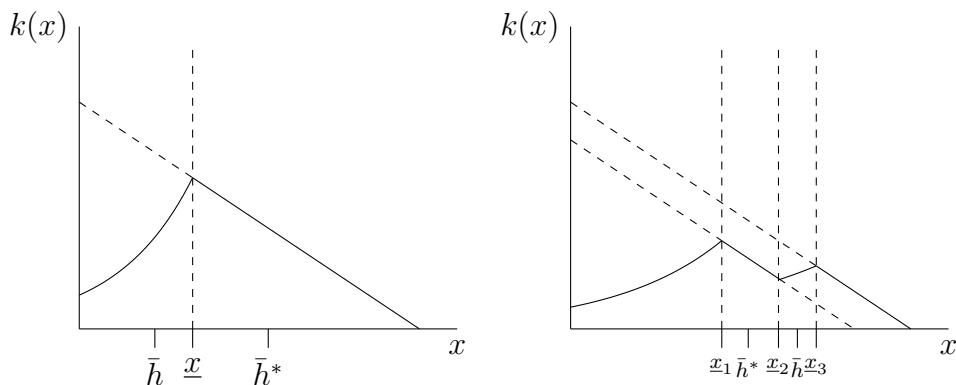


Figure 4: The steady state with limited (*left*) or severe (*right*) storm surges.

In words, the equilibrium structure is as follows. For a ‘small’ extent of the flood zone (low \bar{h}), the steady state features capital build-up to a threshold \underline{x} , which lies strictly beyond the flood zone, and depreciation between this threshold and the advancing shoreline (the left panel in Figure 5). Recalling that we are working in coordinates relative to the (moving) shoreline, this means that a unit of capital built up at the edge of the city gradually ‘moves up’ the city, getting closer to the approaching shoreline, with more capital being constructed around it as the location becomes more desirable. At some point, the approaching shoreline is sufficiently close that the buildings at this location are all left to gradually depreciate. This depreciation is allowed to commence even before the storm surge zone has reached the building. As the flood zone reaches further up, the ‘depreciation zone’ also becomes wider: capital is at risk even further from the shore.

If the flood zone is wide enough, there is a dual-peaked structure (the right panel in Figure 5). A unit of capital built up on the outskirts again moves up as more capital is built around it. There is a zone of depreciation surrounding the furthest extent of the flood zone: it is too risky to maintain a very dense stock of capital in the flood zone. However, the flood zone is wide enough that investment is still profitable within it, as long as the capital density is not too large. Thus there is renewed investment, until the buildings are finally left to depreciate and to be eventually inundated. An increase in the extent of the flood zone pushes the ‘temporary depreciation zone’ outwards, but does not affect the distance at which the final depreciation begins.

It is easy to determine asset prices under sea level rise. For capital in any interval with strictly positive investment, the asset price in equilibrium must equal the exogenous capital price γ . In the depreciation zones, the asset prices fall below γ , equalling the present value of future benefits. With the two-peaked structure, capital prices dip below γ as the first depreciation zone is reached, but then increase back up towards γ from the boundary of the storm surge zone.

5 Insurance subsidies and sea level rise

TO BE WRITTEN.

6 Conclusions

We have presented a simple yet rich model of urban form, insurance and adaptation to sea-level rise. We have shown that the US National Flood Insurance Program offers subsidised insurance to even newly-built low-elevation properties at high risk of flooding due to storm surges. We have calibrated the model to the Tampa Bay region and find that a large number of properties at risk of flooding impose a costly burden on the NFIP and, ultimately, to the taxpayer. We have also discussed how coastal cities should adapt to future sea level rise and consequent increases in the severity of future storm surges.

The model is tractable and can be employed to study alternative insurance policies and questions of whether, for example, flood insurance should be provided on competitive markets instead of the current, government-led system (Michel-Kerjan, 2010). It also has the potential to be extended towards other questions. The distributional effects of cheap flood insurance being provided to owners of valuable homes near the shore has been one of the issues often discussed in the context of the NFIP (Kousky, 2017). Agglomeration effects may make city shapes ‘sticky’ and thus delay adaptation measures, leading to unnecessary social costs. Finally, the model provides a template for empirical analysis using the rich sets of spatially disaggregated data which are available. We leave these extensions for future work.

