Architecture of energy fiscal federalism

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Abstract

We analyze how horizontal and vertical externalities may occur in presence of two distortionary taxes (capital and energy taxes). We also assess the impact of these taxes on local jurisdiction welfare. We identify three effects generated by the inclusion of energy tax as a way to alleviate environmental damages: i) horizontal externalities may occur due both to the mobility of production factors (capital and energy) and to the citizen’s preferences between private and public consumption, ii) jurisdiction size and perfect mobility of inputs can lead to vertical externalities, iii) energy tax may lead a “double dividend” by allowing to finance public good and also by improving environmental quality. Keywords: Fiscal federalism; interactions; Energy. JEL classification: C72, H23, H77.

1 Introduction:

It is important to develop efficient environmental policies to mitigate climate change, preserve biodiversity, and reduce water and air pollution. To this end, the public authorities have a wide range of instruments that are generally regulatory (conventional instruments, aimed at constraining agents’ behavior) or economic ones (an incentive to encourage more virtuous behavior). Environmental taxation appears to be a less coercive tool than the norm, and nevertheless efficient when it allows economically to favor for sustainable behavior as compared to harmful behaviors to the environment. Thus, taxation is nowadays recognized as a powerful lever for modifying individual and collective behavior, owing to the financial incentive it addresses to those who support it.

Furthermore, energy taxation is being mutation in sense that fiscal decisions become increasingly centralized. For instance, in a Community framework, the EU has set drastic energy policy targets, including a 20% reduction in greenhouse gas emissions by 2020 and an increase of 20% of the share of renewable energies in gross final energy consumption. However member states remain sovereign in terms of energy resources and way to achieve the objectives set by Europe. For illustration, it is important to emphasize that energy policy for most European countries has at least three distinct levels of decision: the European Union, the State and local authorities. Therefore, while energy transition is increasingly defined at the upper tier (egg. European level), the actors involved in its implementation will be increasingly decentralized.
and close to consumers. In addition, energy resources are various forms and dispersed in space. Hence, their exploitation modifies the geographical distribution of energy potentials.

Related to energy resources, two main implications are essentials when implementing energy fiscal policy: there are fiscal and environmental considerations. From fiscal considerations, the concentration of energy resources in certain localities may lead to fiscal disparities between local authorities. In sense that local governments derive from natural resources which allow them to compete both in terms of fiscal point of view and provision of public services in order to attract qualified labour. To alleviate these fiscal distortions, central or federal government can levies a federal tax in order to reduce the magnitude of intergovernmental fiscal disparities. Indeed, it is well-known that heterogeneity of energy fiscal endowments can lead to significant fiscal disparities. For illustration, the problematic of fiscal disparities between regions is particularly important in large countries which produce natural resources such as Canada (oil, natural gas, uranium, diamonds, gold, nickel), Australia (coal, uranium, iron, potash, Bauxite) and Russia (natural gas, oil, coal, uranium, aluminum, nickel). In these three federations, the natural resource rent is owned by the regional entities. From an environmental point of view, the exploitation of energy resources can cause significant environmental damage. In this perspective, environmental taxation has attracted increasing attention as taxes can, at least in principle, internalize the external effects of environmental damage. Furthermore, many economists have argued that environmental taxes are an efficient instrument for achieving environmental objectives (see, e.g, Baumol and Oates (1988) and Pearce and Turner (1990).

Furthermore, even if high attention has turned to the key feature of the energy transition, however from a normative point of view, there are few ideas on the notion of "energy fiscal federalism" defined as the imbrication of different levels of government which possess an autonomous taxation in terms of energy. In other words, should the Member States levy alone the energy tax? What would be the effectiveness of an additional energy tax levied on a larger scale at European level? Is it desirable for fiscal policy to be fully transferred to a central level (Brussels in a Community framework, or the central State in the framework of a country) or contrarily, should be given priority to the decentralization of energy tax policy to sub-national communities in order to better account for heterogeneity among jurisdictions, in particular in terms of energy access? Should the distribution of energy taxation instruments depend on the nature of energies? These questions are then asked to determine which level of government should levy what energy taxation.

Thus the identification of the optimal jurisdiction level is crucial when determining the optimal level of taxation of energy resources. In particular, in the context of a decrease in government grants to local authorities, environmental considerations are not always enough to encourage local and regional authorities to place energy transition among the priorities of their public policies. Decentralization of energy taxation should therefore be analyzed in the light of these mechanisms, taking into account the specificities of the bases of energy resources: the environmental consequences of the exploitation of energy, the more or less renewable nature of energy, their (sometimes immediate) impacts on economic growth, the greater or lesser substitutability between the bases or the globalization of energy markets.

