Water quantity management in a heterogeneous landscape with strategic farmers

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Abstract

Agricultural production contributes to many environmental problems. In semi-arid areas, agricultural irrigation causes the so-called waterlogging phenomenon. This phenomenon is both spatial and dynamic since percolations depends on soil quality summed up in landscape heterogeneity and evolves along time. Furthermore, farmers can develop strategies with respect to their contribution to percolation. We study regulation schemes to be implemented to restore the socially optimal spatial and temporal production plan of farmers in such a context. We show that both a temporal tax on percolation and a spatio-temporal tax on inputs (both at the extensive and at the intensive margin) are efficient for the restoration of the socially optimal solution. Furthermore, the error made when implementing a fiscal scheme designed for myopic farmers whereas they are strategic does not always increase with the degree of heterogeneity of the landscape.

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1 Introduction

The economics and management of water and drainage in agriculture is an important topic to which Dinar and Zilberman (1991) dedicated a book. More specifically, in semiarid areas, the problem of waterlogging arises in impermeable or poorly drained soils (see Wichelns, 1999, for more details on the extend of the problem at the international level). The consequence is the soaking of soils above the underlying aquifer that causes plant asphyxia and worsen the process of salinization, defined as the increasing concentration of dissolved salts in soils and waters. In some areas, it is the excess of irrigation that leads to the rise of the water table up to the crop-root, inducing a reduction of crop yields. This excess of irrigation is due to the fact that irrigation water and the saturated zone are not priced correctly to reflect scarcity rents and opportunity cost. Within this framework, the study of corrective policies to be implemented to restore efficiency is of major importance. It is the focus of this paper.

As stressed by Xabadia, Goetz and Zilberman (2004, 2006, 2008), given the dynamic nature of water storage and the spatial nature of soil quality when the landscape is considered heterogeneous, corrective policies should vary over space and over time. To derive such policies, they compare a socially optimal solution to a myopic one. The derivation of the socially optimal allocation over space and time is not obvious since when both the decision and the state variable depend on the two arguments, distributed optimal control is needed (see Calvo Calzada and Goetz, 2001). Goetz and Zilberman (2000) show that in the case where the decision variable depends on two arguments and the state variable on only one, it is possible to decompose the problem into two steps. For instance, in such a case, the first step can consist in solving the spatial problem and the second step in optimizing over time. It is especially appropriated to the waterlogging case as stressed by the serie of papers from Xabadia, Goetz and Zilberman (2004, 2006, 2008). More particularly, in the first step, we determine the optimal level of irrigation and of production for each location within the assumed heterogeneous landscape. The result of this spatial static optimization is summed up by a value function that is independant of space. In the second step, this value function becomes the objective of a dynamic optimization.

Xabadia, Goetz and Zilberman (2004, 2006, 2008) assume that farmers are myopic in the case with no corrective policies. In other words, farmers are assumed static maximisers that do not take into account the effects of their actions on the dynamics of the system. This assumption is commonly justified by the fact that agents are like atoms: they are too small a part of the whole to be able to consider the impact of their decisions on the dynamics of the system. Following Legras and Lifran (2006), we place our study in a setting in which farmers are faresighted, i.e. they are able to consider their impact on the dynamics of the saturated zone. There has been a great effort invested in understanding the processes governing waterlogging since the scientific works dating from the 1990s (see for instance Mérot et al., 1995). In most areas at risk of waterlogging, one can easily find maps of this risk and be aware of the impact of his irrigations on the height of the saturated zone. Each farmer is able to value his contribution to the waterlogging phenomenon and knows that other farmers contribute to the phenomenon and make their choice depending on the height of the saturated zone. Our main contribution is to study how the results from Xabadia, Goetz and Zilberman are modifed in such a strategic setting with respect to the myopic setting.

Another strand of literature studies an opposite problem with respect to the waterlogging one: the groundwater scarcity problem. Provencher and Burt (1993) show that the difference between a myopic solution and a socially optimal one exhibits stock and pumping cost externalities. The stock externality is linked to the finitness of the groundwater stock whereas the pumping cost externality is linked to the cost of pumping groundwater that is proportional to the aquifer height. We exhibit the presence of the same type of effects in the waterlogging problem that we will respectively call a drainage externality and a waterlogging cost externality. Negri (1989) shows that the difference between a strategic solution and a socially optimal one exhibits an additional externality: the strategic one. We bring to the fore the same kind of effect in the waterlogging problem. The intuition of this effect is the following one: since each farmer perfectly anticipates the impact of his decisions on the waterlogging phenomenon and that the other farmers do the same, each one has incentives to increase the waterlogging phenomenon before the others do.

Rubio and Casino (2001) propose the same kind of contribution as ours: they confirm the so-called Gisser and Sanchez effect in a strategic setting. According to the Gisser and Sanchez effect, if the groundwater capacity is large, the difference between the socially optimal and private extraction is negligible. The private extraction is a myopic one in Gisser and Sanchez (1980); it is a strategic one in Rubio and Casino (2001). In the groundwater scarcity problem, the framework is dynamic and aspatial. The spatial nature of the waterlogging problem allows us to test the sensibility of the difference between myopic and strategic assumptions to the landscape heterogeneity.

More particularly, we depart from models developped by Xabadia, Goetz and Zilberman and add a strategic dimension into the temporal problem. Despite the fact that the spatial and temporal problems are separable, we show that in a strategic setting the spatial production plan, i.e. the soil quality threshold from which farmers decide to irrigate, differs from the one of myopic farmers. The effect pass through the shadow value of percolation which is different from zero (its value in the myopic case) in the strategic case. Indeed, strategic farmers anticipate the increase of the saturated zone due to his own percolation impacts his opponents choices. This means that corrective policies to be implemented to restore efficiency when farmers are strategic with respect to waterlogging phenomenon also depend on the spatial distribution of soil quality and must differ from that derived when farmers are assumed myopic. We furthermore bring to the fore that in a strategic case, a fiscal scheme proportionnal to percolation restores efficiency in addition to the fiscal scheme proportionnal to input use proposed by Xabadia, Goetz and Zilberman (2004, 2006, 2008).

In a work rather focusing on interaction between water pollution and the nitrogen pool in the soil, Goetz et al. (2006) show that intensity-oriented instruments (tax on inputs as fertilizers) need to be complemented by regulation at the extensive margin (tax on land-use) to support a socially optimal outcome. They show that this result is driven by the non-linearity of the pollution function in one input. This result is confirmed by Xabadia, Goetz and Zilberman (2006) or Goetz and Zilberman (2007) who bring to the fore the importance of complementing intensity-oriented instruments by instruments that affect choice at the extensive margin, contrary to Xabadia, Goetz and Zilberman (2004, 2008) that do not, assuming linearity in the percolation process. We propose to assume non linearity of the percolation process and to test the impact of the consideration of strategic farmers on the importance of combining intensive and extensive margin regulations. We show that this combination remains relevant but that the amounts of the schemes differ between myopic and strategic assumpions. We also bring to the fore that a tax on percolation is able to restore efficiency.

Based on simulations applied to the central valley of California, Xabadia, Goetz and Zilberman (2008) show that the gains from optimal policies do not always increase with the degree of heterogeneity of the landscape. We run simulations with the same set of parameter values to show that so does the error made when implementing a fiscal scheme designed for myopic farmers whereas they are faresighted and strategic.

