Energy transition with variable and intermittent renewable electricity generation

May 1, 2018

1 Introduction

In the short/medium term, renewables cannot be deployed at a large scale to replace coal and other fossil fuels in electricity generation. Indeed, they are on the one hand on average still more costly than fossil fuels, and on the other hand both variable, which is predictable (night and day, seasons), and intermittent, which is not (cloud cover, etc.). But in the future the production of electricity has to be decarbonized, and producing energy by renewable means seems to be the only possibility to do so[1], before nuclear fusion becomes eventually available.

The literature considering the penetration of renewables in the energy mix consists so far in two separate trends.

On the one hand, macro-dynamic models à la Hotelling consider renewable energy as an abundant and steady flow available with certainty, possibly after an investment in capacity has been made, at a higher unit cost than fossil energy[2]. The issue is the cost –otherwise clean renewable energy would replace polluting fossil fuels immediately. Thus standard models of energy transition ignore variability and intermittency and focus on the cost issue. However, the expansion of renewables will probably not be limited by the direct costs of electricity generation in the (near) future. Costs have already been widely reduced, due to technical progress and learning effects in production and installation, and the decrease is expected to continue, until a limit lower bound of the cost is reached. For instance, according to the International Energy Agency (2011), solar PV costs have been reduced by 20% for each doubling of the cumulative installed capacity. The Energy Information

[1]Carbon Capture and Storage (CCS) is another option, but it is still expensive and can only offer a partial solution as potential carbon sinks are of limited capacity. CCS has already been extensively studied. See for instance Lafforgue, Magne and Moreaux (2008).

[2]See, for early path-breaking papers, Hoel and Kverndokk (1996), and Tahvonen (1997).

Administration reports that the US average levelized cost of electricity in 2012 is $/MWh 95.6 for conventional coal, 66.3 for natural gas-fired combined cycle, 80.3 for terrestrial wind, 204.1 for offshore wind, 130 for solar PV, 243 for solar thermal and 84.5 for hydro. Terrestrial wind and hydro (that we do not consider here because the expansion possibilities are very limited in developed countries) are already competitive, and solar PV is rapidly catching up in the US. In sunny and dry countries it is even more so: solar PV has already obtained grid parity[3] in sunny islands, and is expected to reach grid parity very soon for instance in Italy or California. Hence the real obstacle to a non-marginal expansion of renewables is not their cost but their variability, their intermittency, and maybe also their footprint in terms of land use – especially for wind energy.

Another strand of literature is composed of static models that are not directly interested in energy transition, but focus on the design of the electric mix (fossil fuels and renewables) when intermittency is taken into account, with or without storage devices. Ambec and Crampes (2012, 2015) are representative of this literature. They study the optimal electric mix with intermittent renewable sources, and contrast it to the mix chosen by agents in a decentralized economy where the retailing price of electricity does not vary with its availability. They examine the properties of different public policies and their impacts on renewable penetration in the electric mix: carbon tax, feed-in tariffs, renewable portfolio standards, demand-side management policies.

A recent survey on the economics of solar electricity (Baker *et al.,* 2013) emphasizes the lack of economic analysis of a decentralized clean energy provision through renewable sources. We intend to contribute to fill this lack by putting together the two strands of the literature mentioned above, in order to make macro-dynamic models more relevant for the study of the energy transition. Indeed we believe that the energy transition is by essence a dynamic problem, which cannot be fully understood through static models. On the other hand, dynamic models are so far unable to take into account properly some crucial features of renewables. We plan to extend to a dynamic setting the static models cited above, taking into account variability and intermittency, in order to study to what extent they actually constitute a serious obstacle to energy transition.

In a first step we tackle the variability issue alone. We build a stylized deterministic dynamic model of the optimal choice of the electric mix (fossil and renewable), where the fossil energy, let us say coal for the purpose of illustration, is abundant but CO_2-emitting, and the renewable energy, let us say solar, is variable but clean. The originality of the model is that electricity produced when the renewable source of energy is available, and electricity produced when it is not, are considered two different goods: day-electricity and night-electricity in the case of solar energy. At each period of time, consumers derive utility from the consumption of the two goods. Considering that there are two different goods allows

[3]Grid-parity is reached when the cost of electricity generation with the renewable source is roughly equal to the retailing electricity price.

taking into account intra-day variability. Day-electricity can be produced with coal and/or solar. Night-electricity can be produced with coal, or by the release of day-electricity that has been stored to that effect. Storing energy is costly due to the loss of energy during the restoration process. We consider that coal and solar are available at zero variable costs, in order to focus on the variability and intermittency issues. We also make the assumption that at the beginning of the planning horizon coal-fired power plants already exist so that there is no capacity constraint on the production of electricity by the fossil source, but that the existing solar capacity is small so that investments are to be made in order to build up a sizable capacity.

We solve the centralized program under the constraint of a carbon budget that cannot be exceeded, and derive an optimal succession of regimes. We show that with a low initial solar capacity it is optimal to first use fossil fuels during night and day, then use fossil fuels during night only and finally go for no fossil fuels at all, when the carbon budget is exhausted. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time. We show that storage begins when fossil fuels have been abandoned at day and the solar capacity is large enough. Simulations allow us to analyze the consequences of improvements in the storage and solar power generation technologies and of a more stringent environmental policy on the optimal investment decisions and energy mix.

In a second step, we introduce intermittency[4] in the model and study the design of the power system enabling to accommodate it. With intermittency, day-electricity generation by solar power plants becomes uncertain. We consider that there is only partial generation if solar radiations are too weak due for instance to the cloud system, which occurs with a given probability. We exhibit two very different situations. If the cloud problem is not too severe, we show that intermittency does not matter so much, rejoining there the empirical result of Gowrisankaran *et al.* (2016). The energy transition follows the same succession of phases as under variability only, and intermittency only makes things a little bit worse. On the contrary, if there may be very few sun during the day, the transition is very different as fossil fuels are abandoned later while storage starts earlier. In this case, intermittency leads to a very significant welfare loss.

The structure of the paper is the following. Section 2 sets up the framework, solves the model and studies the sensitivity of the solution to the main parameters in the case where variability only is taken into account. Section 3 introduces intermittency. We compute in Section 4 the value of storage. Section 5 concludes.

[4]This means that clean energy is not only variable but intermittent as well.

2 Variability in renewable electricity generation

One of the novelties of the paper is that day and night electricity are modeled as two different goods that the representative household wants to consume at each point in time. Energy requirements may be satisfied by fossil sources, let's say coal, at day and night. Coal is abundant and carbon-emitting: the issue with coal extraction and consumption is not scarcity but climate change. There are no extraction costs. Climate policy takes the form of a carbon budget that society decides not to exceed, to have a good chance to maintain the temperature increase at an acceptable level -typically 2°C. This carbon budget is consumed when coal is burned. It is also possible to use a renewable source of energy, abundant but clean, provided that a production capacity is built. This energy is *variable* i.e. changes in a predictable way. We consider for the purpose of illustration that it is solar energy, that can be harnessed at day but not at night. Costly investment allows to increase solar capacity. There exists a storage technology that allows to store imperfectly electricity from day to night at no monetary cost but with a physical loss.

Our objective is to determine the optimal electric mix, the path of investment in renewable capacity and the optimal storage policy.

2.1 The optimal solution

The social planner seeks to maximize the discounted sum of the net surplus of the economy. Instantaneous net surplus is the difference between the utility of consuming day and night-electricity and the cost of the investment in solar capacity.[5] Day-electricity can be produced by coal-fired power plants and/or solar plants. A fraction of solar electricity can be stored to be released at night.[6] In addition to fossil electricity, night-electricity can be produced by the release of solar electricity stored during the day, with a loss.[7]

[5]For simplicity and to focus on the variability issue, we ignore the extraction cost of coal and the variable cost of using solar panels. We suppose that a large fossil capacity exists at the beginning of the planning horizon but that the initial solar capacity is low.

[6]It does not make sense to store coal electricity since coal-fired power plants can be operated at night as well and there is no capacity constraint.

[7]For instance, according to Yang (2016), the efficiency of pumped hydroelectric storage (defined as the electricity generated divided by the electricity used to pump water) is lower than 60% for old systems, but over 80% for state-of-the-art ones.

The social planner's programme reads:

$$\max \int_0^{\infty} e^{-\rho t} \left[u\left(e_d(t), e_n(t)\right) - C(I(t)) \right] dt \tag{1}$$

$$\begin{aligned}
e_d(t) &= x_d(t) + (1 - a(t))\overline{\phi} Y(t) \\
e_n(t) &= x_n(t) + k a(t) \overline{\phi} Y(t) \\
\dot{X}(t) &= x_d(t) + x_n(t) \\
\dot{Y}(t) &= I(t) \\
0 \leq a(t) &\leq 1 \\
X(t) &\leq \overline{X} \\
x_d(t) &\geq 0, x_n(t) \geq 0 \\
X_0 \geq 0, &\quad Y_0 \geq 0 \text{ given}
\end{aligned}$$

where u is the instantaneous utility function, supposed to have the standard properties, e_d and e_n are respectively day and night-electricity consumption, x_d and x_n are fossil-generated electricity consumed respectively at day and night, X is the stock of carbon accumulated into the atmosphere due to fossil fuel combustion, $\overline{X}$ is the carbon budget i.e. the ceiling on the atmospheric carbon concentration, Y is solar capacity, $\overline{\phi}$ measures the efficiency of solar electricity generation, I is the investment in solar capacity, $C(I)$ is the investment cost function, a is the share of solar electricity produced at day that is stored to be released at night. The efficiency of the storage technology is represented by the parameter $k \in [0, 1]$ ($1 - k$ is the leakage rate of this technology). ρ is the discount rate.