The present paper contributes to the theoretical literature in three ways. First, it provides additional comprehension elements on the issue of energy fiscal federalism by examining...
the structure of energy taxes aimed at internalizing the external effects from environmental
damage (pollution). Second, it extends earlier analytical work on optimal energy taxation
in the presence of other distortionary tax by considering that energy tax is imposed on
intermediate inputs for improving environmental quality\(^1\). It also takes into account both envi-
ronmental distortions associated with the external effects of environmental damages (pollution)
and, distortions linked to capital market. The third contribution of the paper is its numerical
investigation of optimal energy tax policies in the presence of other distortionary taxes.

2 Literature review

In this section we focus with some background on the several literatures that inform our re-
search. On the one hand, it highlights the notion of fiscal federalism by seeking the optimal
level of taxation. On the other hand, it focuses on issues of environmental taxation.

**Fiscal federalism literature:**

Several studies have emphasized the importance of tax externalities (horizontal and vertical)
for the theory of fiscal federalism. Initially developed by Oates Wallace (1972) and formally
modelled by Wilson (1986) and Zodrow and Mieszkowski (1986), the occurrence of these exter-
nalities depends on the mobility of production factor (generally capital factor). In particular,
horizontal tax externalities occur, when a region increases its capital tax rate, some amount of
capital is reallocated to other regions. This capital movement represents a positive externality,
implying a tendency for taxes and public expenditures to be set inefficiency low in equilibrium.
On the other hand, literature has paid a special attention on vertical tax externalities, which
arise when two or more different levels of government share the same tax base. Each level
of government neglects the adverse effect it has on the other by raising its tax rate, thereby
causing the common tax base to shrink. This tax externality points towards excessively high
state taxes (Keen (1998), Keen (1998), Hoyt (2001) and Dahlby and Wilson (2003)).

For instance, Keen and Kotsogiannis (2004) explore the impact of intensified tax competi-
tion within federal systems characterized by the presence of both horizontal tax externalities
between the states and vertical tax externalities between states and federal government. The
main conclusion from horizontal tax competition has been that horizontal externalities tend
to leave equilibrium lower-level ('state') taxes too low (race to the bottom), since each state
ignores the benefit it confers on other states by raising its tax rate and so inducing outward
movement of its tax base. On the other hand, vertical externalities tend to leave state taxes
too high: each state ignores the harm it does to others by raising its tax rate in so far as
the induced contraction in the federal tax base leads to a reduction in federal spending that
harms other states too. With horizontal externalities pointing towards state taxes that are
inefficiently low and vertical externalities towards state taxes that are inefficiently high, it is
natural to ask which will dominate. In their previous version, Keen and Kotsogiannis (2002),
show that whether equilibrium taxes are too high or too low in equilibrium depends on the

\(^1\)As mentioned in Boovenberg and Goulder (1996), economists typically have analyzed environmental taxes
without taking into account the presence of other, distortionary taxes. The omission is significant because the
consequences of environmental taxes depend fundamentally on the levels of other taxes, including income and
commodity taxes.
elasticities of the demand for capital and the supply of savings.

**About environmental taxation:**

Generally, an emissions tax is used as an environmental policy instrument to reduce local environmental damages. The rents raised from this tax are retained locally, but can be allocated to varying degrees for public finance or for private consumption. In this regard, Alexeev et al. (2016) explore the implications for jurisdictional welfare of sharing environmental rents between private and public consumption. Using three production factor (labor, capital and emissions), they show that jurisdictional welfare increases as environmental rents are initially allocated towards public consumption, yielding a “double dividend”\(^2\), but that this dividend may or may not continue as all rents are shifted to public finance. They consider that a “double dividend” can occur in the sense that welfare of jurisdictional decision-making is highest when all environmental rents are dedicated to public finance. These results illustrate the crucial importance of environmental rent sharing for the efficiency of jurisdictional decision-making.

Linked to the previous point, Kim and Wilson (1997) investigate the possibility of a ‘race to the bottom,’ under which intergovernmental competition for mobile capital leads to inefficiently lax environmental standards. The term ‘race to the bottom’ is often used to describe the possibility that intergovernmental competition for mobile capital will lead to inefficiently lax environmental policies. Their model is based on the Bucovetsky-Wilson model of tax competition with multiple tax instruments, extended to include pollution emissions from production activities. Indeed, in addition to capital and labor as input factors, they include emissions in production function. In particular, according to this input factor, they follow to Oates and Schwab (1988), by making the emissions-labor ratio, \( e = \frac{E}{L} \). They show that decentralized decision-making by independent national governments leads to environmental standards that are inefficiently lax.

According to Wellisch (1995), in the course of jurisdictional Competition, jurisdictions set environmental policies efficiently if there are no other market distortions and the environmental policies capture and return environmental rents to local, immobile residents. A similar kind of result is obtained in the classic model of Oates and Schwab (1988). In the presence of other market distortions or if jurisdictions can not to capture the environmental rents, jurisdictional environmental policymaking is not likely to be efficient.