We present our model in the next section. Then, we detail the closed loop equilibrium that is our main contribution to the literature presiously quoted. We dedicated a section to the study of the inefficiencies of such an equilibrium and another one to the fiscal regulation schemes to be implemented to restore efficiency. Finally, we present the simulation run to test the sensitivity of the fiscal schemes to the assumptions made with respect to the heterogeneity of the landscape.

2 The model

We consider $n \geq 2$ farmers, indexed by i, that share L acres of land as a production input over the unbounded time horizon, $t \in [0, +\infty[$. The quality of land is heterogenous and measured by a parameter $q \in [q_0, q_1]$, summing up the biophysical attributes that reflect the soil capacity to retain water. The landscape is described by a density function, $g(q)$, with $\int_{q_0}^{q_1} g(q)\, dq = 1$. This fucntion indicates the rate of land area L with quality q. Each farmer i chooses to cultivate a share $l_i(t,q) \in \left[0, \frac{1}{n}\right]$ of total cultivble area $g(q)L$, for all (t,q), the upper constraint on l_i amounts saying the farmers are equally endowed with land of quality q.

Beside land, the production requires the use of water per unit of land, denoted by $w_i(t,q)$, for irrigation. The fraction of applied water that is actually utilized by the crop, $k(q)w_i(t,q)$, increases with soil quality that sums up its retention capacity, i.e. $\frac{\partial k(q)}{\partial q} > 0$. We denote by $f(k(q)w_i(t,q))$ the production function per acre, where $\frac{\partial f}{\partial w_i} > 0$ and $\frac{\partial f}{\partial^2 w_i} < 0$. Assuming L is normalized to 1, the instantaneous profit at time t therefore writes:

$$\int_{q_0}^{q_1} \left[\left(pf\left(k\left(q\right) w_i\left(t,q\right)\right) - cw_i\left(t,q\right) - I \right) l_i\left(t,q\right) \right] g\left(q\right) dq := \pi_i(t, q_0, q_1, w_i, l_i) \tag{1}$$

where c denotes the unit cost of input w, and I some fixed costs. The price of the output, denoted by p, is considered as constant (perfect competition).

Irrigation generates a quantity of percolated water. The percolation accumulates above an impermeable layer, which feeds a saturated zone of height $h(t,q)$. The increase of the saturated zone induced by irrigation is given by $\phi(r(q), w_i(t,q))$, where $r(q)$ is a percolation coefficient. Since higher soil quality has higher irrigation efficiency, it generates less percolation and $\frac{\partial r}{\partial q} < 0$. Furthermore, ϕ is twice differentiable, increasing, and strictly convex in $w_i(t,q)$. The water accumulated evacuates through natural drainage at a rate σ. We assume the diffusion of water over q is complete and instantaneous, i.e. $\frac{\partial}{\partial t \partial q} h(t,q) = \frac{\partial}{\partial q} h(t,q) = 0$. As a consequence, the height of the saturated zone in (t,q) depends on past stock solely, and temporal effects are captured by:

$$\frac{\partial}{\partial t} h(t) = \int_{q_0}^{q_1} \sum_{i=1}^{n} \phi(r(q), w_i(t,q)) l_i\left(t,q\right) g\left(q\right) dq - \sigma h\left(t\right) \qquad \forall q \tag{2}$$

Finally, saturation causes a widespread environmental damage, which farmers symmetrically suffer. In particular, waterlogging is known to hinder crops by reducing the subsurface atmosphere at the next period of time. The private cost of waterlogging depends on the saturated zone height and is denoted by $D(h(t))$, with $D(0) = 0$, $\frac{\partial D}{\partial h} > 0$, and $\frac{\partial D}{\partial^2 h} \geq 0$.

Farmers are assumed farsighted, meaning each of them knows he feeds the saturated zone by percolating and that other farmers percolate too. As a consequence, profits are interdependent through opponents' percolation choices, and the damage it causes. We furthermore assume strategies are closed loop, namely each farmer i anticipates that her opponents choose their control variables by taking into account the current height of the saturated zone. When all players are able to observe the impact of the current global saturation height on their own dotation, closed loop strategies are more credible than strategies that depend only on time. It defines a differential game, within which farmer i's maximization program writes:

$$\max_{w_i, l_i} \int_0^{+\infty} e^{-\delta t} \pi_i \left(t, q_0, q_1, w_i, l_i\right) - D\left(h\left(t\right)\right) dt \tag{3a}$$

$$\frac{\partial}{\partial t} h(t) = \int_{q_0}^{q_1} \left(\phi(w_i, r(q)) l_i + \sum_{j \neq i}^{n} \phi(w_j(t, q, h), r(q)) l_j\left(t, q, h\right) \right) g\left(q\right) dq - \sigma h\left(t\right) \tag{3b}$$

$$g\left(q\right) w_i \geq 0 \ , \qquad g\left(q\right) l_i \in \left[0, \frac{1}{n}\right], \qquad \text{and} \qquad h(0) = h_0 \geq 0. \tag{3c}$$

As the underlying model depends on two evolution parameters, the solving for a subgame perfect equilibrium may seem complicated. However, let us remark that the control problem described above does not feature proper spatio-temporal stock phenomenon. Indeed, both the dynamics of saturation and the intertemporal objective depend on variables or functionals that can first be aggregated with respect to space. In particular, consider for later use, farmer i's instantaneous height of percolation aggregated over her private landscape, from q_0 to q, and let us denote it by $\int_{q_0}^{q} \phi\left(r\left(q\right), w_i\right) l_i g\left(q\right) dq =: m_i(w_i, l_i, q)$.

One consequence is that the faresighted control problem is sequentially solvable, which the next section details.

3 The closed loop equilibrium

The derivation of the closed cloop solution over space and time is not obvious. We adapt Goetz and Zilberman (2000) solving method to our strategic framework, which amounts considering the spatial production plan as a control problem separate from the optimal percolation path in time. More particularly, it consists of the two following steps.

First, we consider the program at a fixed point in time, in order to determine the optimal level of irrigation and production for each location within the assumed heterogeneous landscape. The result of this spatial static optimization is summed up by a private landscape value function, which corresponds to the objective π_i evaluated over he landscape summed up in q at the optimal production plan, for a given height of instantaneous percolation. In a second step, one can find the optimal percolation path profile, taking into account the private landscape value function optimized with respect to space. It is defined as the closed loop Nash equilibrium of the differential game in which intertemporal payoffs are expressed in term of the indirect profit function.

3.1 The optimal production plan in space

In a first step, let us assume each famer i considers his instantaneous production plan across q, given some prespecified cap of percolation:

$$m_i(w_i, l_i, q_1) := z_i(t)$$

Put differently, for all $t \geq 0$, the farmer asks himself how much and where he should irrigate, knowing that (i) the output and the percolation depend on the land quality, and (ii) the percolation across the area cultivated should add up to $z_i(t)$ which does not depend on q since it is the aggregated percolation other the private landscape.