We make the following assumptions on the utility and investment cost functions: utility is logarithmic and investment cost is quadratic (because of adjustment costs):

$$u\left(e_d, e_n\right) = \alpha \ln e_d + (1 - \alpha) \ln e_n, \qquad 0 < \alpha < 1 \tag{2}$$

$$C(I) = c_1 I + \frac{c_2}{2} I^2, \qquad c_1, c_2 > 0 \tag{3}$$

With these assumptions we are able to solve the problem analytically. We obtain the following results.

Proposition 1 *In the case where only variability of renewable energy is taken into account and the initial solar capacity is low, the optimal solution consists in 4 phases:*

(1) production of day and night-electricity with fossil fuel-fired power plants complemented at day by solar plants, no storage, investment in solar panels to increase solar capacity (from 0 *to* $\underline{T}$*);*

(2) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, no storage (from $\underline{T}$ *to* T_i*);*

(3) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, progressive increase of storage from 0 to its maximal value, which depends on preferences for day and night-electricity (from T_i to $\overline{T}$);

(4) production of day and night-electricity with solar plants only, storage at its maximum value at day to produce night-electricity, and investment in solar panels to increase capacity, up to a steady state (from $\overline{T}$ to ∞). This last phase begins when the carbon budget is exhausted.

Proof. See Appendix A. ■

Proposition 1 shows that it is always optimal to begin installing solar panels immediately and to use them to complement fossil energy at daytime. However, it is never optimal to begin storing immediately. Storage would allow saving fossil energy at night, but at the expense of more fossil at day to compensate for the solar electricity stored; it would also cause a physical loss of electricity. Even if the storage technology is available, as it is the case in the model, storage must only begin after fossil has been abandoned at day because the installed solar capacity has become high enough. Full storage coincides with the final abandonment of fossil.

We show analytically in Appendix A that the four phases identified in Proposition 1 are characterized by the following equations.

- Evolution of the shadow value λ of the atmospheric carbon stock (the carbon value) before the carbon budget is exhausted:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{4}$$

- Evolution of solar capacity over the whole horizon:

$$\dot{Y}(t) = \frac{1}{c_2}(\mu(t) - c_1) \tag{5}$$

  where μ is the shadow value of solar capacity.

- Evolution of the value of solar capacity in each phase:

$$\begin{aligned}
&\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - \overline{\phi}\lambda(t) \\
&\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{\alpha}{Y(t)} \\
&\text{Phase (3)} \qquad \dot{\mu}(t) = \rho\mu(t) - k\overline{\phi}\lambda(t) \\
&\text{Phase (4)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{1}{Y(t)}
\end{aligned} \tag{6}$$

- Fossil fuel use, storage and total electricity consumption in each phase:

$$\begin{array}{lllll}
\text{Phase (1)} & x_d(t) = \dfrac{\alpha}{\lambda(t)} - \overline{\phi}Y(t) & x_n(t) = \dfrac{1-\alpha}{\lambda(t)} & a(t) = 0 & \\
 & e_d(t) = \dfrac{\alpha}{\lambda(t)} & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (2)} & x_d(t) = 0 & x_n(t) = \dfrac{1-\alpha}{\lambda(t)} & a(t) = 0 & (7) \\
 & e_d(t) = \overline{\phi}Y(t) & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (3)} & x_d(t) = 0 & x_n(t) = \dfrac{1}{\lambda(t)} - k\overline{\phi}Y(t) & a(t) = 1 - \dfrac{\alpha}{k\lambda(t)\overline{\phi}Y(t)} & \\
 & e_d(t) = \dfrac{\alpha}{k\lambda(t)} & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (4)} & x_d(t) = 0 & x_n(t) = 0 & a^* = 1 - \alpha & \\
 & e_d(t) = \overline{\phi}\alpha Y(t) & e_n(t) = (1-\alpha)k\overline{\phi}Y(t) & &
\end{array}$$

The carbon value follows the Hotelling rule (Eq. (4)).

The joint paths of solar capacity Y and its shadow value μ are determined by the four dynamic systems composed of Eq. (5) and each of the equations in system (6). They are represented on the phase diagram of Figure 1, constructed for a given $\lambda(0)$ that reflects the stringency of the climate constraint. The last phase is a saddle path leading to a steady state $\mu^* = c_1$, $Y^* = 1/(\rho c_1)$. Moving backward from the steady state along the stable branch, date $\overline{T}$ is reached. The relevant path then corresponds to phase (3) where the dynamics of Y and μ are independent, until date T_i is reached. From T_i on, the saddle path of phase (2), corresponding to the steady state $\mu^* = c_1$, $Y^{**} = \alpha/(\rho c_1)$ is followed until date $\underline{T}$ (the steady state is never reached).[8] After $\underline{T}$ the path followed corresponds to phase (1) where Y and μ are independent, until date 0. The initial shadow value of solar panels, $\mu(0)$ then obtained (as a function of $\lambda(0)$) matches the initial condition $Y(0) = Y_0$.

For a given value of $\lambda(0)$, the joint evolutions of λ and Y trigger the phase switchings, i.e. give dates $\overline{T}$, T_i and $\underline{T}$ (see Figure 2 and Eqs. (4) and (5)).

The climate constraint then pins down $\lambda(0)$.

In phase (1), $e_d(t) = \alpha/\lambda(t)$ and $e_n(t) = (1-\alpha)/\lambda(t)$: electricity consumptions are only driven by the carbon value, i.e. by climate policy. In phase (2) (when there is no storage), it is still the case for night electricity consumption, but during daytime, electricity only depends on installed solar capacity.[9] In phase (3) storage occurs and no fossil is used

[8]Phase (2) does not exist if there is no loss in the storage technology ($k = 1$).

[9]It implies that there is overcapacity of fossil generation during daytime.

during daytime; but due to storage (driven by the climate constraint), it is again only the climate constraint that determines electricity consumption at each period. In phase (4), when electricity production is totally carbon-free, electricity consumption at night and day only depends on installed solar capacity. The amount of electricity stored at day to be consumed at night only depends on the preference for night-electricity: $a^* = 1 - \alpha$. If $\alpha > 1/2$, consumers prefer consuming at day, when there is sun. This means that peak time consumption coincides with the availability of solar electricity. It is obviously the most favorable case. If on the contrary $\alpha < 1/2$, sun is shining at off-peak time. This would correspond to the case that gives rise to the Californian duck documented by CAISO[10] and recently analyzed by Fowlie[11] and Wolfram:[12] in California, there has been more and more solar generation during day in the recent years, while consumption is mainly in the evening, meaning that sun shines off-peak. These dynamics generate a daily net generation (i.e. electricity generation net of electricity consumption) profile that evolves to look like a duck.

[10] What the duck curve tells us about managing a green grid, https://www.caiso.com/Documents/FlexibleResourcesHelpRenewables_FastFacts.pdf

[11] See "The duck has landed", the Energy Institute Blog, https://energyathaas.wordpress.com/2016/05/02/the-duck-has-landed/

[12] See "What's the Point of an Electricity Storage Mandate?" https://energyathaas.wordpress.com/2013/07/29/whats-the-point-of-an-electricity-storage-mandate/