Bovenberg and De Mooij (1994) aim to explore under which conditions environmental taxes do indeed reduce the efficiency costs of financing public spending in presence of other distortion tax (in particular, labor tax). To do end, they formulate a simple general equilibrium model of a small open economy. Besides labor \((L)\) and capital \((K)\), they also include in production

\(^2\)The double dividend hypothesis holds that it is possible to improve the quality of the environment and the efficiency of the tax system. In other words the double dividend assumption implies that there would be environmental and non-environmental gains in the application of an environmental tax when revenues from the environmental tax are used to reduce a form of taxation that creates more distortions. A ‘double dividend’, namely, both an improvement in environmental quality, but also a reduction in the efficiency costs associated with raising public revenue. In the literature, the double dividend is known into two forms: weak and strong forms. According to Goulder et al. (1999), the weak form states that it is cheaper to use green tax revenues to reduce a pre-existing distortionary tax than to redistribute those revenues into lump-sum transfers. The strong form implies that taxing a polluting good and recycling its revenues in order to reduce a representative distortionary tax does not entail any cost, or even brings income to the state.
function, a third input (E) which causes environmental damage when used in production. It is called the ‘polluting’ input and can be thought off as energy. In their model, they formalize an inverse relationship between the quality of the natural environment and the demand for, respectively, polluting consumption commodities and polluting inputs into production. They find that environmental taxes typically render the overall tax system a less efficient instrument to finance public spending. They also find that pre-existing tax distortions in the labor market reduce rather than enhance the attractiveness of environmental policy, in general, and of a heavy reliance on environmental taxes, in particular. The fundamental reason is that the environment is a collective good; all residents benefit - irrespective of the amount of labor they supply.

Bovenberg and Goulder (1996) extend the model in Bovenberg and De Mooij (1994) by incorporating intermediate inputs. They also consider three inputs in production function, namely, labor (L), "clean" and "dirty" intermediate goods. They examine how optimal environmental tax rates deviate from rates implied by the Pigovian principle in a second-best setting where other, distortionary taxes are present. They show that, in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle—even when revenues from environmental taxes are used to cut distortionary taxes.

3 Model

The starting point for the analysis is the work of Keen and Kotsogiannis (2004) with multiple tax instruments. Their model is extended by including energy component (as pollution emissions or fuel consumption) from production activities. Hence, the particular case of this study is twofold. First, we consider environmental quality as a local public good in sense that environmental quality in each jurisdiction is a function only of the quantity of pollution emitted in that jurisdiction. Therefore local governments may levy energy taxes in order to correct environmental damages. Then, we also access the existence of horizontal externalities. Second, environmental quality is a pure public good which allows to include federal tax and testing the optimal tax structure and its impact on jurisdiction welfare.

As a point of departure, we take a simple microeconomic approach in order to well understand our analysis context. In this framework, there are n identical jurisdictions, numerous enough and we also assume that there is an unique firm in each jurisdiction which supplies a single commodity (Y).

3.1 The representative household

Let \( U(C_i, g_i, G, E) \) be the utility of the representative household of locality i derives from the provision of local public goods, \( g_i \), central public goods \( G \), Environment quality, \( E \) and from the consumption of a private good, denoted by \( C_i \):

\[
U(C_i, g_i, G, E) = U(C_i) + v(g_i) + V(G) + U(E)
\]  

(1)
where the utility functions $v(\cdot)$, $V(\cdot)$, and $U(\cdot)$ are increasing in its argument, twice differentiable and strictly concave. Citizens are assumed to be identical and immobile. Like most papers on capital tax competition, our representative citizen is both the owner of a unique firm located in its locality and the owner of exogenous amounts $k$ of capital and $e$ of energy. The profit is, in its entirety, transferred to the usual owner of the firm in this type of model, that is, the representative citizen. Hence, firms maximize profits taking prices and tax rates parametrically. Net-of-tax profits read:

$$\pi_j = F(k_j, e_j) - (\rho_k + t_j + T_k)k_j - (\rho_e + \tau_j + T_e)e_j$$  \hspace{1cm} (2)

The private consumption $C$ thus amounts to the sum of the profit of the firm, denoted by $\pi_i$, and the net remuneration of the capital and energy endowments. Thus, the representative household’s budget constraint is given by:

$$C_i = \pi_i + \rho_k k + \rho_e e$$  \hspace{1cm} (3)

### 3.2 Capital and energy markets

Output can be devoted to public consumption ($G$ or $g_i$), to household consumption $C$, or to environmental quality good, using labor, capital and energy as input factors. Labor is not mobile between jurisdictions while capital and energy are perfectly mobile. We assume that labor is normalized to unity in sense that jurisdictional output in state $i$ is $F_i = F(k_i, e_i)$, where $k_i$ and $e_i$ denote respectively the capital and energy factor located in state $i$. The production function, identical across jurisdictions, is assumed to be strictly concave in $k$ and $e$ with positive and diminishing factor productivities ($F_k > 0, F_e > 0, F_{kk} < 0, F_{ee} < 0$).