7 Appendix

7.1 Proof of proposition 3

Proof. Consider an arbitrary $\bar{h} \geq 0$. Recall that an equilibrium requires $V(x) \leq \gamma$, $I \geq 0$, with complementary slackness. Integrating (3), the value of a unit of capital at location x is given by

$$V(x) = \int_0^x e^{\frac{\rho+\kappa+\mu(z)}{\kappa}(z-x)} \frac{1}{\kappa} (\bar{\alpha} - \alpha x - \beta k(z)) dz$$

The value function is clearly continuous in x , \bar{h} and the steady-state capital distribution $k(z)$.

First note if investment takes place on an interval, the value function must satisfy $V(x) = \gamma$, $V'(x) = 0$ across this interval. From (3), this implies that the associated capital stock profile must satisfy

$$k^I(x) \equiv \frac{1}{\beta} (\bar{\alpha} - \alpha x - (\rho + \delta + \mu(x))\gamma). \quad (8)$$

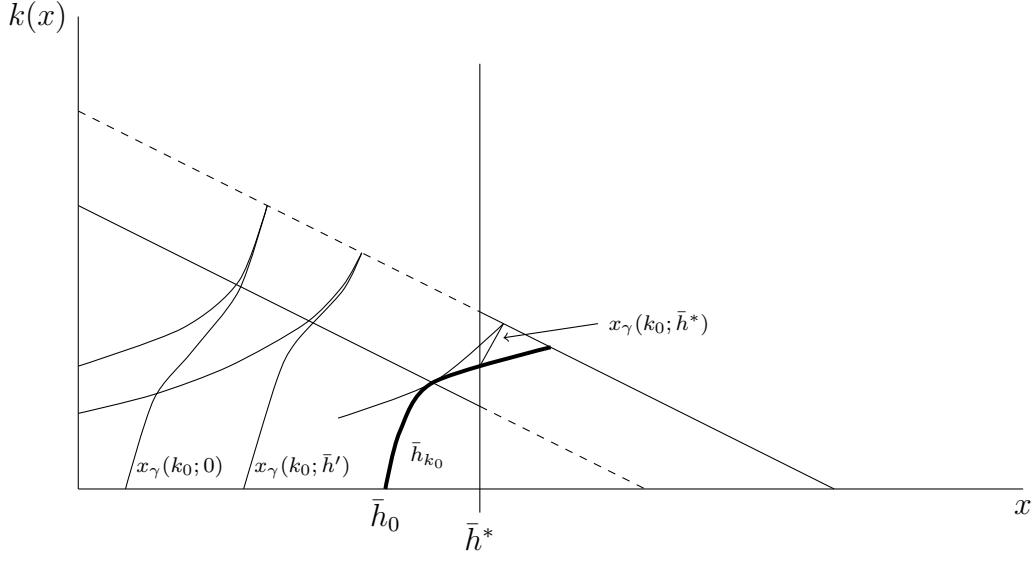


Figure 5: Constructing the steady state

Such ‘continuous investment lines’ are plotted on Figure 5 for both $x < \bar{h}$, $x > \bar{h}$.

As $\lim_{x \downarrow 0} V(x) = 0$, close to the shoreline there is no investment and the capital stock just depreciates over time. We call paths on which capital just depreciates the entire way to $x = 0$ ‘depreciation paths’, and index them by the capital stock at $x = 0$, denoted k_0 . Given this, the path of the capital stock is $k^D(x; k_0) = k_0 e^{\frac{\delta}{\kappa} x}$. The value of capital along a depreciation path is given by