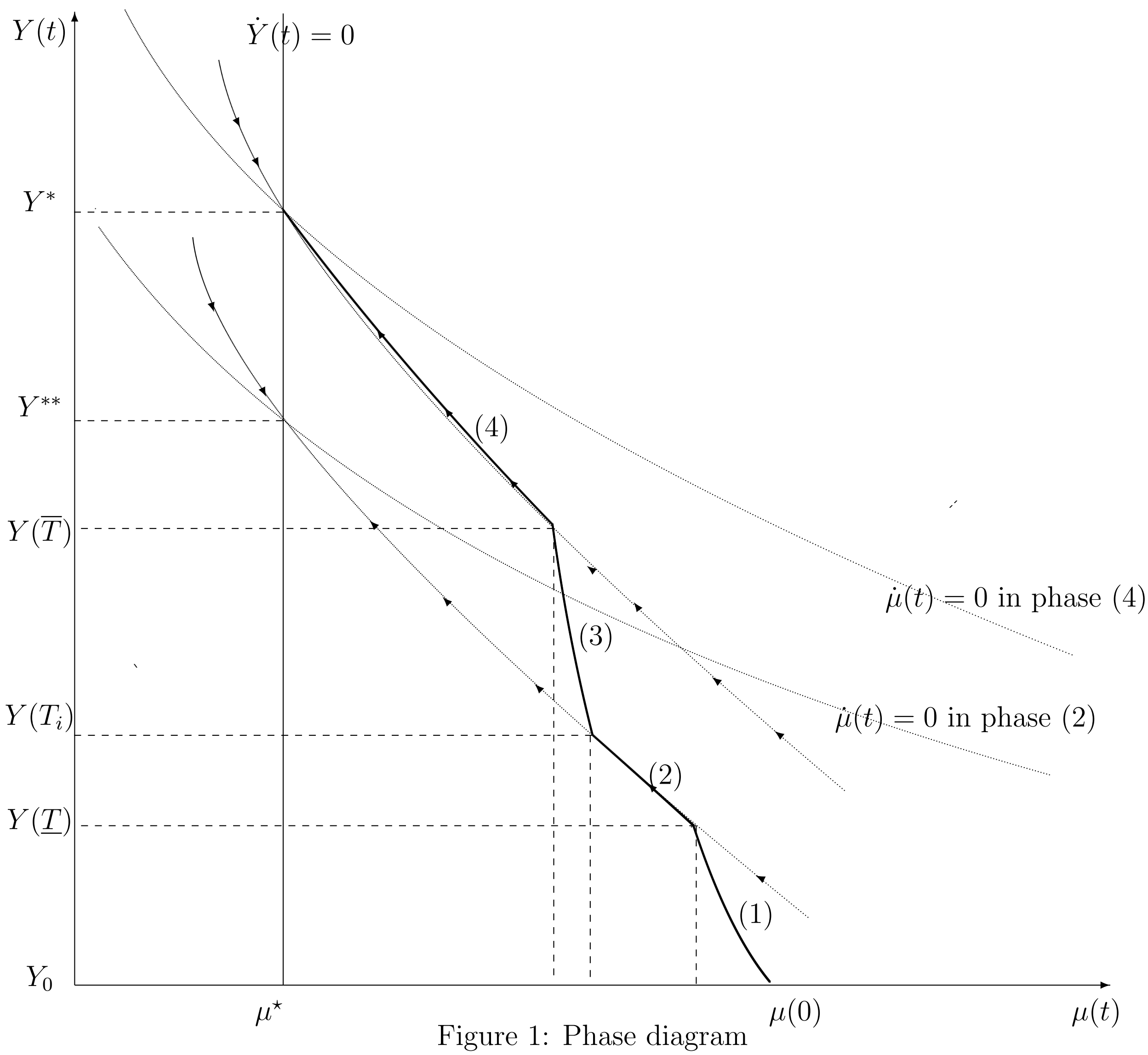


Figure 1: Phase diagram

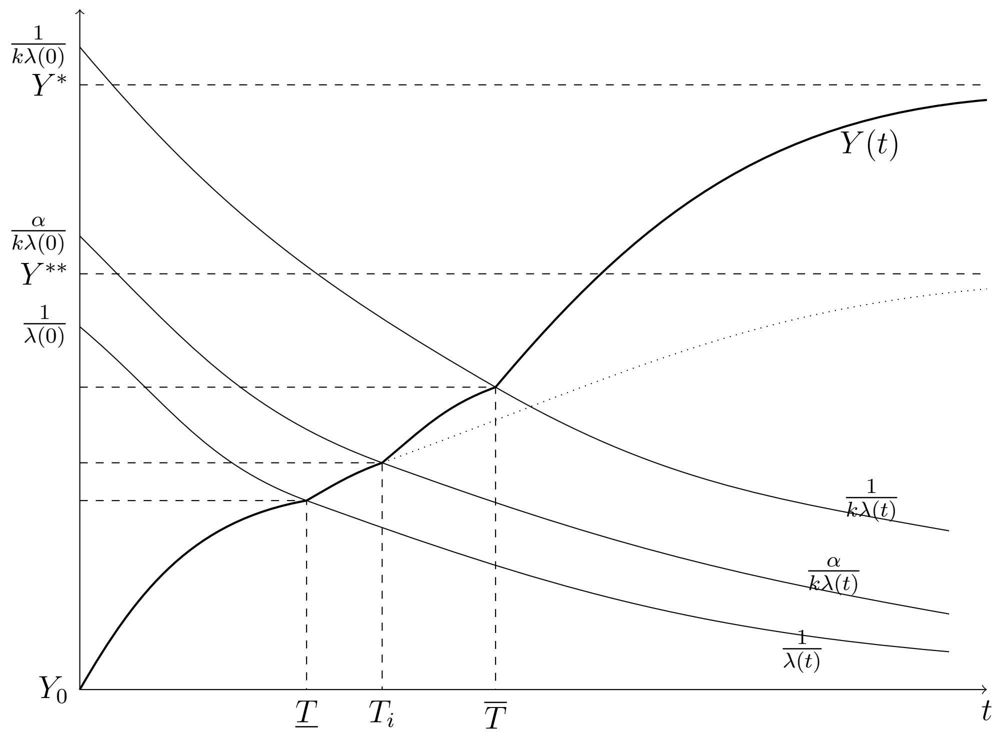


Figure 2: Solar capacity and carbon value before the ceiling

2.2 Numerical illustrations

We now perform some comparative dynamics exercises, to assess the impact of the stringency of climate policy and of the value of the parameters on the level and the time profile of electricity consumption, storage and solar capacity. The parameters of the reference simulation are given in Table 1. They are chosen for illustrative purposes only, without any pretension of realism.

ρ	k	α	$\overline{\phi}$	c_1	c_2	Y_0	X_0	$\overline{X}$
0.04	0.6	0.8	0.76	1	20	0	0	50

Table 1: Parameters in the reference simulation, variability only

In the reference simulation, day-electricity consumption is W-shaped (V-shaped if $k = 1$). It is first decreasing because of the rise of the carbon value (phase (1)); it is then increasing as fossil fuel is abandoned at day and more solar panels are installed (phase (2)); next, storage begins and increases, at the expense of day-electricity consumption, which decreases (phase (3)); finally, the increasing use of solar panels joint with a constant share of day-electricity stored generates a rise in day-electricity consumption (phase (4)). Night-electricity consumption is V-shaped. Indeed, when fossil energy is used at night,

night-electricity consumption is driven by the carbon value i.e. by climate policy, hence it is decreasing (phases (1)-(3)); then, when fossil fuel is abandoned at night, night consumption increases with the stock of solar panels and the development of storage.

The results of the comparative statics exercises are represented on Figures 3 to 6.

- *Less stringent climate policy* (Figure 3):

  In the short run energy consumption at day and night are higher than in the reference case, storage occurs later, the switch to clean energy is postponed. Investment in solar panels is lower,[13], therefore solar capacity is smaller at each date. This explain why energy consumption becomes lower than in the reference case in the medium run: there is an hysteresis effect. Even in the absence of explicit damages due to climate change, a lenient climate policy has adverse effects in the medium run because it delays investment in clean energy.

- *Less efficient storage technology* (Figure 4):

  A less efficient storage technology translates in the model in a higher loss rate $1 - k$. Then, the date at which storage begins is postponed, which allows to consume more at day in phase (2). Of course night-electricity consumption is smaller. Again an hysteresis effect appears: consumption is lower in the long run, because the development of solar panels has been slower (except at the very beginning of the planning horizon, because storage and therefore electricity loss occur later).

- *Off-peak sun* (Figure 5):

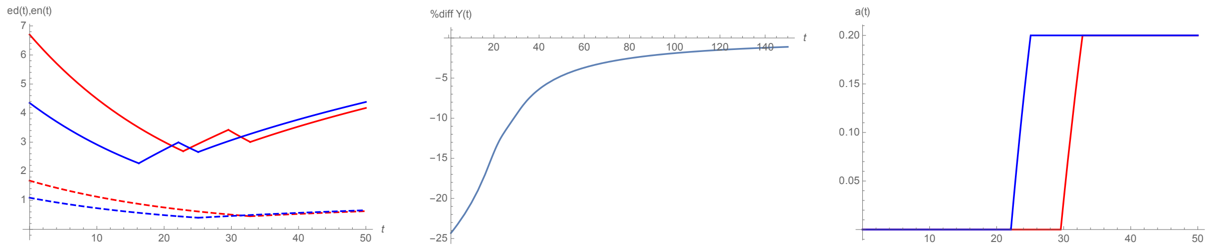
  In the reference simulation, consumers prefer to consume electricity when the sun is shining and solar panels can harness its radiation, i.e. there is sun at peak time. We make in this simulation the opposite assumption: consumers prefer to consume electricity when there is no sun (it corresponds to the Californian duck case). Clearly, the situation is now less favorable. At each date, total electricity consumption (over day and night) is reduced. The date at which fossil is not used at day anymore is brought forward so that fossil consumption at night may be higher, and storage occurs earlier. The long run level of storage has to be higher, which means more overall electricity loss. Solar panel accumulation is delayed. It is particularly salient when storage has not started (this explains why the solar capacity in % diff curve is non monotonous).

- *Less efficient solar electricity generation* (Figure 6):

  At any time, energy consumption at day and night are both lower. Investment in solar panels is lower and storage is delayed, which postpones the switch to clean energy. This

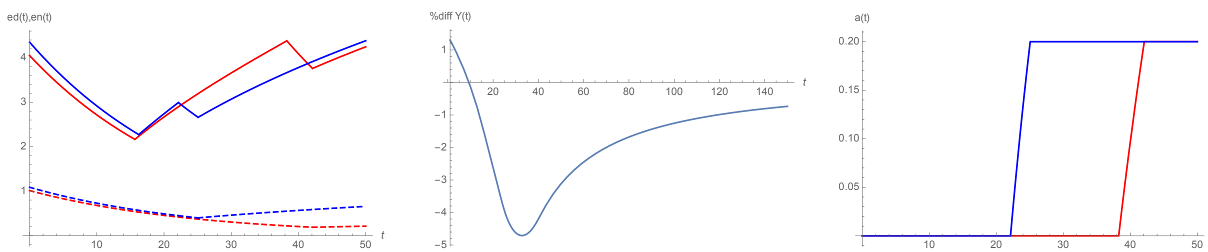
[13]See the panel "solar capacity" on Figure 3 which represents the difference in percentage between solar capacity in the simulation and in the reference case.

result stresses the importance of R&D and scale effects in solar electricity generation for the energy transition.