For simplicity and without loss of generality, we assume that rents are untaxed. Since the producer acts as a price taker, the first-order conditions:

$$F_k = \rho_k + t_j + T_k$$ and $$F_e = \rho_e + \tau_j + T_e$$ with $t_j$ and $\tau_j$ are source-based capital and energy taxes. $\rho_k$ and $\rho_e$ are the pre-tax rate-of-return on capital and energy respectively, common across all jurisdictions. The first-order conditions represent the implicit demands for respectively, capital and energy inputs. Differentiating the first-order conditions of profit maximization, the resulting demand for capital $k$ and energy $e$ are:

$$\frac{dk_i}{dr_i} = \frac{f_{ee}}{f_{kk}f_{ee} - f_{ek}f_{ke}}$$  \hspace{1cm} (4)

$$\frac{dk_i}{dr_i} = \frac{-f_{ke}}{f_{kk}f_{ee} - f_{ek}f_{ke}}$$  \hspace{1cm} (5)
\[
\frac{de_i}{dt_i} = \frac{f_{kk}}{f_{kk}f_{ee} - f_{ek}f_{ke}} 
\]

\[
\frac{de_i}{dr_i^e} = \frac{-f_{ee}}{f_{kk}f_{ee} - f_{ek}f_{ke}} 
\]

Normalizing the price of the private good to one, the national supplies of capital and energy are respectively the sum of the initial endowment \( k \) and \( e \) of the \( n \) representative citizens of the country. Hence, \( \forall i \), given that \( r_i^k = \rho_k + t_i + T_k \) and \( r_i^e = \rho_e + \tau_i + T_e \), the capital and energy market-clearing conditions\(^3\) in the federation are:

\[
\sum_{i=1}^{n} k(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = nK, \\
\sum_{i=1}^{n} e(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = nE 
\]

which characterize the capital (resp. energy) market equilibrium, i.e. it defines in a symmetric setting the net return \( \rho_k(t_1, \ldots, t_n, \tau_1, \ldots, \tau_n) \) (resp. \( \rho_e(t_1, \ldots, t_n, \tau_1, \ldots, \tau_n) \)) as a decreasing function of the local tax rate on capital (resp. on energy). Differentiating these market-clearing conditions yields, at the symmetric equilibrium with respect to \( \rho_k, \rho_e, t_i, \tau_i \):

\[
\frac{dp_k}{dt_i} = \frac{e_{r_i^k} \sum k_{r_i^k} - k_{r_i^k} \sum e_{r_i^k}}{\sum k_{r_i^k} \sum e_{r_i^k} - \sum k_{r_i^e} \sum e_{r_i^e}} = \frac{-f_{ek} * (n f_{ke}) - f_{ee} * (n f_{kk})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = -\frac{1}{n} 
\]

\[
\frac{dp_e}{d\tau_i^e} = \frac{k_{r_i^e} \sum e_{r_i^e} - e_{r_i^e} \sum k_{r_i^e}}{\sum k_{r_i^e} \sum e_{r_i^e} - \sum k_{r_i^e} \sum e_{r_i^e}} = \frac{-nf_{ke}(-nf_{ek}) - (-nf_{kk}nf_{ee})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = -\frac{1}{n} 
\]

\[
\frac{dp_k}{d\tau_i} = \frac{e_{r_i^k} \sum k_{r_i^k} - k_{r_i^k} \sum e_{r_i^k}}{\sum k_{r_i^k} \sum e_{r_i^k} - \sum k_{r_i^e} \sum e_{r_i^e}} = \frac{-nf_{kk}f_{ke} - (-nf_{ke}f_{kk})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = 0 
\]

\[
\frac{dp_e}{dt_i} = \frac{k_{r_i^e} \sum e_{r_i^e} - e_{r_i^e} \sum k_{r_i^e}}{\sum k_{r_i^e} \sum e_{r_i^e} - \sum k_{r_i^e} \sum e_{r_i^e}} = \frac{-nf_{ee}f_{ke} - (-nf_{ke}f_{ee})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = 0 
\]

The demand functions of capital and energy depend on all fiscal remunerations of production factors \( (\rho_k, \rho_e, t_j, \tau_j) \) implying that each firm in local jurisdiction take into account both