Therefore, the individual height of percolation in q, given by $m_i(w_i, l_i, q)$, can be considered as the state variable of a finite control problem over the interval $[q_0, q_1]$, which brings farmer i into maximizing:

$$\max_{w_i, l_i} \int_{q_0}^{q_1} \left(pf\left(k\left(q\right)w_i\right) - cw_i - I\right) l_i g\left(q\right) dq \tag{4a}$$

$$\text{st.} \qquad \frac{\partial m_i}{\partial q}(w_i, l_i, q) = \phi\left(r\left(q\right), w_i\right) l_i g\left(q\right), \qquad m_i(q_0) = 0, \tag{4b}$$

$$w_i g\left(q\right) \geq 0, \qquad l_i g\left(q\right) \geq 0, \qquad l_i \leq \frac{1}{n}, \tag{4c}$$

under the terminal state constraint: $m_i(q_1) = z_i$. As percolation does not impact the instantaneous profit, the farmer i does not care about the height of percolation of opponents, denoted by $\{z_j\}_{j \neq i}^n$, which implies the resulting program is not strategic. The Lagrangian reads as[1] :

$$L_i = \left(pf\left(k\left(q\right)w_i\right) - cw_i - I\right) l_i g\left(q\right) - \lambda_i \phi(r\left(q\right), w_i) l_i g(q) + \left(\mu_i w_i + \left(\varphi_i - \eta_i\right) l_i + \eta_i \frac{1}{n}\right) g\left(q\right)$$

The necessary conditions on controls for an interior irrigation equilibrium, that is assuming $\mu_i = 0$ and the upper constraint on w_i is not binding, are given by:

$$p\frac{\partial f}{\partial w_i}\left(k\left(q\right)w_i\right) - c - \lambda_i \frac{\partial \phi(r(q), w_i)}{\partial w_i} = 0, \qquad \forall i \tag{5a}$$

$$pf\left(k\left(q\right)w_i\right) - cw_i - I - \lambda_i \phi\left(r\left(q\right), w_i\right) + \varphi_i - \eta_i = 0. \tag{5b}$$

Besides, the condition on the adjoint variable implies: $\frac{d\lambda_i}{dq} = \frac{dL_i}{dm_i} = 0$. Added to the transversality conditions, $\lambda_i(q_1) \geq 0$ and $\lambda_i(q_1)\left(z_i - m_i(w_i, l_i, q_1)\right) = 0$, we obtain that the adjoint variable must be a constant over cultivated lands, i.e. over q at which production is strictly positive. Let us keep denoting such a constant by λ_i for simplicity, and assume $\lambda_i > 0$, i.e. that the unconstrained optimum is such that $m_i(w_i, l_i, q_1) \geq z_i(t)$[2]. The costate λ_i indicates, for all q along the optimal trajectory, the rate of change of farmer i's q-stream of profits subsequent to a variation of the stock of percolation, m_i. In other words, λ_i is interpreted as the shadow cost of the prespecified cap of percolation.

[1] A negative sign in front of the adjoint variable λ_i has been introduced to facilitate its interpretation.

[2] If one states, in contrast, that $\lambda_i = 0$ for $z_i(t)$ given, it would mean agents can fulfill $z_i(t)$ by behaving myopically, which is not a case of interest for the present paper that precisely studies the impact of relaxing the assumption of myopic agents.

Now, coming back to equation 5a, we obtain an optimal irrigation of $w^*(q, \lambda_i)$, where index i is dropped for symmetry. This optimal quantity of water applied is such that at every location, the value of the marginal product per acre equals the sum of the marginal cost of water and of the marginal cost of generated percolation per acre.

Remark, then, that both the dynamics and the objective are linear in the control l_i. Let us denote by $A(q, \lambda_i)$ the condition on land use 5b evaluated at the optimal irrigation, $w^*(q, \lambda_i)$:

$$A(q, \lambda_i) = pf\left(k\left(q\right)w^*\left(q, \lambda_i\right)\right) - cw^*\left(q, \lambda_i\right) - I - \lambda_i \phi(r\left(q\right), w^*\left(q, \lambda_i\right)) \tag{6}$$

Following Hartl and Feichtinger (1995), we know the optimal control is given by:

$$l^*(q, \lambda_i) = \begin{cases} 0 & \text{if } A(q, \lambda_i) < 0 \Rightarrow q < q^*(\lambda_i) \\ \frac{1}{n} & \text{if } A(q, \lambda_i) \geq 0 \Rightarrow q \geq q^*(\lambda_i) \end{cases} \tag{7}$$

where $q^*(\lambda_i)$ denotes the unique solution of $A_i(q, \lambda_i) = 0$ in $[q_0, q_1]$. This result means that farmer i's optimal land use is such that it exclusively cultivate lands with a quality above some treshold, $q^*(\lambda_i)$, which is a function of the shadow cost of the prespecified cap of percolation aggregated over the private landscape. Added to the dynamics given by 4b, we obtain the following equality:

$$z_i = \frac{1}{n} \int_{q^*(\lambda_i)}^{q_1} \phi\left(r\left(q\right), w^*\left(q, \lambda_i\right)\right) g\left(q\right) dq \tag{8}$$

which implicitly characterizes farmer i's constant shadow cost of percolation, as a function of the prespecified amount of percolation aggregated over the private landscape, $\lambda^*(z_i)$ which is constant over space.

3.2 The closed loop percolation path

The second step considers the issue of percolation in time, knowing the dynamics of the saturated zone (see equation 3b) and the optimal spatial plan. In particular, let us define farmer i's private landscape value function as the indirect profit function corresponding to his instantaneous objective (see equation 1) when he follows the optimal spatial production plan defined in the first step:

$$V\left(\lambda^*(z_i), q, q_1\right) = \int_{q^*(\lambda^*(z_i))}^{q_1} \left(pf\left(k\left(q\right)w^*\left(q, \lambda^*(z_i)\right)\right) - cw^*\left(q, \lambda^*(z_i)\right) - I\right) g\left(q\right) dq \tag{9}$$

The farmers are assumed farsighted. Each of them thus seeks for the percolation path that maximizes his intertemporal profit, given the strategy profile of opponents, $\{z_j(h(t))\}_{j \neq i}$, and the optimal spatial production plan. It amounts to maximizing the following intertemporal problem:

$$\max_{z_i} \int_0^{+\infty} e^{-\delta t} \left(V\left(\lambda^*(z_i), q, q_1\right) - D\left(h(t)\right)\right) dt \tag{10a}$$

$$\frac{dh(t)}{dt} = z_i + \sum_{j \neq i}^{n} z_j(h(t)) - \sigma h(t), \qquad h(0) = 0 \tag{10b}$$

Introducing again a negative sign in front of the adjoint variable ψ_i for interpretation facilitation, the Hamiltonian is given by $H_i = V\left(\lambda^*(z_i), q, q_1\right) - D(h(t)) - \psi_i \left(z_i + \sum_{j \neq i}^{n} z_j\left(h(t)\right) - \sigma h(t)\right)$, which yields the following optimality conditions, for all i:

$$\begin{cases} \dfrac{\partial V\left(\lambda^*(z_i), q, q_1\right)}{\partial z_i} = \psi_i & \text{(11a)} \\ \dfrac{d\psi_i}{dt} = \psi_i \left(\delta + \sigma - \displaystyle\sum_{j \neq i}^{n} \dfrac{dz_j(h(t))}{dh} \right) - D_h(h(t)). & \text{(11b)} \end{cases}$$

Conjointly with the n transversality conditions,

$$\lim_{t \to +\infty} \int_0^{+\infty} e^{-\delta t} \psi_i h(t) dt = 0, \qquad \forall i,$$

these $2n$ equalities characterize a closed loop equilibrium.