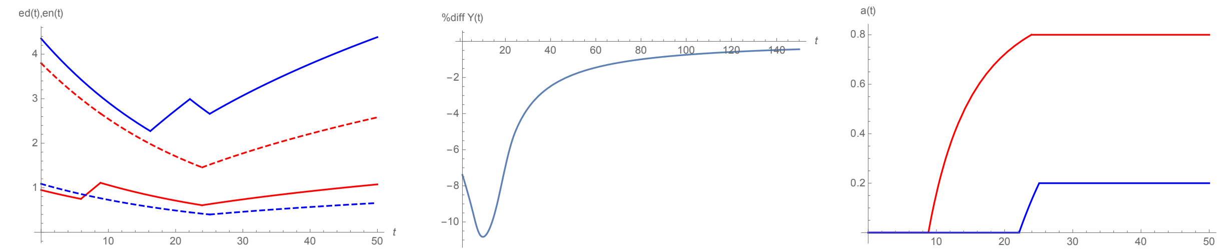
Energy consumption day (plain) and night (dashed) | Solar capacity (% diff.) | Storage

Figure 3: Effect of a less stringent climate policy under variability only ($\overline{X} = 50$ in blue and $\overline{X} = 100$ in red)



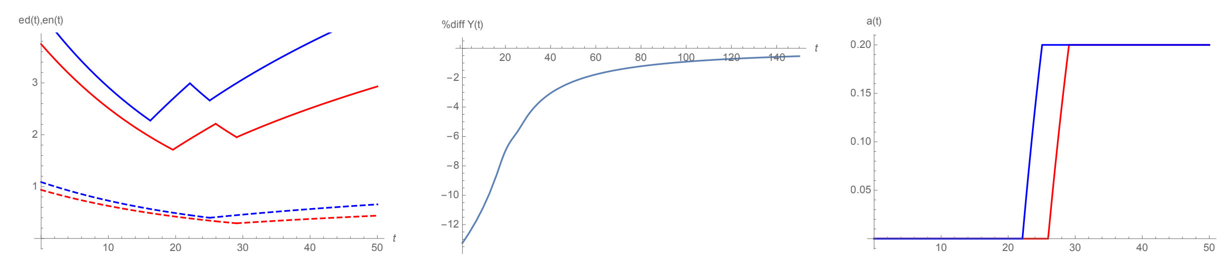
Energy consumption day (plain) and night (dashed) | Solar capacity (% diff.) | Storage

Figure 4: Effect of a less efficient storage technology under variability only ($k = 0.6$ in blue and $k = 0.2$ in red)



Energy consumption day (plain) and night (dashed) | Solar capacity (% diff.) | Storage

Figure 5: Effect of a smaller preference in day electricity under variability only ($\alpha = 0.8$ in blue and $\alpha = 0.2$ in red)



Energy consumption day (plain) and night (dashed) | Solar capacity (% diff.) | Storage

Figure 6: Effect of less efficient solar electricity generation under variability only ($\overline{\phi} = 0.76$ in blue and $\overline{\phi} = 0.52$ in red)

3 Intermittency in renewable electricity generation

We now account for the fact that renewable electricity generation is not only variable but *intermittent* as well, i.e. some of its variations are not predictable. For the purpose of illustration, we again consider the example of solar energy. Solar radiations can be fully harnessed during the day if there is sun, but can only be partially harnessed during the day if there are clouds. No harnessing can happen during the night.

3.1 The optimal solution

During the day, the weather is sunny with a probability q and solar panels are then producing electricity at full capacity Y. With a probability $(1-q)$ the weather is cloudy and solar panel only produce ϕY electricity with $0 < \phi < 1$.[14] As before, there exists a storage technology that allows to store imperfectly electricity from day to night at no monetary cost but with a physical loss. With intermittency, the sequences of storage, solar panel accumulation and electricity consumptions are decided at the beginning of the program, accounting for the fact that weather will be uncertain. e^u denotes electricity consumption when the sun is shining, while it is noted e^l when there are clouds. The social planner's programme becomes:

$$\max \int_0^{\infty} e^{-\rho t} \left[qu\left(e_d^u(t), e_n^u(t)\right) + (1-q)u\left(e_d^l(t), e_n^l(t)\right) - C(I(t))\right] dt \tag{8}$$

$$\begin{aligned}
e_d^u(t) &= x_d(t) + (1-a(t))Y(t), \quad e_d^l(t) = x_d(t) + (1-a(t))\phi Y(t) \\
e_n^u(t) &= x_n(t) + ka(t)Y(t), \quad e_n^l(t) = x_n(t) + ka(t)\phi Y(t) \\
\dot{X}(t) &= x_d(t) + x_n(t) \\
\dot{Y}(t) &= I(t) \\
0 \leq\ & a(t) \leq 1 \\
E(X(t)) &\leq \overline{X} \\
x_d(t) &\geq 0, x_n \geq 0, \\
X_0 &\geq 0, \quad Y_0 \geq 0 \text{ given}
\end{aligned}$$

The characteristics of the optimal solution are described in Proposition 2.

Proposition 2 *When the intermittency of renewable energy is taken into account and the initial solar capacity is low, there exist two different types of optimal solution:*

[14] To ensure that comparisons can be made between this model and the one with variability only, we impose in the simulations $\overline{\phi} = q + (1-q)\phi$: the efficiency of solar panel in the variability only case is equal to their expected efficiency in the case where intermittency is taken into account.

(1) For $\phi > \tilde{\phi}$ defined below, the optimal solution under intermittency and variability exhibits the same succession of phases as under variability only;

(2) For $\phi < \tilde{\phi}$, the optimal solution under intermittency and variability significantly differs from the one under variability only: storage optimally begins before fossil has been abandoned at day and solar panels accumulate more slowly.

The threshold $\tilde{\phi}$ is the real positive and smaller than 1 root of the following second degree equation:

$$\phi^2 + \frac{k\left(q^2 + (1-q)^2\right) - 1}{kq(1-q)}\phi + 1 = 0 \tag{9}$$

It is an increasing function of k, which implies that the more efficient the storage technology is the smaller is the range of ϕs for which intermittency may be safely ignored.

Proof. See Appendix B. ■

Intuition runs as follows. There are two different methods to make sure that night-electricity demand is satisfied under a climate constraint that prevents to use as much fossil as necessary. The first one is to abandon fossil at day to "save" fossil for night when solar capacity is high enough to ensure that day-electricity needs are satisfied. The second one is storage, which allows to transfer electricity from day to night, at the expense of a loss. With variability only, and with intermittency characterized by a small cloud problem (ϕ is high, corresponding to case (1) in Proposition 2), it is optimal to use the first method first, that is to begin storage after fossil has been abandoned at day, to avoid incurring the loss. With intermittency characterized by a severe cloud problem (ϕ is low, corresponding to case (2) in Proposition 2), the second method is used first, in spite of the loss. Fossil is abandoned at day later, to make sure that, in the case of no or few sun, day-electricity consumption can be satisfied. To compensate for the smaller quantity of fossil left available for night, a part of day-electricity production by solar panels is stored.

3.2 Numerical illustrations

These simulations are for illustrative purpose only. Parameters are the same as in the variability only case (see Table 1), except for $\overline{\phi}$. The discount rate is $\rho = 4\%$, and the loss rate of the storage technology is $1-k = 0.4$. Solar is shining at peak time: $\alpha = 0.8 > 1/2$. As for the new parameters characterizing intermittency, we consider that the probability that there are no clouds at daytime is $q = 0.7$, and the parameter representing the magnitude of the cloud problem ϕ is chosen to illustrate the two situations identified in the resolution (see Proposition 2):

- In Case I, where the cloud problem is moderate, we choose $\phi = 0.5 > \tilde{\phi} = 0.201$. Then $\overline{\phi} = q + (1-q)\phi = 0.85$.

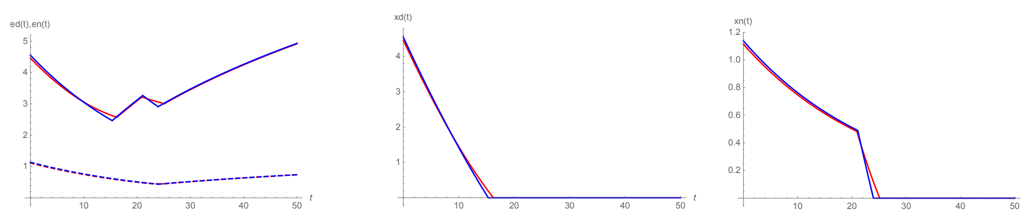
- In Case II, where the cloud problems severe, we consider the extreme case where there may be no solar generation at all during daytime $\phi = 0 < \tilde{\phi} = 0.201$. Then $\overline{\phi} = q + (1-q)\phi = 0.7$.

Results drastically differ depending on the case.

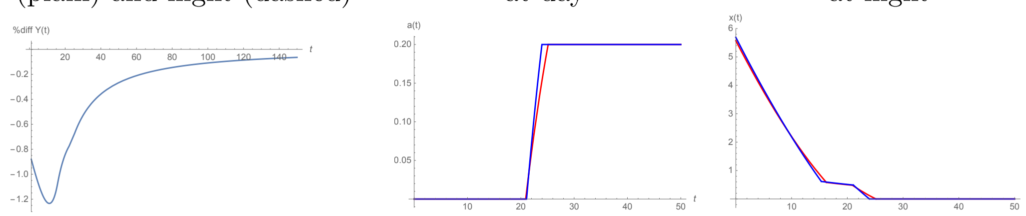
3.2.1 Case I: Moderate cloud problem

It is very striking there are only very small differences in the numerical simulations of case I and variability only (see Figure 7). For instance, day-electricity consumption follows a path very close to a W shape as exhibited under variability only. In addition, intermittency reduces welfare by only 4.5%. This is consistent with the empirical findings in Gowrisankaran and Reynolds (2016). In the latter paper, the social costs of different levels of large scale renewable energy capacity are estimated on southeastern Arizona data. Not accounting for offset CO_2, the social costs for 20% solar generation are \$138.4/MWh, of which variability and intermittency accounts for \$46 and intermittency alone for \$6.1.