\(^3\)The capital (resp. energy) market clearing condition implies that aggregate demand for capital (resp. demand for energy) must equal capital supply (resp. energy supply)
local and neighboring characteristics when determining demand functions. Eq. (8) and (9) imply that the interest rate moves as follows:

\[
\frac{dr^i}{dt_i} = 1 + \frac{d\rho}{dt_i} = 1 - \frac{1}{n} \quad \text{and} \quad \frac{dr^e}{d\tau_i} = 1 + \frac{d\rho}{d\tau_i} = 1 - \frac{1}{n}
\]

Following to eq. (8) to (11), local tax choices affect the location choice of capital or energy. Indeed, since the net return on capital (resp. on energy) decreases when the tax rate on capital (resp. on energy) \( \forall i, \forall j \) increases \((\frac{d\rho}{dt_i} = -\frac{1}{n} \) and \( \frac{d\rho}{d\tau_i} = -\frac{1}{n} \)). Therefore, it should be noted that horizontal tax externalities emerge among local governments based on the mobility of the tax base on capital (resp. on energy). We also found that the impact of federal taxation on the net return has no impact at the local level due to the nondistortionary of federal taxation on capital and energy.

### 3.3 Federal and local governments

Pure public goods are provided at both local and regional tiers, with no spillovers and no scale economies. Each local government \( i \) provides a local public good in quantity \( g_i \), which is financed by the taxation at rates \( t_i \) and \( \tau_i \) of the amount of capital \( K_i \) and \( e_i \) invested in its location. The local budget constraint is thus given by

\[
g_i = t_i k_i + \tau_i e_i \quad (12)
\]

Federal government levies a unit tax common of each input to all states at the rates \( T_k \) and \( T_e \) which served to finance central public good \( G \) in jurisdiction \( i \) (i.e. federal spending in this jurisdiction). The federal budget constraint is thus given by:

\[
G_i = \frac{1}{n} T_k \sum_{i=1}^{n} k(\rho_k + t_i + T_k) + T_e \sum_{i=1}^{n} e(\rho_e + \tau_i + T_e) \quad (13)
\]

It is noteworthy that capital and energy can be taxed by both levels of government. State \( j \) levies a source-based tax \( t_j \) and \( \tau_j \) on each unit of capital (resp. of energy) in its jurisdiction while the federal government levies a unit tax—common to all states—at the rate \( T_k \) (resp. \( T_e \)). We assume here that the federal government allocates its total tax receipts equally across states and recall jurisdictions are identical within countries. We assume, further, that there are no intergovernmental transfers neither central-local government transfers nor between local governments. Both federal and regional governments are benevolent. Each regional government acts so as to maximize the utility of the representative household located in its location.

### 3.4 Environmental quality

As noted above, there are three consumption commodities. One of these commodities is the so-called environmental quality:
\[ E = h(e) \]  

with \( h_e < 0 \) which link up to energy resource. This expression formalizes the inverse relationship between the quality of the natural environment and the demand for energy inputs into production. We assume that energy consumption \((e)\) harms environmental quality \((E)\). Indeed, in the process of production, the use of energy input is associated with less environmental quality for final good (high level of emissions). Following to Bovenberg and Goulder, 1996, we consider that environment quality is a collective good and weakly separable from private goods. Accordingly, households take the quality of the environment as given and they ignore the adverse effect of their demand for polluting goods on the quality of the environment. In other words, energy resource decreases utility of final good.

4 Benchmark cases:

As is usual in the literature using the symmetric competition model, the analysis is conducted for a "representative" jurisdiction. Before illustrating the subgame-perfect equilibrium, we first present the outcome of the first-best solution which will serve for comparison purposes. In this benchmark case, a benevolent social planner aims to maximize the aggregated welfare \( W(t_i, \tau_i, T_k, T_e) = U(C_i, g_i, G_i, E_i) \)

subject to the budget constraints (3), (12) and (13) and (14). Suppose that central and local governments play a Nash game. In this situation, the federal government and the local governments simultaneously select their budgetary choices.

4.1 Local governments program:

Local government planners are assumed to choose taxes and expenditure policies so as to maximize a social welfare function which includes the utility level of its existing residents. To derive the optimal tax rates, they solve the government's problem of maximizing household utility subject to the government budget constraint and the decentralized optimizing behavior of firms and households, taking as given the tax choices of other localities. It thus solves the problem:

\[
\begin{align*}
\text{Max} & \ W(t_i, \tau_i, T_k, T_e) = U(C_i, g_i, G_i, E_i) \\
C_i & = \pi_i + \rho_k k_i + \rho_e e_i, g_i = t_i k_i + \tau_i e_j, G_i = \frac{1}{n} T_k \sum_{i=1}^{n} k_i \rho_k + t_i + T_e \sum_{i=1}^{n} e_i \rho_e + \tau_i \\
E_i & = h(e_i) \\
\end{align*}
\]