$$\begin{aligned}
 V^D(x, k_0; \bar{h}) &= \int_0^x e^{\int_x^z \frac{\rho + \delta + \mu(z')}{\kappa} dz'} \frac{1}{\kappa} (\bar{\alpha} - \alpha z - \beta k(z)) dz \\
 &= e^{\frac{\rho + \delta}{\kappa} (\min\{\bar{h}, x\} - x)} \int_0^{\min\{\bar{h}, x\}} e^{\frac{\rho + \delta + \lambda}{\kappa} (z - \min\{\bar{h}, x\})} \frac{1}{\kappa} (\bar{\alpha} - \alpha z - \beta k_0 e^{\frac{\delta}{\kappa} z}) dz \\
 &\quad + \int_{\min\{\bar{h}, x\}}^x e^{\frac{\rho + \delta}{\kappa} (z - x)} \frac{1}{\kappa} (\bar{\alpha} - \alpha z - \beta k_0 e^{\frac{\delta}{\kappa} z}) dz.
 \end{aligned} \tag{9}$$

Some partial derivatives of such a depreciation path are

$$\frac{\partial V^D}{\partial x} = -\frac{\rho + \delta + \mu(x)}{\kappa} V^D(x, k_0; \bar{h}) + \frac{1}{\kappa} \left(\bar{\alpha} - \alpha x - \beta k_0 e^{\frac{\delta}{\kappa} x} \right), \quad (10)$$

$$\frac{\partial^2 V^D}{\partial x^2} = -\frac{\rho + \delta + \mu(x)}{\kappa} \frac{\partial V^D}{\partial x} - \frac{1}{\kappa} \left(\alpha + \frac{\delta}{\kappa} \beta k_0 e^{\frac{\delta}{\kappa} x} \right), \quad (11)$$

$$\frac{\partial V^D}{\partial k_0} = -\frac{\beta}{\kappa} \int_0^x e^{\int_x^z \frac{\rho + \delta + \mu(z')}{\kappa} dz'} e^{\frac{\delta}{\kappa} z} dz, \quad (12)$$

$$\frac{\partial^2 V^D}{\partial k_0^2} = 0, \quad (13)$$

$$\frac{\partial V^D}{\partial \bar{h}} = \begin{cases} 0 & \text{if } x < \bar{h}, \\ \left(e^{\frac{\delta}{\kappa}(\bar{h}-x)} - 1 \right) e^{\frac{\rho + \delta}{\kappa}(\bar{h}-x)} \frac{1}{\kappa} \left(\bar{\alpha} - \alpha \bar{h} - \beta k_0 e^{\frac{\delta}{\kappa} \bar{h}} \right) & \text{otherwise.} \end{cases} \quad (14)$$

From these, we can readily deduce several things. First, clearly $V_x^D(0, k_0; \bar{h}) > 0$. Second, at any point such that $V_x^D = 0$, $V_{xx}^D < 0$, so that V^D is quasi-concave in x . Third, $V_{k_0}^D < 0$ for $x > 0$. Fourth, $V_{\bar{h}}^D < 0$ for $x > \bar{h}$. Fifth, note that for a very large x , $V^D(x, k_0; \bar{h})$ becomes negative, i.e. the depreciation path value must have a local (and thus also global) maximum somewhere.

For any given k_0 and \bar{h} , we define $x_\gamma(k_0; \bar{h})$ implicitly by

$$V^D(x_\gamma, k_0; \bar{h}) = \gamma, \quad (15)$$

$$V_x^D(x_\gamma, k_0; \bar{h}) \geq 0. \quad (16)$$

In other words, starting on a depreciation path at $(x_\gamma, k_0 e^{\frac{\delta}{\kappa} x_\gamma})$ will yield a payoff of exactly γ . As V^D is quasi-concave in x , condition (16) ensures that for any k_0 , x_γ is uniquely defined (if it exists).