Less surprisingly, the general result is that intermittency globally makes matters slightly worse (see Figure 7). Electricity consumption is always smaller with intermittency than under variability only. The full storage capacity and the no-fossil economy are reached later. Night-electricity consumption begins to increase later.



Energy consumption day (plain) and night (dashed)	Fossil fuel consumption at day	Fossil fuel consumption at night



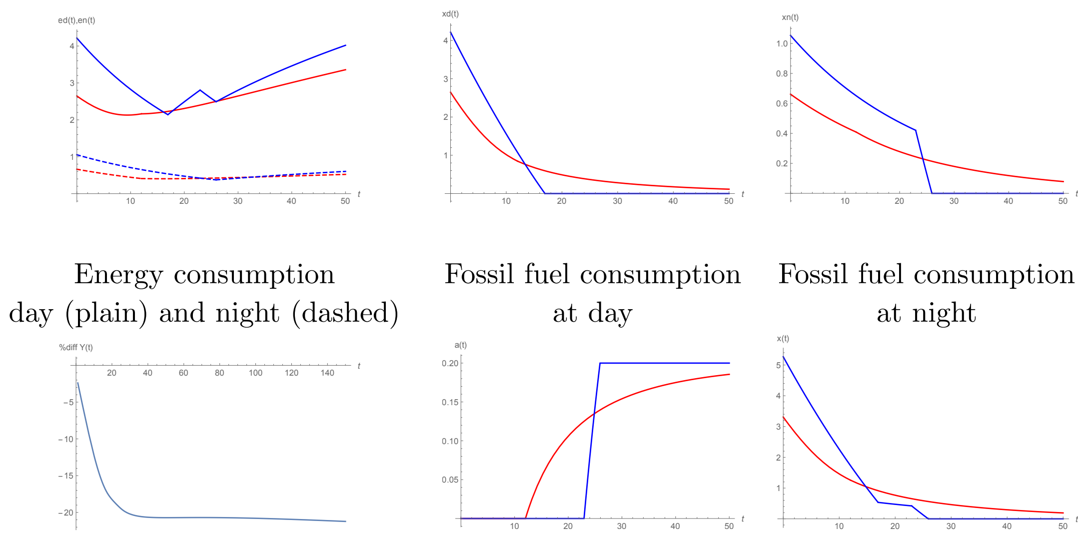
Solar capacity	Storage	Stock of carbon

Figure 7: Dynamics of variables of interest under intermittency in Case I (red) and variability only (blue)

3.2.2 Case II: Severe cloud problem

A risk of bad solar generation during day completely changes the story and the differences are significant between Case II and both case I and the case with variability only (see Appendix C and Figure 8). Energy consumption is significantly lower under case II than under variability only. There is less fossil fuel use at the beginning to save some for later times and be able to face days with no solar generation. As a result, the carbon budget is consumed more slowly. The transition to clean energy never actually occurs: fossil fuel consumption tends towards zero asymptotically but is never totally abandoned, to compensate a total absence of sun. As expected, storage starts earlier in case II than in the other cases, to deal with the smaller use of fossil at night. It converges to the steady-state level asymptotically. The solar panel capacity is always smaller in case II, in the short/medium run and at the steady state as well, where it is equal to $\tilde{Y} = q/(\rho c_1)$. Finally, intermittency reduces welfare by $92,6\%$ which 20 times more than in case I.

We can therefore deduce that intermittency only matters if the bad realization of the solar generation is bad enough (i.e. ϕ is low enough). In our numerical illustration, a bad realization corresponding to dividing by half ($\phi = 0.5$) the solar generation is not enough to trigger a significant effect of intermittency. With our calibration, intermittency starts playing a non-negligible role when $\phi \leq 0.201$.



Energy consumption day (plain) and night (dashed) | Fossil fuel consumption at day | Fossil fuel consumption at night

Solar capacity | Storage | Stock of carbon

Figure 8: Dynamics of variables of interest under intermittency in Case II (red) and variability only (blue)

Another interpretation is that intermittency is likely to be negligible compared to variability in those countries in the world where the sunshine is guaranteed. The map in Figure 9 gives an idea of the countries where intermittency should be accounted for in addition to variability. There also exist indices of cloudiness that rank cities over the world. Finally, the World Meteorological Organization provides a map for cloudiness and rain (see Figure 10) suggesting that in France intermittency should be accounted for while it does not really matter in North-Africa.

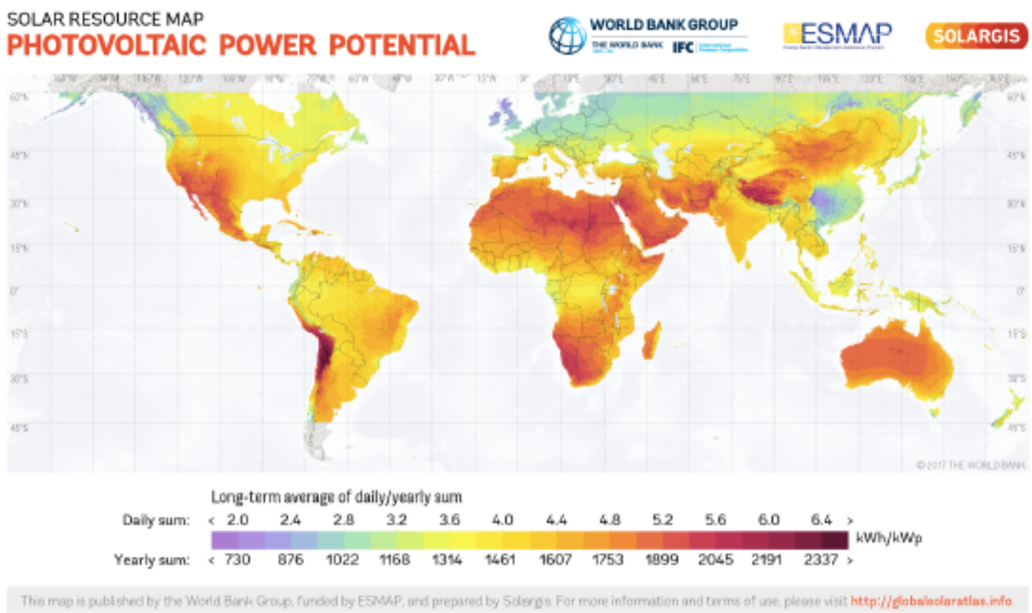


Figure 9: Photovoltaic Electricity Potential. Source: The World Bank 2017, Solar resource data: Solargis

4 The value of storage

As storage is suspected to play a major role in tackling variability and intermittency, we appraise its importance by comparing the model with variability only and storage analyzed above with a model with variability only and no storage possibilities. It will allow us to determine he value of storage, in terms of the welfare gain its availability brings.

In the case with variability only and no storage, the social planner program reads:

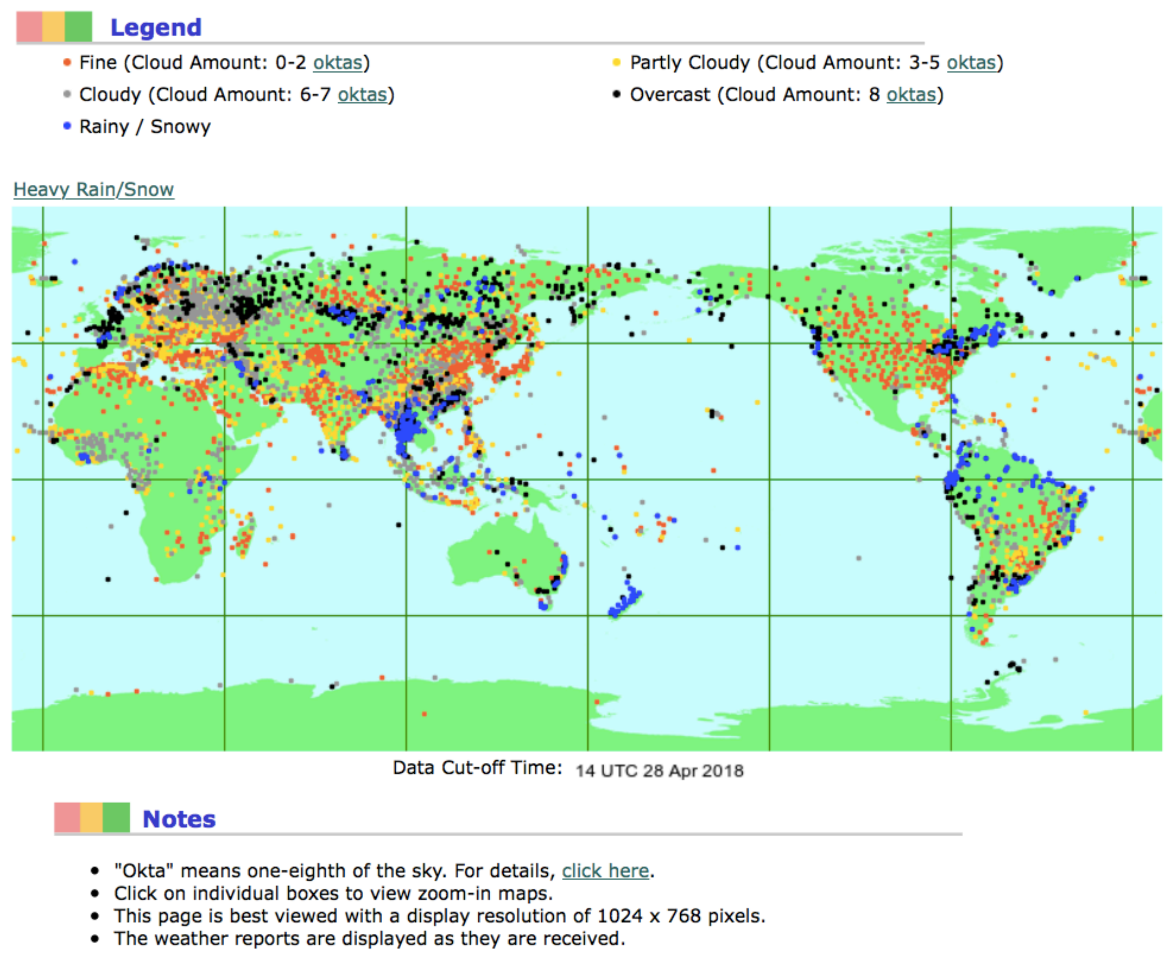


Figure 10: Global observations- Cloudiness and Rain

$$
\begin{aligned}
&\max \int_0^{\infty} e^{-\rho t}\left[u\left(e_d(t), e_n(t)\right) - C(I(t))\right] dt \\
&e_d(t) = x_d(t) + \overline{\phi} Y(t) \\
&e_n(t) = x_n(t) \\
&\dot{X}(t) = x_d(t) + x_n(t) \\
&\dot{Y}(t) = I(t) \\
&X(t) \leq \overline{X} \\
&x_d(t) \geq 0, x_n(t) \geq 0 \\
&\quad X_0 \geq 0,\ Y_0 \geq 0 \text{ given}
\end{aligned}
$$

Only two phases appear. In Phase (1), fossil fuels are used night and day, whereas they are used only during night in Phase (2). During these two phases, the carbon value λ still follows the Hotelling rule. The dynamic equation driving solar capacity accumulation over the whole horizon is the same as in the general model as well. The evolution of the value of solar capacity in each phase is:

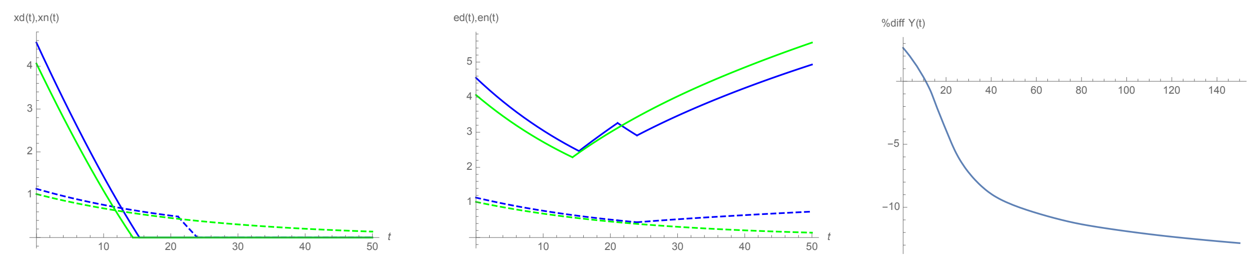
$$
\begin{aligned}
&\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - \overline{\phi}\lambda(t) \\
&\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{\alpha}{Y(t)}
\end{aligned}
$$

Fossil fuels use and total electricity consumption in each phase are:

$$
\begin{aligned}
&\text{Phase (1)} \qquad x_d(t) = \frac{\alpha}{\lambda(t)} - \overline{\phi} Y(t) \qquad && x_n(t) = \frac{1-\alpha}{\lambda(t)} \\
&\qquad\qquad\qquad e_d(t) = \frac{\alpha}{\lambda(t)} && e_n(t) = \frac{1-\alpha}{\lambda(t)} \\
&\text{Phase (2)} \qquad x_d(t) = 0 && x_n(t) = \frac{1-\alpha}{\lambda(t)} \\
&\qquad\qquad\qquad e_d(t) = \overline{\phi} Y(t) && e_n(t) = \frac{1-\alpha}{\lambda(t)}
\end{aligned}
$$

Numerical simulations are performed for $\phi = 0.5$. The dynamics for fossil fuels use and electricity consumption are shown in Figure 11. Phase (2) is a saddle path leading to a steady state $\mu^* = c_1$, $Y^{**} = \alpha/(\rho c_1)$. Along this path, daytime electricity consumption is determined by solar capacity, but night electricity consumption still uses coal as there exists no mean to transfer daytime solar generation towards the night. Therefore electricity generation is never carbon-free and night consumption (that is equal to fossil fuels use) asymptotically tends toward zero as it is driven by the climate constraint. It implies overcapacity of fossil generation during day, as in the general model (but opposite to what happens if variability is ignored). Moving backward from the steady state along the stable branch, a date is reached when there is a switch to Phase (1). In this phase, the dynamics of Y and μ are independent; electricity consumptions night and day use fossil fuels and are determined by climate policy.[15]

Comparisons between the two models are provided in Figure 11. Thanks to storage, it is possible to transfer electricity generated at day using solar capacity towards the night. Benefits from solar generation are therefore higher with storage, which explains that solar capacity at the steady state is higher with storage ($Y^* = 1/(\rho c_1)$ compared to $Y^{**} = \alpha/(\rho c_1)$ without storage). Finally, the value of storage can be appraised by computing the welfare in the two models: we obtain that introducing a storage technology increases welfare by 16.4%.



Fossil fuel use
day (plain) and night (dashed)

energy consumption
days (plain) and night (dashed)

Solar capacity
(% diff)

Figure 11: The role of storage (variability and storage in blue, variability and no storage in green)

5 Conclusion

In this paper, we build a stylized dynamic model of the optimal choice of the electric mix, where the fossil energy, coal, is abundant but CO_2-emitting, and the renewable energy, solar, is variable and intermittent but clean. We solve the centralized program under the constraint of a carbon budget that cannot be exceeded and derive an optimal succession of regimes. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time. Simulations allow us to show that a

[15] As in the previous models, the initial value of solar panels, $\mu(0)$ then provided matches the initial condition Y_0. For a given value of $\lambda(0)$ the joint evolutions of λ and Y trigger the phase switchings and the climate constraint then pins down $\lambda(0)$.

lenient climate policy delays both storage and the switch to clean energy, as does a higher probability of bad weather. Off-peak sun, rather than peak sun, increases storage and hinders consumption. We also obtain that, compared with variability only, intermittency worsens the situation although not so significantly when the cloud problem is moderate, but changes drastically the optimal solution when it is severe.

This work can be considered as a first step in the study of energy transition under variability and intermittency of the clean sources. Next steps should concern the account of an endogenous capacity for fossil fuel generation. In addition, a decentralized version of the model would be interesting and challenging as the energy market exhibits several peculiar features, and it would allow designing policy instruments. Finally, R&D investments improving storage technologies, and learning effects allowing a decrease in the cost of solar energy may also be considered.

References

Ambec S. and C. Crampes (2012), "Electricity Provision with Intermittent Sources of Energy", *Resource and Energy Economics* 34:319–336.

Ambec S. and C. Crampes (2015), "Decarbonizing Electricity Generation with Intermittent Sources of Energy", TSE Working Paper 15-603.

Baker E., M. Fowlie, D. Lemoine and S. S. Reynolds (2013), "The Economics of Solar Electricity", *Annual Review of Resource Economics*, 5:387-426.

Gowrisankaran G. and S. S. Reynolds (2016), "Intermittency and the Value of Renewable Energy", *Journal of Political Economy*, 124(4), 1187-1234.

Heal G. (2016), "Notes on the Economics of Energy Storage", NBER Working Paper 22752, October.

Hoel M., and S. Kverndokk (1996), "Depletion of Fossil Fuels and the Impacts of Global Warming", *Resource and Energy Economics*, 18: 115-136.

International Energy Agency (2011), Solar Energy Perspectives.

Joskow, P. (2011), "Comparing the costs of intermittent and dispatchable generating technologies ", *American Economic Review*, 101(3), 238-241.

Lafforgue G., B. Magne and M. Moreaux (2008), "Energy substitutions, climate change and carbon sinks", *Ecological Economics*, 67: 589-597.

Lazkano I., L. Nostbakken and M. Pelli (2016), "From Fossil Fuels to Renewables: The Role of Electricity Storage", Working Paper.

Sinn H. W. (2016), “Buffering volatility: A study on the limits of Germany’s energy revolution”, CESifo Working Papers n 5950, June.

Tahvonen O. (1997), “Fossil fuels, stock externalities, and backstop technology”, *Canadian Journal of Economics*, 30: 855-874.

Yang C.-J. (2016), “Pumped hydroelectric storage”, in *Storing Energy*, T.M. Letcher ed., Elsevier.