Solving first-order conditions for all localities, using Cramer's rule gives:

\[
\begin{align*}
\begin{align*}
\text{Max} & \ W(t_i, \tau_i, T_k, T_e) = U(C_i, g_i, G_i, E_i) \\
C_i & = \pi_i + \rho_k k_i + \rho_e e_i, g_i = t_i k_i + \tau_i e_j, G_i = \frac{1}{n} T_k \sum_{i=1}^{n} k_i \rho_k + t_i + T_e \sum_{i=1}^{n} e_i \rho_e + \tau_i \\
E_i & = h(e_i) \\
\end{align*}
\end{align*}
\]
\[ t^*_i = \frac{n}{n-1} \frac{U_C - \sigma_g}{\sigma_g} (f_{kk} k_i + f_{ke} e_i) \] (15)

\[ \tau^*_i = \frac{n}{n-1} \frac{U_C - \sigma_g}{\sigma_g} (f_{ke} k_i + f_{ee} e_i) - \frac{U_E}{\sigma_g} E_e \] (16)

The term \( \frac{U_C}{\sigma_g} \) denotes the marginal rate of substitution of private consumption (C) for the local public good (g), while \( \frac{U_E}{\sigma_g} \) is the marginal rate of substitution of environmental quality (E) for the local public good (g).

**Proposition 1:** In the presence of two distortionary taxes, local governments set taxe rates taking into account the size of the jurisdiction, the marginal rate of substitution (between private and public goods) and the endowments in input factors (capital and energy).

4.2 Central government program

The maximization program faced by the central government is the following:

Max \( W(t_i, \tau_i, T_k, T_e) = U(C_i, g_i, G_i, E_i) \)

\[ C_i = \pi_i + \rho_k k_i + \rho_e e_i, \quad g_i = t_i k_i + \tau_i e_j, \quad G_i = \frac{1}{n} T_k \sum_{i=1}^{n} k(\rho_k + t_i) + T_e \sum_{i=1}^{n} e(\rho_e + \tau_i) \text{ and } E_i = h(e_i). \]

**Proposition 2:** Each government equalizes the marginal rate of substitution (MRS) to the marginal rate of fiscal transformation (MRT) between the private and the public goods.

At the first-best optimum, we found that \( -U_C \tilde{k} + \sigma_G k_i = 0 \) and \( -U_C \tilde{e} + \sigma_G e_i = 0 \) implying that at the symmetric equilibrium: the marginal rate of substitution of the public good (G) for private consumption (C) (\( \frac{\sigma_G}{\tilde{C}_G} = \frac{\tilde{k}}{k_i} = \frac{\tilde{e}}{e_i} = 1 \)) equals to 1.

4.3 Analytical Model with Cobb–Douglas Functional Forms

In this section, we establish our second main result about energy fiscal federalism. Our purpose is to show the tractability of our method with commonly-used production functions. To provide more insight, Cobb–Douglas functional forms are used to demonstrate the way solutions respond to production and utility function parameters. In this regards, we consider the following production function:

\[ f(k, e) = k^{\sigma_k} e^{\sigma_e} \] (17)

The variables \( k \) and \( e \) are defined as before; \( \sigma_k \) and \( \sigma_e \) are the share parameters for capital and energy, respectively.

Jurisdictional utility is represented as: \( U(t_i, \tau_i) = C_i^{\sigma_C} g_i^{\sigma_g} G^{\sigma_G} E^{\sigma_E} \)
Optimizing utility function for local governments and using Cramer’s rule give:

\[ t^*_i = \frac{\sigma_p \sigma_k (\sigma_k - 1) + \sigma_p \sigma_k \sigma_e \frac{1}{E} + \sigma_k (\sigma_k - 1) \left( \frac{1}{k_i} - \frac{1}{c_i} \right)}{\text{det} D} \]  

(18)

\[ \tau^*_i = \frac{-\sigma_p \sigma_k (\sigma_k - 1) - \sigma_p \sigma_k \sigma_e \frac{1}{E} - [\sigma_k (\sigma_k - 1) + \sigma_e (\sigma_e - 1)] \frac{1}{c_i}}{\text{det} D} \]  

(19)

with \( \text{det} D = \sigma_c \kappa \omega \frac{k_i}{C_i} (f_{kk} - f_{ek}) + \sigma_c \kappa \omega \frac{2 f_{kk} f_{ek}}{E f_{kk} f_{ee} - f_{ke} f_{ek}} \frac{n-1}{n} + \sigma_c \omega \frac{f_{kk} f_{ee} - f_{ke} f_{ek}}{E f_{kk} f_{ee} - f_{ke} f_{ek}} \frac{n-1}{n} + \sigma_c \omega \left( \frac{-k^2}{C_i} f_{kk} - \frac{-e^2}{C_i} f_{ee} \right) + \frac{f_{kk} f_{ee} + f_{ke} f_{ek}}{f_{kk} f_{ee} - f_{ke} f_{ek}} \frac{n-1}{n} \)

5 Stackelberg game with regional decentralized leadership

We now introduce decentralized leadership and analyze the impact of fiscal scheme on regional budgetary incentives (in progress...)