Here, the costate variable ψ_i indicates, for all t along the optimal trajectory, the rate of change of the private landscape value subsequent to a variation of $h(t)$, i.e. it is the shadow cost of the saturation height. Applying the dynamic enveloppe theorem with respect to the fixed-end point $m_i(q_1) = z_i$ (see Seierstad 1982) in the first solving step, we know the left hand side (LHS) of condition (11a) rewrites:

$$\frac{\partial V\left(\lambda^*(z_i), q, q_1\right)}{\partial z_i} = \lambda^*(z_i) \mid_{q_1} = \lambda^*(z_i) \tag{12}$$

since the costate in the spatial optimization problem is constant over space. In other words, the impact of the control z_i on the objective in the intertemporal problem of saturation, is summed up by the shadow cost of the prespecified cap of percolation aggregated over the private landscape, $\lambda^*(z_i)$.

It implies the shadow cost of saturation must equalizes the shadow cost of instantaneous percolation at the closed loop equilibrium, i.e. the condition (11a) becomes $\lambda^*(z_i) = \psi_i$, which we substitute into the optimality condition (11b).

Proposition 1. *Assuming a symmetric closed loop equilibrium denoted by $\hat{z}(h)$, this latter is thus known to verify:*

$$\frac{\partial \lambda^*(\hat{z})}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial h} \frac{\partial h}{\partial t} = \lambda^*(\hat{z}) \left(\delta + \sigma - \frac{\partial \hat{z}}{\partial h}(n-1) \right) - D_h(h(t)) \tag{13}$$

Equation (13) indicates that the shadow cost of the prespecified cap of percolation aggregated over the private landscape evolves along time through the temporal motion of the cap of percolation. This temporal motion is driven by the discount rate, δ, the drainage rate, σ, the private marginal cost of waterlogging, $D_h(h(t))$, and the private cost of the increase the saturation height due to strategic behavior of opponents, $\frac{\partial \hat{z}}{\partial h}(n-1)$. This last effect is linked to our assumption of farsighted farmers: each farmer knows the impact of his decisions on the waterlogging phenomenon and that other farmers contribute to the waterlogging phenomenon and make their choice according to the height of the saturated zone. The consequence is that each one has incentives to increase the waterlogging phenomenon before the others do.

4 The inefficiencies of the closed loop equilibrium

To study if the closed loop equilibrium is socially optimal, we need to compare it with the cooperative equilibrium. As for the cooperative case, it is assumed that a benevolent social planner maximizes the sum of benefits of the n farmers, with respect to water and land, perfectly knowing the impact of these choices on the dynamics of the height of the saturated zone. It is obvious to see that the first step (production plan in space) of the resolution process is unchanged. It is the second step (percolation temporal path) that is quite different since a benevolent social planner internalizes the impact of the rise of the social cost of percolation on all farmers.

Before we study the cooperative percolation path, let us state some preliminary results on the optimal production plan in space.

Lemma 1. *Provided that some cap z_i has to be reached, the positive shadow cost of increasing percolation is decreasing in z_i. This result comes from the fact that (i) the optimal production plan in space is such that the optimal water use is decreasing with respect to the shadow cost, λ, and (ii) the minimum quality threshold increases, i.e. the optimal production shrinks towards higher quality land, as the constant shadow cost λ^* increases.*

This result shows that, if the constant costate λ is positive, a higher shadow cost means that farmer i constrains himself to a percolation cap that is more stringent (equivalently, to a smaller aggregated height of percolation). Accordingly, the more the farmer i constrains his aggregate percolation target, z_i, the less he irrigates, whatever the quality of the land, and the highest the quality of the land he chooses to cultivate.

4.1 The cooperative percolation path

Let us now study the cooperative percolation path which is socially optimal. Assuming a symmetric solution, denoted by $z^*(t)$, the Hamiltonian is given by: $H = nV(\lambda^*(z(t)), q, q_1) - nD(h(t)) - \psi(nz(t) - \sigma h(t))$. Comparing with the closed loop equilibrium, $nz(t)$ is the percolation aggregated over the public landscape, i.e. the sum of the percolations over the private landscapes. The two corresponding optimality conditions are given by:

$$\begin{cases} n\dfrac{\partial}{\partial z}V(\lambda^*(z(h)), q, q_1) - n\psi = 0 & \text{(14a)} \\ \dfrac{d\psi}{dt} = \psi(\delta + \sigma) - nD_h(h(t)) & \text{(14b)} \end{cases}$$

with the transversality condition, $\lim_{t\to+\infty} \int_0^{+\infty} e^{-\delta t}\psi h(t)dt = 0$.

Remark 1. *It follows $z^*(t)$ is characterized, in turn, by the following differential equation:*

$$\frac{d\lambda^*(z^*)}{dt} = \lambda^*(z^*)(\delta + \sigma) - nD_h(h(t)) \tag{15}$$

Comparing with the condition (13), observe that, in the (closed loop) strategic case, farmer i internalizes the discount effect and the drainage externality summed up respectively in δ and σ. However, he does not internalize the impact of the rise of the saturation height on the waterlogging cost of his opponents, $(n-1)D_h(h(t))$. This is what we can call a waterlogging cost externality. In our faresighted framework summed up in equation (13), this effect is emphasized by farmer's anticipation that the increase of the saturated zone due to his own percolation impacts his opponents choices. We propose to call such an effect a strategic externality.

4.2 Steady state comparison

To go further, let us focus on the steady state. The height of the saturated zone is constant at the steady state, which means $z_i + \sum_{j \neq i} z_j = \sigma h$. Assuming symmetry, we obtain a condition on the individual height of percolation:

$$z_\infty = \frac{\sigma h_\infty}{n}. \tag{16}$$

At the cooperative steady state, the individual height of percolation must be proportional to the height of natural drainage. Substituting the expression for z_∞ into the closed loop equilibrium condition, we obtain that the stationary height of the saturated zone, denoted by $\hat{h}_\infty$, is characterized by:

$$\begin{aligned} \frac{\partial \lambda^*(z)}{\partial z}\mid_{z_\infty} \frac{\partial z}{\partial h}\mid_{z_\infty} \frac{d\hat{h}_\infty}{dt} &= \lambda^*(z_\infty)\left(\sigma + \delta - (n-1)\frac{\partial z}{\partial h}\mid_{z_\infty}\right) - D_h(\hat{h}_\infty) \\ \Leftrightarrow \qquad 0 &= \lambda^*\left(\frac{\sigma \hat{h}_\infty}{n}\right)\left(\sigma + \delta - (n-1)\frac{\sigma}{n}\right) - D_h(\hat{h}_\infty) \\ \Leftrightarrow \qquad \lambda^*\left(\frac{\sigma \hat{h}_\infty}{n}\right) &= \frac{D_h(\hat{h}_\infty)}{\frac{1}{n}\sigma + \delta} \end{aligned} \tag{17}$$

In the case of the cooperative solution, similar computations lead to h^*_∞, characterized by:

$$\lambda^*\left(\frac{\sigma h^*_\infty}{n}\right) = n\frac{D_h(h^*_\infty)}{(\sigma + \delta)} \tag{18}$$

At the steady state, the strategic externality is summed up in the natural rate of drainage, $\frac{1}{n}\sigma$.