Still taking \bar{h} as fixed, implicitly differentiating (15)

$$\left. \frac{dk_0}{dx} \right|_{x=x_\gamma(k_0; \bar{h})} = -\frac{V_x^D}{V_{k_0}^D} \geq 0,$$

where the inequality again follows from (16). Thus, there is a monotonic relationship between x_γ and k_0 . As $k(x_\gamma) = k_0 e^{\frac{\delta}{\kappa} x_\gamma}$, we can obtain a relationship between $k(x_\gamma)$ and x_γ . This also has a positive slope, as

$$\frac{dk(x_\gamma)}{dx_\gamma} = e^{\frac{\delta}{\kappa} x_\gamma} \left(\frac{dk_0}{dx_\gamma} + \frac{\delta}{\kappa} k_0 \right) \geq \frac{\delta}{\kappa} k(x_\gamma) \geq 0. \quad (17)$$

We call such a locus, for a given \bar{h} , a γ -locus. The slope begins at $k(x_\gamma) = 0$ and extends up, terminating at $(x_\gamma, k(x_\gamma))$ such that

$$V_x^D(x_\gamma(k_0; \bar{h}), k_0; \bar{h}) = 0. \quad (18)$$

We denote by $\bar{k}_0(\bar{h})$ the value of k_0 which satisfies (18).

Our assumptions and the definition of V^D guarantee that, for all \bar{h} , $\exists x_\gamma(0; \bar{h}) > 0$. Consider, for now, only loci satisfying $x_\gamma(k_0; \bar{h}) < \bar{h}$. It is straightforward to see that, for any \bar{h} , the point $(x_\gamma(\bar{k}_0(\bar{h})), \bar{k}_0(\bar{h}))$ lies on the continuous investment line for $x > \bar{h}$.

At this point, the γ -locus has slope $\frac{\delta}{\kappa}k(x_\gamma)$, which coincides with the slope of the depreciation path at that point. As an example, see the locus $x_\gamma(k_0; 0)$ on Figure 5.

We will now determine how the loci $k = k(x_\gamma) = k_0 e^{\frac{\delta}{\kappa}x_\gamma(k_0; \bar{h})}$ change with \bar{h} . Holding $k(x_\gamma)$ fixed implies $dk_0/d\bar{h} = -(\delta/\kappa)k_0 dx_\gamma/d\bar{h}$. Totally differentiating (15) gives

$$\frac{dx_\gamma}{d\bar{h}} \left(1 + \frac{\delta}{\kappa}k_0 \frac{dx_\gamma}{dk_0} \right) = -\frac{V_{\bar{h}}^D}{V_{x_\gamma}^D} \geq 0,$$

implying x_γ grows with \bar{h} , holding $k(x)$ fixed. In other words, the loci move to the right as \bar{h} grows (Figure 5).

There will exist a fixed point \bar{h}_0 , satisfying $x_\gamma(0; \bar{h}_0) = \bar{h}_0$ (our assumptions guarantee it is possible to reach the value γ even with a very large \bar{h} , for some low capital stock, guaranteeing the fixed point will exist). Increasing \bar{h} further will not shift x_γ , i.e. for $\bar{h} > \bar{h}_0$, $x_\gamma(0; \bar{h}) = \bar{h}_0$. Such a fixed point will also exist for some higher levels of k_0 ; we trace the locus of such fixed points, denoted \bar{h}_{k_0} , as the thick line in Figure 5. From each of these points, a locus $x_\gamma(k_0; \bar{h}_{k_0})$ will extend to the continuous investment line.

Note that, along the locus $V^D(\bar{h}, k_0; \bar{h}) = \gamma$, the depreciation value is given by (8):

$$V^D(\bar{h}, k_0; \bar{h}) = \int_0^{\bar{h}} e^{\frac{\rho+\delta+\lambda}{\kappa}(z-\bar{h})} \frac{1}{\kappa} \left(\bar{\alpha} - \alpha z - \beta k_0 e^{\frac{\delta}{\kappa}z} \right) dz,$$

so that

$$\frac{dV^D}{d\bar{h}} = -\frac{\rho + \delta + \lambda}{\kappa} V^D + \frac{1}{\kappa} \left(\bar{\alpha} - \alpha \bar{h} - \beta k_0 e^{\frac{\delta}{\kappa} \bar{h}} \right).$$

This is clearly zero on the lower continuous investment line. As the locus \bar{h}_{k_0} satisfies

$$\left. \frac{dk_0}{d\bar{h}} \right|_{x_\gamma(k_0; \bar{h}) = \bar{h}} = -\frac{V_{\bar{h}}^D}{V_x^D},$$

the the depreciation path (with slope given by (17)) is tangent to the locus where the latter crosses the lower continuous investment line. We can now trace that particular depreciation trajectory up to where it meets the upper continuous investment line. This point is the topmost point of some locus $x = x_\gamma(\bar{k}_0(\tilde{h}; \tilde{h}))$.