6 Conclusion:

Based on the classical work of Keen and Kotsogiannis (2004) with multiple tax instruments, we analyze the nature of externalities in presence of two distortionary taxes (capital and energy taxes). Our results are resumed as follow. Firstly, we found that horizontal externalities may occur due both to the mobility of production factors (capital and energy) and to the citizen’s preferences between private and public consumption. Secondly, federal taxation (both on capital and energy) on the net return has no impact at the local level due to its nondistortionary. Finally, we also investigate various functions in order to show the tractability of our method.

7 Appendix

7.1 Appendix A: Profit maximization and market clearing conditions

Profit maximization

\[
\begin{aligned}
f_{k_i} &= \rho_k + t_j \\
f_{e_j} &= \rho_e + \tau_j
\end{aligned}
\]

Differentiating the first-order conditions of profits with respect to \( t_j \) and \( \tau_j \) gives:

\[
\begin{aligned}
f_{kk} \frac{\partial k}{\partial t_i} + f_{ke} \frac{\partial e}{\partial t_i} &= \frac{\partial \rho_k}{\partial t_i} + 1 \\
f_{ke} \frac{\partial k}{\partial t_i} + f_{ee} \frac{\partial e}{\partial t_i} &= \frac{\partial \rho_e}{\partial t_i}
\end{aligned}
\]
\[
\begin{align*}
\begin{cases}
\frac{\partial k}{\partial \tau_i} + \frac{\partial e}{\partial \tau_i} = \frac{\partial \rho_k}{\partial \tau_i} \\
\frac{\partial e}{\partial \tau_i} + \frac{\partial e}{\partial \tau_i} = \frac{\partial \rho_e}{\partial \tau_i} + 1
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\frac{\partial k}{\partial \tau_i} + \frac{\partial e}{\partial \tau_i} = \partial r_k \\
\frac{\partial e}{\partial \tau_i} + \frac{\partial e}{\partial \tau_i} = \partial r_e
\end{cases}
\end{align*}
\]

Using Cramer's rule gives:

\[
dk_i = \frac{f_{ee} \partial r_k - f_{ke} \partial r_e}{f_{kk} f_{ee} - f_{ke} f_{ke}} \tag{20}
\]

\[
di_i = \frac{f_{kk} \partial r_e - f_{ke} \partial r_k}{f_{kk} f_{ee} - f_{ke} f_{ke}} \tag{21}
\]

**Market clearing conditions** Differentiating market clearing conditions with respect to \( \rho_k, \rho_e, t_i, \tau_i, T_k \) et \( T_e \), yields the following system of equations:

\[
\begin{align*}
\begin{cases}
\sum_{i=1}^{n} k_{ri} d\rho_k + \sum_{i=1}^{n} e_{ri} d\rho_e + k_{ri} dt_i + k_{ri} dT_k + \sum_{i=1}^{n} k_{ri} dT_e = 0 \\
\sum_{i=1}^{n} e_{ri} d\rho_k + \sum_{i=1}^{n} e_{ri} d\rho_e + e_{ri} dt_i + e_{ri} dT_k + \sum_{i=1}^{n} e_{ri} dT_e = 0
\end{cases}
\end{align*}
\]

### 7.2 Appendix B: intermediate details of maximization program

**Impact of taxes on private consumption:**

\( \pi_i = f(k_i, e_i) - r^k_i k_i - r^e_i e_i \Rightarrow \frac{d\pi_i}{dt_i} = \pi'_{k_i} = f'\left(\frac{dk_i}{dt_i}\right) + f'\left(\frac{de_i}{dt_i}\right) - k_i - \frac{\partial \rho}{\partial t_i} - \frac{e_i}{\partial t_i} \)

\( \pi'_{k_i} = (f' - r^k_i) \frac{dk_i}{dt_i} + (f' - r^e_i) \frac{de_i}{dt_i} - k_i \Rightarrow \pi'_{k_i} = -k_i \)

Likewise \( \pi'_{e_i} = (f' - r^k_i) \frac{dk_i}{dt_i} + (f' - r^e_i) \frac{de_i}{dt_i} - e_i \Rightarrow \pi'_{e_i} = -e_i \)

\( C_i = \pi_i + \rho_k \bar{k} + \rho_e \bar{e} \)