A Variability only

The current value Hamiltonian associated to the social planner's programme (1) reads, dropping the time index:

$$\mathcal{H} = u\left(x_d + (1-a)\overline{\phi}Y, x_n + ka\overline{\phi}Y\right) - C(I) - \lambda\left(x_d + x_n\right) + \mu I$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \underline{\omega}_a a + \overline{\omega}_a(1-a) + \omega_d x_d + \omega_n x_n + \omega_X\left(\overline{X} - X\right)$$

The first order conditions are:

$$u_1 = \lambda - \omega_d \tag{10}$$

$$u_2 = \lambda - \omega_n \tag{11}$$

$$Y\left(u_1 - ku_2\right) = \underline{\omega}_a - \overline{\omega}_a \tag{12}$$

$$C'(I) = \mu \tag{13}$$

$$-\omega_X = \dot{\lambda} - \rho\lambda \tag{14}$$

$$-(1-a)\overline{\phi}u_1 - ka\overline{\phi}u_2 = \dot{\mu} - \rho\mu \tag{15}$$

and the complementarity slackness conditions read:

$$\begin{aligned}
\underline{\omega}_a a = 0, \underline{\omega}_a \geq 0, a \geq 0 \\
\overline{\omega}_a(1-a) = 0, \overline{\omega}_a \geq 0, 1-a \geq 0 \\
\omega_d x_d = 0, \omega_d \geq 0, x_d \geq 0 \\
\omega_n x_n = 0, \omega_n \geq 0, x_n \geq 0 \\
\omega_X\left(\overline{X} - X\right) = 0, \omega_X \geq 0, \overline{X} - X \geq 0
\end{aligned}$$

Before the ceiling, $X < \overline{X}$ and $\omega_X = 0$. Then FOC (14) reads $\dot{\lambda}/\lambda = \rho$, i.e.:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{16}$$

The shadow price of carbon concentration (the carbon value) follows a Hotelling rule before the ceiling, as long as fossil fuel is used.

Moreover, Eq. (13) reads: $c_1 + c_2 I = \mu$, which, together with $\dot{Y} = I$, yields:

$$\dot{Y} = \frac{1}{c_2}\left(\mu - c_1\right) \tag{17}$$

A.1 Fossil night and day

This phase is necessarily the first one, if it exists. We denote by $\underline{T}$ the date at which it ends. We have $x_d > 0$, $x_n > 0$, $\omega_d = 0$, $\omega_n = 0$. The FOC read:

$$\frac{\alpha}{x_d + (1-a)\overline{\phi}Y} = \lambda \tag{18}$$

$$\frac{1-\alpha}{x_n + ka\overline{\phi}Y} = \lambda \tag{19}$$

$$(1-k)\lambda\overline{\phi}Y = \underline{\omega}_a - \overline{\omega}_a \tag{20}$$

$$-\left((1-a) + ka\right)\overline{\phi}\lambda = \dot{\mu} - \rho\mu \tag{21}$$

to which we add Eqs. (16) and (17).

The left-hand side member of (20) is necessarily positive. Hence the case $\underline{\omega}_a = 0$ and $\overline{\omega}_a > 0$, i.e. $a = 1$ (full storage) is excluded. The interior case $\underline{\omega}_a = 0$ and $\overline{\omega}_a = 0$, i.e. $0 < a < 1$, is possible iff $Y = 0$, which means that no intermittent source of energy is used. But it does not make sense to have a positive capacity of storage absent any intermittent source of energy. This case is also excluded. The only possibility is thus $\underline{\omega}_a > 0$ and $\overline{\omega}_a = 0$, i.e. $a = 0$: no storage.

With no storage, Eq. (21) simplifies into:

$$\dot{\mu} - \rho\mu = -\overline{\phi}\lambda \tag{22}$$

Using Eq. (16), this equation can be integrated, to obtain $\mu(t)$ as a function of $\mu(0)$, $\lambda(0)$ and time. It allows us to obtain $Y(t)$ as a function of the same variables by integration of Eq. (17). We suppose that Y_0 is low enough, s.t. $I(0) > 0$, which requires $\mu(0) > c_1$.

Eqs. (18) and (19) with $a = 0$ allow to compute x_d and x_n:

$$x_d = \frac{\alpha}{\lambda} - \overline{\phi}Y \tag{23}$$

$$x_n = \frac{1-\alpha}{\lambda} \tag{24}$$

At the end of this phase, $x_d(\underline{T}) = 0$ ($x_n(\underline{T}) = 0$ is impossible, since it would require $\lambda(\underline{T}) = +\infty$). Hence:

$$\lambda(\underline{T}) = \frac{\alpha}{Y(\underline{T})} \tag{25}$$

Day and night electricity consumption are given by:

$$e_d = \frac{\alpha}{\lambda}, \qquad e_n = \frac{1-\alpha}{\lambda} \tag{26}$$

A.2 Fossil at night only

This or these phases are intermediate. We have here $x_d = 0$, $x_n > 0$, $\omega_d > 0$, $\omega_n = 0$ and the FOC read:

$$\frac{\alpha}{(1-a)\overline{\phi}Y} = \lambda - \omega_d \tag{27}$$

$$\frac{1-\alpha}{x_n + ka\overline{\phi}Y} = \lambda \tag{28}$$

$$\frac{\alpha}{1-a} - k\lambda\overline{\phi}Y = \underline{\omega}_a - \overline{\omega}_a \tag{29}$$

$$-\frac{\alpha}{Y} - ka\overline{\phi}\lambda = \dot{\mu} - \rho\mu \tag{30}$$

to which we add Eqs. (16) and (17).

A.2.1 No storage

In the case of no storage, the FOC become:

$$\frac{\alpha}{\overline{\phi}Y} = \lambda - \omega_d \tag{31}$$

$$\frac{1-\alpha}{x_n} = \lambda \tag{32}$$

$$\alpha - k\lambda\overline{\phi}Y = \underline{\omega}_a \tag{33}$$

$$-\frac{\alpha}{Y} = \dot{\mu} - \rho\mu \tag{34}$$

Eqs. (17) and (34) yield a saddle-path dynamic system in (μ, Y). The steady state values of μ and Y are:

$$\mu^* = c_1 \tag{35}$$

$$Y^{**} = \frac{\alpha}{\rho c_1} \tag{36}$$

According to (31) and (33), we must have $\frac{\alpha}{\overline{\phi}Y} \leq \lambda \leq \frac{\alpha}{k\overline{\phi}Y}$. Hence this phase begins at $\underline{T}$ (see Eq. (25)), and it ends at T_i defined by:

$$\lambda(T_i) = \frac{\alpha}{k\overline{\phi}Y(T_i)} \tag{37}$$

Moreover, according to (32), we have:

$$x_n(\underline{T}) = \frac{1-\alpha}{\alpha}\overline{\phi}Y(\underline{T}), \qquad x_n(T_i) = \frac{1-\alpha}{\alpha}k\overline{\phi}Y(T_i)$$

Day and night electricity consumption are given by:

$$e_d = \overline{\phi}Y, \qquad e_n = \frac{1-\alpha}{\lambda} \tag{38}$$

A.2.2 Interior storage

In the case of an interior solution on a, Eq. (29) yields:

$$a = 1 - \frac{\alpha}{k\lambda\overline{\phi}Y} \tag{39}$$

Eq. (30) reads:

$$-k\overline{\phi}\lambda = \dot{\mu} - \rho\mu \tag{40}$$

This equation can be integrated to obtain $\mu(t)$ as a function of $\mu(T_i)$, T_i, $\lambda(0)$ and time, which allows us to obtain $I(t)$ and, by integration, $Y(t)$ as a function of the same variables and $Y(T_i)$.

From Eq. (39), $a \geq 0$ requires $\frac{\alpha}{k\overline{\phi}Y} \leq \lambda$, which shows that this phase begins at T_i (see Eq. (37)). The date at which this phase ends is given by the fact that fossil fuel consumption at night becomes nil. Then Eq. (28) shows that $a = 1-\alpha$ at the end of this phase, and Eq. (29) that the date $\overline{T}$ of the end of this phase is defined by:

$$\lambda(\overline{T}) = \frac{1}{k\overline{\phi}Y(\overline{T})} \tag{41}$$

Day and night electricity consumption are given by:

$$e_d = \frac{\alpha}{k\lambda}, \qquad e_n = \frac{1-\alpha}{\lambda} \tag{42}$$

A.2.3 Full storage

Impossible.

A.3 No fossil

This phase is necessarily the last one, and it always exists. It begins at the date at which the ceiling is reached.

We have $x_d = 0$, $x_n = 0$, $\omega_d > 0$, $\omega_n > 0$ $\forall t \geq \overline{T}$.

The FOC then read:

$$\frac{\alpha}{(1-a)\overline{\phi}Y} = \lambda - \omega_d \tag{43}$$

$$\frac{1-\alpha}{ka\overline{\phi}Y} = \lambda - \omega_n \tag{44}$$

$$\frac{a-(1-\alpha)}{a(1-a)} = \underline{\omega}_a - \overline{\omega}_a \tag{45}$$

$$\dot{\mu} - \rho\mu = -\frac{1}{Y} \tag{46}$$

to which we have to add Eqs. (14) and (17).

The no storage ($a = 0$) or complete storage ($a = 1$) cases cannot occur because the marginal utility of consumption at night or day would become infinite. Hence the solution is an interior solution on a, with $\underline{\omega}_a = \overline{\omega}_a = 0$. Eq. (45) yields:

$$a^* = 1 - \alpha \tag{47}$$

There is a constant rate of storage all along this phase, depending on the weight of night-electricity in utility.

Eqs. (17) and (46) yield a saddle-path dynamic system in (μ, Y). The values of μ and Y at the steady state are:

$$\mu^* = c_1 \tag{48}$$

$$Y^* = \frac{1}{\rho c_1} \tag{49}$$

What determines the long run stock of solar panels is the discount rate and the marginal investment cost.