Proposition 2. *The stationary height of the saturated zone at the closed loop equilibrium is higher than the one in the cooperative case: $\hat{h}_\infty \geq h^*_\infty$. It follows that the stationary percolation heights, $\hat{z}_\infty$ and z^*_∞, verify in turn:*

$$\hat{z}_\infty = \frac{\sigma \hat{h}_\infty}{n} \geq z^*_\infty = \frac{\sigma h^*_\infty}{n}. \tag{19}$$

Likewise, compared to the social optimum:

$$w^*\left(q, \lambda^*\left(z_i\right)\right)\mid_{z_i = \frac{\sigma \hat{h}_\infty}{n}} \geq w^*\left(q, \lambda^*\left(z_i\right)\right)\mid_{z_i = \frac{\sigma h^*_\infty}{n}} \tag{20}$$

$$\text{and} \quad q^*\left(\lambda^*\left(z_i\right)\right)\mid_{z_i = \frac{\sigma \hat{h}_\infty}{n}} \leq q^*\left(\lambda^*\left(z_i\right)\right)\mid_{z_i = \frac{\sigma h^*_\infty}{n}}, \tag{21}$$

i.e. the production plan at the closed loop equilibrium increases both intensively (irrigation increases) and extensively (the cultivated area increases).

In other word, consistently with lemma 1, the strategical effect related to percolation impacts spatial choices of farmers by leading them to extend the cultivated area on lower quality lands with respect to what is socially optimal. Besides they use, for all q, more water than in the cooperative case.

5 The fiscal regulation scheme

Once the inefficiencies of the closed loop equilibrium brought to the fore, let us turn to the study of the fiscal scheme that should be implemented to restore a socially optimal solution. Since the inefficiencies occur in the second step of the resolution, a suited regulation should be differentiated with respect to time only and be proportional to percolation aggregated over the private landscape, z_i. Since the regulator is not always able to monitor the percolation aggregated over the landscape, we also study a fiscal scheme on inputs which acts both at intensive (water consumption) and at extensive (land-use) margins. Such a fiscal scheme will be differentiated both in space and in time because of being implemented in the first step of our resolution process.

5.1 Temporal fiscal scheme on percolation

Consider a unit tax, denoted by $x(t)$ under strategic assumption and by $y(t)$ under myopic assumption, on farmer i's aggregate percolation at time t

In the closed loop problem, the Hamiltonian is now given by:

$$H_i = V\left(\lambda^*(z_i), q, q_1\right) - xz_i - D(h(t)) - \psi_i\left(z_i + \sum_{j\neq i}^{n} z_j\left(h(t)\right) - \sigma h(t)\right). \tag{22}$$

The optimality conditions therefore reads as follows, for all i:

$$\begin{cases} \dfrac{\partial V\left(\lambda^*(z_i), q, q_1\right)}{\partial z_i} - x - \psi_i = 0 & \text{(23a)} \\ \dfrac{d\psi_i}{dt} = \psi_i\left(\delta + \sigma - \displaystyle\sum_{j\neq i}^{n} \dfrac{dz_j(h(t))}{dh}\right) - D_h(h(t)) & \text{(23b)} \end{cases}$$

Proposition 3. *Again, taking into account the equivalence between the shadow cost of percolation and the marginal value over space, as well as assuming symmetry, we obtain the following condition:*

$$\frac{d\lambda^*(z)}{dz}\frac{dz}{dh}\frac{dh}{dt} - \frac{d}{dt}x = \left(\lambda^*\left(z\right) - x\right)\left(\delta + \sigma - (n-1)\frac{dz}{dh}\right) - D_h(h(t)). \tag{24}$$

Note that this last condition is equivalent to condition (13) with the fiscal scheme on percolation added.

Let us denote $h^*(t)$ the saturated zone's intertemporal trajectory induced by the closed loop equilibrium characterized by equation (13). When farmers are farsighted, the tax on percolation, $x^p(t)$, that induces the first best, solves the differential equation:

$$\frac{d}{dt}x = x(\delta + \sigma) - (n-1)\frac{\partial D}{\partial h}\mid_{h^*(t)} + \left(\lambda^*\left(z(h^*(t))\right) - x\right)(n-1)\frac{dz}{dh}\mid_{h^*(t)}. \tag{25}$$

The latter is obtained by identification of equation (13) to equation (24). This tax gives farmers incentives to internalize both the stock externality, $(n-1)\frac{\partial D}{\partial h}$, and the strategic externality, $(n-1)\frac{dz}{dh}$.

At the steady state, the tax on percolation becomes the constant tax rate:

$$x_\infty^p = \frac{\delta}{\left(\delta + \frac{1}{n}\sigma\right)} \frac{(n-1)}{(\sigma + \delta)} D_h(h_\infty^*). \tag{26}$$

The first term in the expression for x_∞ internalizes the strategic externality at the steady state.

Let us turn to the myopic assumption, as a benchmark already derived by Xabadia, Goetz and Zilberman (2004, 2006, 2008). Within our framework, the spatial production plan is assessed for some z_i given in a first step, but the objective does not depend on the state in the second step.

Remark 2. *The symmetric Lagrangian is given by $L = V\left(\lambda^*(z(t)), q, q_1\right) - yz(t)$ for all t, which yields the first order condition $\lambda^*(z(t)) - y = 0$.*

It follows that the optimal tax on percolation is simply given by $y_p(t) = \lambda^*(z^*(t))$, obtained by identification of:

$$\frac{d}{dt}y(t) = y\,(\delta + \sigma) - n\frac{\partial D}{\partial h}\mid_{h^*(t)}, \tag{27}$$

to the differential equation (15). At the steady state, it becomes:

$$y_\infty^p = \frac{n}{\delta + \sigma} D_h(h_\infty^*). \tag{28}$$

Under myopic assumption farmers do not take into account discounting effect, drainage externality, and waterlogging cost externality. The optimal tax gives them incentive to take them into account.

Since x_∞^p is obviously smaller than 1, the comparison with the myopic tax rate, given by equation 28, allows us to state the following result.

Proposition 4. *When the regulator chooses to implement a percolation tax, the stationnary tax rate is higher for myopic agents than for strategic agents.*

Indeed, in the myopic case, the tax must fully stand for $\lambda^*(z^*(t))$. Conversely, in the fare-sighted case, farmers already partly take into account the impact of their percolation on utilities, hence the optimal tax should only compensate for the lowering of the shadow cost which is due to the strategical effect and to the social cost of waterlogging.

Remark that, under both assumptions (myopic and strategic), the percolation tax does not need to be be spatially differentiated since the shadow cost of percolation is the same, whatever the quality $q \in [q_0, q_1]$ of the land from which it is emitted. However, depending on the monitoring technology available, the regulator may want to convert the tax on percolation into a fiscal scheme on inputs.