To construct the equilibrium for $\bar{h} \leq \tilde{h}$, note first that the capital stock of the economy must switch onto the depreciation path at a location x such that $V^D(x) \geq \gamma$; otherwise, investment at (or just before) x would not be profitable. Of course, free entry also requires that this cannot yield excess profits, i.e. $V^D(x) \geq \gamma \leq \gamma$. Thus, the depreciation path must start on the locus $x_\gamma(k_0; \bar{h})$. Denote the point at which this locus crosses the upper continuous investment line, which is also the top of the locus, by $x^*(\bar{h})$. Suppose the economy gets on the depreciation path at some

$x < x^*(\bar{h})$. Then $V^D(x) = \gamma$, and in the neighbourhood to the right of x , the flow payoff exceeds the flow payoff on the continuous investment line. Thus,

$$\begin{aligned} V(x + \Delta) &= \int_x^{x+\Delta} e^{\frac{\rho+\delta}{\kappa} z} \frac{1}{\kappa} (\bar{\alpha} - \alpha z - \beta k(z)) dz + e^{-\frac{\rho+\delta}{\kappa} \Delta} V^D(x) \\ &\geq \int_x^{x+\Delta} e^{\frac{\rho+\delta}{\kappa} z} \frac{1}{\kappa} (\bar{\alpha} - \alpha z - \beta k^I(z)) dz + e^{-\frac{\rho+\delta}{\kappa} \Delta} \gamma \\ &= \gamma, \end{aligned}$$

where the inequality follows from the fact that the flow payoff is lower on the continuous investment line and the continuation payoff must be below γ on a higher depreciation path at x (as x cannot be the start of the depreciation path), and the equality just follows from the definition of the continuous investment line (note that $x > \bar{h}$ by supposition here). The same argument can be used to show that the capital profile must, in fact, coincide with $k^I(x)$ for all $x > x^*(\bar{h})$.

For $\bar{h} > \bar{h}^*$, the depreciation path cannot begin at the point $x_\gamma(\bar{k}_0(\bar{h}); \bar{h})$, as the depreciation path beginning here reaches below the locus \bar{h}_{k_0} , where it will violate the condition $V^D \leq \gamma$. Using arguments as before, it can also not begin anywhere on the locus $x_\gamma(k_0; \bar{h})$ satisfying $x \geq \bar{h}$. Thus, the depreciation path must begin somewhere on \bar{h}_{k_0} with $x < \bar{h}$. The only possible point is for the depreciation path to begin at the point at which the depreciation path starting at $(x_\gamma(\bar{h}^*; \bar{h}^*), k^I)$ is tangent to the locus \bar{h}_{k_0} (arguments as used in the previous part apply here). Before this point, investment is continuous on the lower locus $k^{I\lambda}$. However, unless $\bar{h} > \bar{x}$, then there is an earlier phase of depreciation investment connecting the two continuous investment loci. At both the beginning \underline{x}_3 and the end \underline{x}_2 of this intermediate depreciation locus, the value of a unit of capital must equal γ . We must then have $\underline{x}_2 < \bar{h} < \underline{x}_3$. Suppose $\underline{x}_2 \geq \bar{h}$; then the capital profile yields higher flow payoffs everywhere between \underline{x}_3 and \underline{x}_2 than $k^I(x)$ would, and both yield γ at \underline{x}_2 , so that the proposed intermediate depreciation would yield strictly higher value at \underline{x}_3 . ■

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