\( \frac{dC_i}{dt_i} = \pi'_{k_i} \frac{dk_i}{dt_i} + \bar{k} \frac{d\rho_k}{dt_i} \Rightarrow \frac{dC_i}{dt_i} = -k_i (1 + \frac{d\rho_k}{dt_i}) + \bar{k} \frac{d\rho_k}{dt_i} \Rightarrow \)

\( \frac{dC_i}{dt_i} = -k_i \) \( \tag{22} \)
\[
\frac{dC_i}{dt_i} = \pi_i \cdot \frac{dr_i}{dt_i} + \tau_i \frac{d\rho_i}{dt_i} \Rightarrow \frac{dC_i}{dt_i} = -k_i (1 + \frac{d\rho_i}{dt_i}) + \tau_i \frac{d\rho_i}{dt_i} \Rightarrow \\
\frac{dC_i}{dt_i} = -e_i
\]

(23)

\[
\frac{dC_i}{dT_k} = -\bar{k} \\
\frac{dC_i}{dT_e} = -\bar{\tau}
\]

(24)

(25)

**Impact of taxes on local public good:**

\[
g_i = t_i k_i + \tau_i e_i \Rightarrow \frac{dg_i}{dt_i} = k_i + t_i \frac{dk_i}{dt_i} + \tau_i \frac{d\rho_i}{dt_i} = k_i + (t_i k_i + \tau_i e_i) \frac{d\rho_i}{dt_i}
\]

\[
\frac{dg_i}{dt_i} = k_i + n - 1 \cdot n \cdot \frac{1}{\int_{kk} f_{ee} - f_{ke} f_{ek}} (t_i f_{ee} - \tau_i f_{ek})
\]

(26)

\[
\frac{dg_i}{dt_i} = e_i + t_i \frac{dk_i}{dt_i} + \tau_i \frac{d\rho_i}{dt_i} = e_i + (t_i k_i + \tau_i e_i) \frac{d\rho_i}{dt_i}
\]

\[
\frac{dg_i}{dt_i} = e_i + n - 1 \cdot n \cdot \frac{1}{\int_{kk} f_{ee} - f_{ke} f_{ek}} (-t_i f_{ke} + \tau_i f_{kk})
\]

(27)

\[
\frac{dg_i}{dT_k} = \frac{dg_i}{dT_e} = 0
\]

(28)

**Impact of taxes on federal public good:**

\[
G = \frac{1}{n} T_k \sum_k (r_{i_k}^k, r_{i_e}^k) + \frac{1}{n} T_e \sum_e (r_{i_k}^e, r_{i_e}^e) \Rightarrow \\
\frac{dG}{dt_i} = \frac{1}{n} T_k \sum_k \frac{d r_{i_k}^k}{dt_i} + \frac{1}{n} T_e \sum_e \frac{d r_{i_e}^e}{dt_i} = \frac{1}{n} T_k \sum_k k_i (1 + \frac{d\rho_i}{dt_i}) + \frac{1}{n} T_e \sum e_i (1 + \frac{d\rho_i}{dt_i}) \Rightarrow \\
\frac{dG}{dt_i} = \frac{1}{n} T_k \frac{1}{f_{kk} f_{ee} - f_{ke} f_{ek}} (f_{ee} + n. f_{ee} (\frac{1}{n})) + \frac{1}{n} T_e \frac{1}{f_{kk} f_{ee} - f_{ke} f_{ek}} (-f_{ek} - n. f_{ek} (\frac{1}{n}))
\]

\[
\frac{dG}{dT_k} = \frac{dG}{dT_e} = 0
\]

(29)

\[
\frac{dG}{dT_k} = \frac{1}{n} \sum k_i (r_{i_k}^k, r_{i_e}^k) = k_i
\]

(30)

\[
\frac{dG}{dT_e} = \frac{1}{n} \sum e_i (r_{i_k}^e, r_{i_e}^e) = e_i
\]

(31)
8 Environment-energy relationship:

\( E_i = h(e_i) \)

\[
\frac{dE}{dt_i} = \frac{dh}{de_i} \frac{de_i}{dt_i} = E_{e_i} e_i \frac{n-1}{n}
\]

\[
\frac{dE}{dt_i} = -E_e \frac{n-1}{n} \frac{f_{ee}}{f_{kk} f_{ee} - f_{ke} f_{ek}}
\]  \hspace{1cm} (32)

\[
\frac{dE}{d\tau_i} = E_{e_i} e_i \frac{dr_i}{d\tau_i}
\]

\[
\frac{dE}{d\tau_i} = E_e \frac{n-1}{n} \frac{f_{kk}}{f_{kk} f_{ee} - f_{ke} f_{ek}}
\]  \hspace{1cm} (33)

\[
\frac{dE}{dT_k} = \frac{dE}{dT_e} = 0
\]  \hspace{1cm} (34)
References