5.2 Perfectly differenciated fiscal scheme on inputs

Consider now that the regulator does not tax percolation, but that she taxes the use of water and land in the production plan. Such a fiscal scheme is equivalent to a combination of intensive (water tax) and extensive (land tax) margin regulations. Let us denote $Cm_w := \frac{\partial \phi(r(q),w)}{\partial w}$ the marginal contribution of irrigation to percolation and $Cm_l := \phi\left(r(q), w\right) g(q)$ the marginal contribution of land use to percolation. Specifically, assume that the regulator applies the following rates:

$$X^w(t,q) = x(t)Cm_w \mid_{w^*(q,\lambda^*(z^*(t)))} \tag{29}$$

$$X^l(t,q) = x(t)\left(Cm_l - wCm_w\right) \mid_{w^*(q,\lambda^*(z^*(t)))} \tag{30}$$

Then, remark that, when evaluated under the closed loop assumption, the optimality conditions in the first step of the solving rewrite:

$$\begin{cases} p\frac{\partial f}{\partial w_i}\left(k(q)w\right) - c - (\lambda + x)\,Cm_w = 0 \\ pf\left(k(q)w\right) - I - cw - (\lambda + x)\,Cm_l = 0 \end{cases} \tag{31}$$

These conditions amount to a variable change which implies that $w^x(\lambda) = w^*(\lambda + x)$ and $q^x(\lambda) = q^*(\lambda + x)$. Likewise, we obtain that:

$$z_i = \frac{1}{n}\int_{q^*(\lambda+x)}^{q_1} \phi\left(r(q), w^*(\lambda + x)\right) g(q)dq, \tag{32}$$

implying, in turn, that $\lambda^*(z_i) = \lambda^x(z_i) + x$. In other word, the shadow cost of percolation under the perfectly differentiated tax scheme is equal to the shadow cost at the laisser-faire, net of the tax rate. Let us denote by $V^x(\lambda^x(z_i), q, q_1)$ the corresponding value. It follows that, at the step of caracterizing the closed loop equilibrium, the shadow cost equivalence in equation (12) rewrites:

$$\frac{\partial V^x(\lambda^x(z_i), q, q_1)}{\partial z} = \lambda^x(z_i) = \lambda^*(z_i) - x$$

and the optimality conditions of the temporal problem therefore become:

$$\begin{cases} \lambda^*(z_i) - x - \psi_i = 0 \\ \frac{d}{dt}\psi_i = \psi_i\left(\delta + \sigma - \sum_{j\neq i}^n \frac{dz_j(h(t))}{dh}\right) - D_h(h(t)), \end{cases}$$

which are strictly equivalent to the conditions (23a) and (23b) that characterize the level of optimal tax on percolation, $x_p(t)$.

Finally, the fiscal scheme on inputs simply weight the tax on percolation by the share to which each input contribute to the percolation, namely its marginal contribution. Accordingly, at the steady state, these tax rates are given by:

$$X^w_\infty(q) = x_{p,\infty}Cm_w \mid_{w^*(q,\lambda^*(z^*_\infty))}, \tag{33}$$

$$\text{and} \quad X^l_\infty(q) = x_{p,\infty}\left(Cm_l - wCm_w\right) \mid_{w^*(q,\lambda^*(z^*_\infty))}, \tag{34}$$

for irrigation and land use, respectively, where $x_{p,\infty}$ is given by equation (26).

For comparison purposes, let us compute the input tax rates of the perfectly differentiated tax scheme in the case of myopic agents. As tax does not apply to percolation, but to w_i and l_i instead, the maximization problem restricts itself to the first step, and the optimal conditions, assuming an interior solution, are given by $pf_{w_i}(k(q)w_i) - c = 0$ and $pf(k(q)w_i) - cw_i - I = 0$, respectively. Then it is obvious that, at the steady state, tax rates on water and land of:

$$Y^w_\infty(q) = y_{p,\infty}Cm_w \mid_{w^*(q,\lambda^*(z^*(t)))} \tag{35}$$

$$\text{and} \quad Y^l_\infty(q) = y_{p,\infty}\left(Cm_l - wCm_w\right) \mid_{w^*(q,\lambda^*(z^*(t)))} \tag{36}$$

induce the first best quantity of water and quality threshold, where $y_{p,\infty}$ is given by equation (28).

Proposition 5. *When the regulator chooses to implement a tax on inputs, she combines intensive (water tax) and extensive margin (land tax) regulations. The stationnary tax rates are higher for the myopic agents than for strategic agents.*

The last section assesses the efficiency losses entailed by mistaking strategic farmers for myopic ones, and how such losses depend on the landscape heterogeneity.

6 Impact valuation of bad regulation scheme implementation

The last two propositions show that depending on whether agents are strategic or myopic, the regulation scheme changes. We now examine what happens when the regulator assume agents are myopic while they actually behave strategically. In particular, implementing the tax rate $y^p(t) = \lambda^*(z^*(t))$ on percolation within strategic agents imply the symmetric closed loop equilibrium is characterized by:

$$\frac{d\lambda^*(z)}{dz}\frac{dz}{dh}\frac{dh}{dt} - \frac{d}{dt}\lambda^*(z^*(t)) = \left(\lambda^*(z) - \lambda^*(z^*(t))\right)\left(\delta + \sigma - (n-1)\frac{dz}{dh}\right) - D_h(h(t)). \tag{37}$$

As the combined taxes on inputs, $X^w(t,q)$ and $X^l(t,q)$, are strictly equivalent to the spatially indifferenciated tax on percolation $x^p(t)$, we choose to restrict the analysis on this latter case. The next proposition summarizes the results at the steady state.

Proposition 6. *If the regulator regulates strategic agents' percolation as if they were myopic, the height of the saturated zone at the (closed loop equilibrium) steady state is lower than h^*_∞. Specifically, it is implicitely characterized by:*

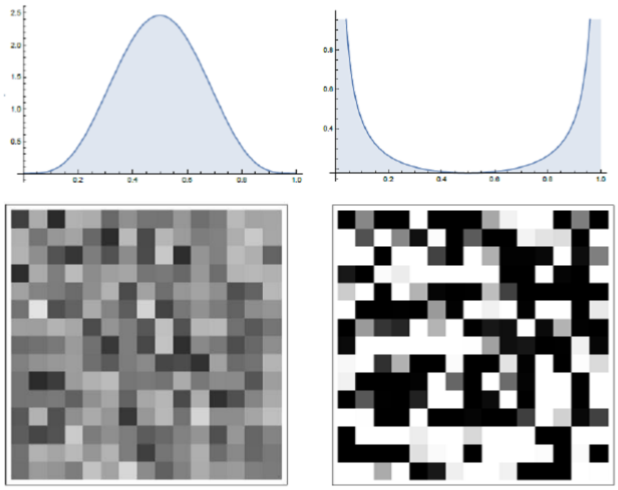
$$\lambda^*\left(\frac{\sigma h_\infty}{n}\right) = \frac{nD_h(h^*_\infty)}{(\delta+\sigma)} + \frac{D_h(h_\infty)}{\left(\delta+\frac{\sigma}{n}\right)}, \tag{38}$$

and causes underproduction, both at an extensive (a higher quality threshold) and an intensive level (a lower use of water).

The extent to which such a tax rate difference actually feeds back the production depends on q and, more precisely, it depends on the variance of q, i.e. the heterogeneity of the landscape. We now turn to the measurement of the impact of landscape heterogeneity on the error made. To do so, we need to run simulations. We depart from Khanna, Isik and Zilberman (2002) empirical study since they provide specifications and parameter values appropriate for our model (see Appendix for more details). Their study is based on cotton production in the San Joaquim Valley in California which faces a waterlogging problem. The irrigation area concerned represents 400,000 acres. To better suit to our strategic context and contrary to Khanna, Isik and Zilberman (2002) who considers 2 irrigation technologies, we assume that farmers only irrigate based on a modern technology (drip irrigation).

To measure the heterogeneity of the land, we follow Xabadia, Goetz and Zilberman (2008) and Barraquand and Martinet (2011) who focus on the beta distribution that allows a wide variety of different shapes of the distribution. As a first step of analysis, we propose to consider two shapes illustrated on Figure 1: the normal and the polarized one. In the normal case, the landscape is normally distributed around the mean quality whereas in the polarized case, land are either of good or of bad quality. The spatial traduction of both these distributions is represented on grids where each pixel stands for the quality of the land (the darker the color the higher the quality).

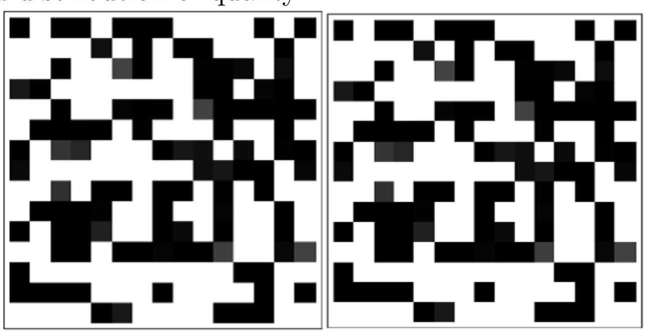
Figure 1: Normal versus polarized distribution of land quality



Let us use the same kind of grids to illustrate the main result of our sensitivity analysis according to which the error made when implementing a wrong regulation scheme, i.e. designed for myopic farmers whereas they are strategic does not always increase with the degree of heterogeneity of the landscape.

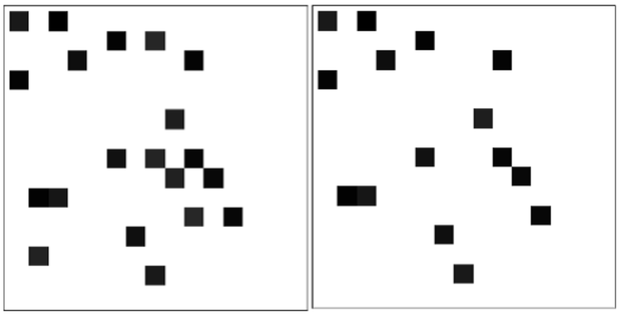
In the polarized case, Figure 2 represents the pixels of land that are cultivated at the cooperative steady state versus under the wrong regulation scheme. The same pixels of lands are cultivated in both cases which means that the error is not significant.

Figure 2: Pixel of land cultivated in the cooperative case versus under the wrong regulation scheme for a polarized distribution of quality



In the normal case, Figure 3 represents the pixels of land that are cultivated at the cooperative steady state versus under the wrong regulation scheme. Some pixels are not cultivated with the wrong regulation scheme whereas they should be for the cooperative solution to be reached.

Figure 3: Normal versus polarized distribution of land quality



7 Conclusion

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APPENDIX

A Proof of Lemma 1

In order to determine the way the shadow cost of percolation impacts the use of water, we apply the implicit function theorem to condition (5a):

$$\frac{\partial}{\partial \lambda} w^* (q, \lambda) = \frac{\frac{\partial \phi(r(q),w)}{\partial w}}{p f_{ww}(k(q)w) - \lambda \frac{\partial \phi(r(q),w)}{\partial^2 w}} \leq 0 \qquad \text{for} \quad \lambda \geq 0,$$

since, by assumption, $f_{ww} < 0$ and $\frac{\partial \phi(r(q),w)}{\partial^2 w} \geq 0$.

Still using the implicit functions theorem applied to condition (6) ccshad we obtain the next results about the use of land:

$$\frac{d}{d\lambda} q^*(\lambda) = \frac{\phi_i(r(q), w^*(q,\lambda))}{p \frac{f_w(k(q)w^*(q,\lambda))}{k(q)} k'(q,\lambda) - \lambda r'(q) \frac{\partial \phi(r(q),w^*(q,\lambda))}{\partial r(q)}} \geq 0$$

We can therefore conclude:

$$\frac{d}{dz_i} \lambda^*(z_i) = \frac{n}{\int_{q^*(\lambda)}^{q_1} \frac{\phi(r(q),w^*(\lambda,q))}{\partial w^*(\lambda,q)} \underbrace{\frac{\partial w^*(\lambda,q)}{\partial \lambda}}_{\leq 0} g(q)dq - \phi\left(r(q^*(\lambda)), w^*\left(q^*(\lambda), \lambda\right)\right) g\left(q^*(\lambda)\right) \underbrace{\frac{\partial q^*(\lambda)}{\partial \lambda}}_{\geq 0}} \leq 0 \tag{39}$$

B Proof of Proposition 2

The stationary height of the saturated zone can be ranked from the assumption $D_{hh} \geq 0$, Lemma 1 and the fact that $\frac{n}{\sigma+\delta} > \frac{n}{\sigma+n\delta}$. So does the production plan from Lemma 1.

C Proof of Proposition 6

On one hand, the LHS of equation (38) is decreasing in h_∞, from lemma 1 and the fact that z_∞ is decreasing in h_∞. Besides, observe that $\lambda^*(0) \geq \frac{nD_h(h^*_\infty)}{(\delta+\sigma)} \geq 0$ since $\frac{nD_h(h^*_\infty)}{(\delta+\sigma)} = \lambda^*(\frac{\sigma h^*_\infty}{n}) \geq 0$, $z_\infty = 0$ when $h_\infty = 0$, and $z^M(t) \geq 0$. On the other hand, the assumption on the social costs imply the right hand side (RHS) of this equation is increasing in h_∞. Moreover we know that:

$$\frac{nD_h(h^*_\infty)}{(\delta+\sigma)} + \frac{D_h(h_\infty)}{\left(\delta+\frac{\sigma}{n}\right)} > \frac{nD_h(h_\infty)}{(\delta+\sigma)} + \frac{D_h(h_\infty)}{\left(\delta+\frac{\sigma}{n}\right)} \quad \forall h_\infty \in [0, h^*_\infty[$$

and that $\frac{nD_h(h_\infty)}{(\delta+\sigma)} + \frac{D_h(h_\infty)}{\left(\delta+\frac{\sigma}{n}\right)} > \frac{D_h(h_\infty)}{\left(\delta+\frac{\sigma}{n}\right)} + x_\infty$.

We can therefore conclude that if $\frac{nD_h(h^*_\infty)}{(\delta+\sigma)} + \frac{D_h(0}{\left(\delta+\frac{\sigma}{n}\right)} \leq \lambda^*(0)$, then there exists $h_\infty < h^*_\infty$ such that condition (38) is satisfied.

D Specifications and parameter values for simulations

We specify the production function as quadratic:

$$f\left(k\left(q\right)w_i\left(t,q\right)\right) = K + aqw_i\left(t,q\right) - b\left(qw_i\left(t,q\right)\right)^2$$

with $p > 0$, $c > 0$, $K \geq 0$, $f_{w_i} = aq - 2bqw_i(t,q) > 0$ hence $w_i(t,q) < \frac{a}{2b}$ and ,$f_{w_i w_i} = -2bq < 0$. We assume quality q ranges from $q_0 = 0$ to $q_1 = 1$ and $k(q) = q$. We also choose a linear percolation function, $\phi(r(q)w_i(t,q)) = r(q)w_i(t,q)$ with a percolation coefficient $r(q) = 1 - rq$, with $r > 0$, so that we have $r'(q) = -r < 0$. Let us furthermore assume a linear social cost $D(h) = \theta h$ with a damage parameter $\theta > 0$. It follows: $D(0) = 0$, $D_h > 0$ and $D_{hh} = 0$. The symmetric individual spatial production problem therefore writes:

$$\max_{w(q),l(q)} \int_0^1 \left[\left(p\left(K + aqw(q) - b\left(qw(q)\right)^2\right) - cw(q) - J\right) l(q)\right] g(q)\, dq$$

$$w(q)\,g(q) \geq 0 \qquad l(q)\,g(q) \geq 0\,, \qquad l(q) \leq \frac{1}{n}, \qquad m(1) = z_i$$

where the percolation flow generated in q is given by $(1-rq)w(q)\,l(q)\,g(q)$. Assuming an interior solution, the FOC on the use of water yield: $w^*(\lambda, q) = \frac{-c+apq-\lambda+qr\lambda}{2bpq^2}$.

Substituting into the FOC on l, we obtain a polynom of degree 2:

$$-J + Kp - w\left(c + (bqw - a)\,pq + (1-qr)\,\lambda\right) = 0$$

which yields two roots, one from which the profit evaluated at $w^*(\lambda, q)$ in the myopic case becomes positive, that is:

$q^*(\lambda) = \frac{ap(c+\lambda)-2\sqrt{bp(J-Kp)(c+\lambda)^2}+r\lambda(c+\lambda)}{4bp(Kp-J)+(ap+r\lambda)^2}$.

As a result, the optimal land use is known to be a piecewise function defined by $l^*(q,\lambda) = 0$ as long as $q < q^*(\lambda)$ and $l^*(q,\lambda) = 0$ otherwise. We can now substitute the optimal input use into the constraint on aggregate percolation:

$$z = \frac{1}{n}\int_{q^*(\lambda)}^{1} \frac{(1-rq)\left(-c+apq-\lambda+qr\lambda\right)}{2bpq^2} g(q)\,dq$$

Remark that the optimal input use in the myopic case is given by $w^*(0,q) = \frac{-c+apq}{2bpq^2}$ and $q^*(0) = \frac{apc-2\sqrt{bp(J-Kp)c^2}}{4bp(Kp-J)+(ap)^2}$
.

We assume the quality is distributed according to a beta distribution with parameters α and β, and we denote L the total number of acres. It follows the primitive of the previous integral is given by:

$$\frac{1}{Beta[\alpha,\beta]2bnp(-2+\alpha)(-1+\alpha)\alpha} Lq^{\alpha-2}(1-\alpha)\alpha(c+\lambda)Hypergeometric2F1[-2+\alpha, 1-\beta, -1+\alpha, q]$$

$$+\frac{1}{Beta[\alpha,\beta]2bnp(-2+\alpha)(-1+\alpha)\alpha} Lq^{\alpha-2}q(-2+\alpha)\alpha(ap+r(c+2\alpha))Hypergeometric2F1[\alpha-1, 1-\beta, \alpha, q]$$

$$-\frac{1}{Beta[\alpha,\beta]2bnp(-2+\alpha)(-1+\alpha)\alpha} Lq^{\alpha-2}qr(-1+\alpha)(ap+r\lambda)Hypergeometric2F1[\alpha, 1-\beta, 1+\alpha, q]$$

Evaluated within $[q^*(\lambda), 1]$, namely the area of cultivated lands, this integral indicates the aggregated percolation per farmer for a given value of the shadow cost λ.

We now turn to the second solving step and consider stationnary states. In the cooperative case, the stationnary shadow cost is given by $\lambda^*_\infty = \frac{n\theta}{\delta+\sigma}$ and, substituting into the optimal input use we obtain

$$w^*_\infty(n,\theta,q,\delta,\sigma) = \frac{-c+apq+\frac{n\theta}{\delta+\sigma}(qr-1)}{2bpq^2}$$

$$q^*_\infty(n,\theta,\delta,\sigma) = \frac{ap(c+\frac{n\theta}{\delta+\sigma}) - 2\sqrt{bp(J-Kp)\left(c+\frac{n\theta}{\delta+\sigma}\right)^2} + r\frac{n\theta}{\delta+\sigma}\left(c+\frac{n\theta}{\delta+\sigma}\right)}{4bp(Kp-J)+\left(ap+r\frac{n\theta}{\delta+\sigma}\right)^2}$$

Likewise, at the closed loop equilibrium, we know that $\hat{\lambda}_\infty = \frac{\theta}{\delta+\frac{\sigma}{n}}$, from which we get

$$\hat{w}_\infty(n,\theta,q,\delta,\sigma) = \frac{-c+apq+\frac{\theta}{\delta+\frac{\sigma}{n}}(qr-1)}{2bpq^2}$$

$$\hat{q}_\infty(n,\theta,\delta,\sigma) = \frac{ap(c+\frac{\theta}{\delta+\frac{\sigma}{n}}) - 2\sqrt{bp(J-Kp)\left(c+\frac{\theta}{\delta+\frac{\sigma}{n}}\right)^2} + r\frac{\theta}{\delta+\frac{\sigma}{n}}\left(c+\frac{\theta}{\delta+\frac{\sigma}{n}}\right)}{4bp(Kp-J)+\left(ap+r\frac{\theta}{\delta+\frac{\sigma}{n}}\right)^2}$$

Finally, let us compute the regulation schemes. When the regulator taxes the output, the stationnary tax rate under an assumption of closed loop strategies, is given by $X_\infty = \frac{(n-1)\theta\delta}{(\delta+\sigma)\left(\delta+\frac{\sigma}{n}\right)}$. In the myopic case, it simply corresponds to λ^*_∞.

When the regulator chooses to implement the differenciated regulation scheme, the stationary tax rates under a closed loop assumption are given by :

$$\text{stationary tax rate on water} \qquad \frac{(n-1)\,\theta\delta}{(\delta+\sigma)\left(\delta+\frac{\sigma}{n}\right)}(1-rq) \tag{40}$$

$$\text{stationary tax rate on lands} \qquad \left(\frac{(n-1)\,\theta\delta}{(\delta+\sigma)\left(\delta+\frac{\sigma}{n}\right)}(1-rq)\,\frac{-c+apq+\frac{n\theta}{\delta+\sigma}(qr-1)}{2bpq^2}\right)g(q) \tag{41}$$

When agents are myopic:

$$\text{stationary tax rate on water} \qquad \frac{n\theta}{(\delta+\sigma)}(1-rq) \tag{42}$$

$$\text{stationary tax rate on lands} \qquad \left(\frac{n\theta}{(\delta+\sigma)}(1-rq)\,\frac{-c+apq+\frac{n\theta}{\delta+\sigma}(qr-1)}{2bpq^2}\right)g(q) \tag{43}$$

Parameters	Values	Units
j	-1589	-
a	2311	-
b	462	-
p	0.65	\$/pound
c	55	\$/acre-foot
I	633	\$
θ	10	\$
δ	0.4	-
σ	0.2	-

Table 1: Parameter values