Finally, Eqs. (43) and (44) imply $\frac{1}{\overline{\phi}Y} \leq \lambda$ and $\frac{1}{k\overline{\phi}Y} \leq \lambda$. The second condition is more stringent than the first one. It shows that this phase begins at date $\overline{T}$ (see Eq. (41)).

Day and night electricity consumption are given by:

$$e_d = \alpha\overline{\phi}Y, \qquad e_n = (1-\alpha)k\overline{\phi}Y \tag{50}$$

A.4 Respect of the carbon budget

The model is completed by specifying that the total quantity of fossil fuel burned cannot exceed the carbon budget:

$$\overline{X} = \int_0^{\underline{T}} (x_n(t) + x_d(t))dt + \int_{\underline{T}}^{T_i} x_n(t)dt + \int_{T_i}^{\overline{T}} x_n(t)dt \tag{51}$$

B Variability and intermittency

The current value Hamiltonian associated to the social planner's programme reads:

$$\mathcal{H} = qu\left(x_d + (1-a)Y, x_n + kaY\right) + (1-q)u\left(x_d + (1-a)\phi Y, x_n + ka\phi Y\right) - C(I) - \lambda\left(x_d + x_n\right) + \mu I$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \underline{\omega}_a a + \overline{\omega}_a(1-a) + \omega_d x_d + \omega_n x_n + \omega_X\left(\overline{X} - X\right)$$

The first order conditions are:

$$q\frac{\alpha}{x_d + (1-a)Y} + (1-q)\frac{\alpha}{x_d + (1-a)\phi Y} = \lambda - \omega_d \tag{52}$$

$$q\frac{1-\alpha}{x_n + kaY} + (1-q)\frac{1-\alpha}{x_n + ka\phi Y} = \lambda - \omega_n \tag{53}$$

$$qY\left[\frac{\alpha}{x_d + (1-a)Y} - k\frac{1-\alpha}{x_n + kaY}\right] + (1-q)\phi Y\left[\frac{\alpha}{x_d + (1-a)\phi Y} - k\frac{1-\alpha}{x_n + ka\phi Y}\right] = \underline{\omega}_a - \overline{\omega}_a \tag{54}$$

$$C'(I) = \mu \tag{55}$$

$$-\omega_X = \dot{\lambda} - \rho\lambda \tag{56}$$

$$-q\left[(1-a)\frac{\alpha}{x_d + (1-a)Y} + ka\frac{1-\alpha}{x_n + kaY}\right] - (1-q)\phi\left[(1-a)\frac{\alpha}{x_d + (1-a)\phi Y} + ka\frac{1-\alpha}{x_n + ka\phi Y}\right] = \dot{\mu} - \rho \tag{57}$$

The complementarity slackness conditions read:

$$\underline{\omega}_a a = 0, \underline{\omega}_a \geq 0, a \geq 0$$
$$\overline{\omega}_a (1-a) = 0, \overline{\omega}_a \geq 0, 1-a \geq 0$$
$$\omega_d x_d = 0, \omega_d \geq 0, x_d \geq 0$$
$$\omega_n x_n = 0, \omega_n \geq 0, x_n \geq 0$$
$$\omega_X \left(\overline{X} - X\right) = 0, \omega_X \geq 0, \overline{X} - X \geq 0$$

Before the ceiling, $X < \overline{X}$ and $\omega_X = 0$. Then FOC (56) reads $\dot{\lambda}/\lambda = \rho$, i.e.:

$$\lambda(t) = \lambda(0) e^{\rho t} \tag{58}$$

Moreover, Eq. (55) reads: $c_1 + c_2 I = \mu$, which, together with $\dot{Y} = I$, yields:

$$\dot{Y} = \frac{1}{c_2} (\mu - c_1) \tag{59}$$

B.1 Fossil night and day

We have $x_d > 0$, $x_n > 0$, $\omega_d = 0$, $\omega_n = 0$. The FOC read:

$$q \frac{\alpha}{x_d + (1-a)Y} + (1-q) \frac{\alpha}{x_d + (1-a)\phi Y} = \lambda \tag{60}$$
$$q \frac{1-\alpha}{x_n + kaY} + (1-q) \frac{1-\alpha}{x_n + ka\phi Y} = \lambda \tag{61}$$

to which we add Eqs. (54), (57), (58) and (59).

B.1.1 No storage

$\underline{\omega}_a > 0$, $\overline{\omega}_a = 0$, $a = 0$.

Eqs. (60) and (61) read:

$$q \frac{\alpha}{x_d + Y} + (1-q) \frac{\alpha}{x_d + \phi Y} = \lambda \tag{62}$$
$$\frac{1-\alpha}{x_n} = \lambda \tag{63}$$

Eq. (62) gives implicitly x_d as a function of Y and λ. It is a quadratic equation which positive root is:

$$x_d = \frac{Y}{2}\left[\left(\frac{\alpha}{\lambda Y} - (1+\phi)\right) + \left(\left(\frac{\alpha}{\lambda Y} - (1+\phi)\right)^2 + 4\phi\left(\frac{\alpha}{\underline{\phi}\lambda Y} - 1\right)\right)^{\frac{1}{2}}\right]. \tag{64}$$

with $\underline{\phi} = \frac{\phi}{\phi q + (1-q)}$.

Eq. (57) simplifies into:

$$\dot{\mu} - \rho\mu = -q\frac{\alpha}{x_d + Y} - (1-q)\phi\frac{\alpha}{x_d + \phi Y}$$

i.e., using (62):

$$\dot{\mu} - \rho\mu = -\phi\lambda - (1-\phi)q\frac{\alpha}{x_d + Y} \tag{65}$$

Eq. (54) reads, using (62):

$$\left(\overline{\phi}k - \phi\right)\lambda < (1-\phi)q\frac{\alpha}{x_d + Y} \tag{66}$$

with $\overline{\phi} = q = (1-q)\phi$.

With our assumption that initial solar capacity is nil, this phase is necessarily the first one. It starts at date 0.

At the end of this phase, either $x_d = 0$ ($x_n = 0$ is impossible, since it would require $\lambda \to +\infty$) or Eq. (66) is satisfied as an equality, meaning that the next phase will be a phase with positive storage. Therefore the end of this phase is $\underline{T}$ or $\underline{T}_a$ respectively defined by:

$$\lambda(\underline{T}) = \frac{\alpha}{\underline{\phi}Y(\underline{T})} \text{ and } x_d(\underline{T}) = 0 \tag{67}$$

$$\lambda(\underline{T}_a) = \frac{(1-\phi)q}{\overline{\phi}k - \phi}\frac{\alpha}{x_d(\underline{T}_a) + Y(\underline{T}_a)} \tag{68}$$

A necessary condition for the existence of $\underline{T}_a$ is:

$$k\overline{\phi} - \phi > 0 \Longleftrightarrow \phi < \frac{1}{1 + \frac{1-k}{qk}} = \tilde{\phi}_0 \tag{69}$$

meaning that when ϕ is small enough, reflecting a serious cloud problem, storage must begin before fossil is abandoned at day. See Figure 12.

If this condition is not satisfied the relevant date of end of this phase is $\underline{T}$. It is $\underline{T}$ as well if this condition is satisfied but $\underline{T} < \underline{T}_a$.

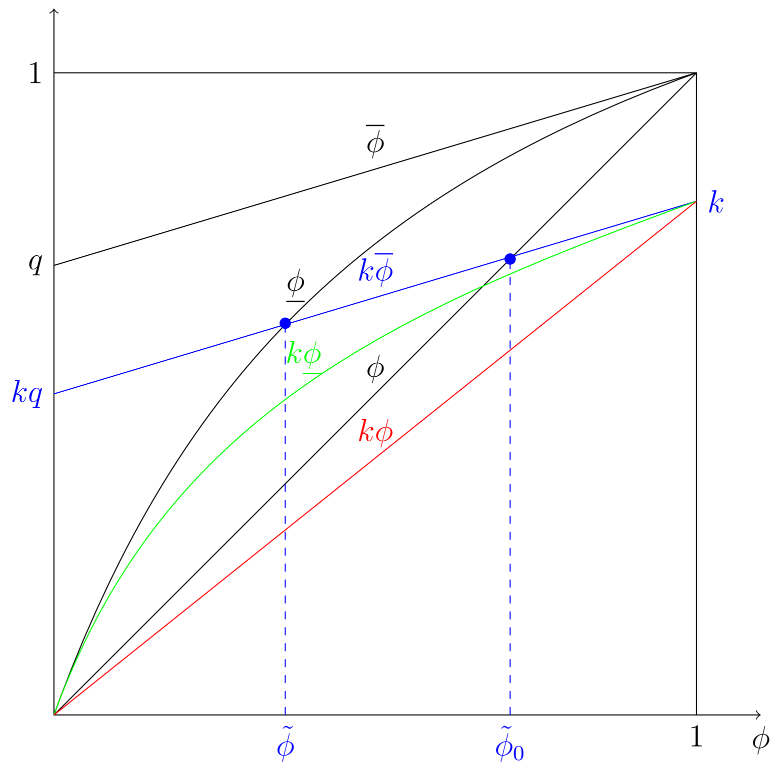


Figure 12: Boundaries of the two possible solutions in the fossil night and day and no storage case

B.1.2 Interior storage

This phase starts at $\underline{T}_a$. We have $\omega_d = \omega_n = \underline{\omega}_a = \overline{\omega}_a = 0$, $x_d > 0$, $x_n > 0$ and $0 < a < 1$. The FOC read:

$$q\frac{\alpha}{x_d + (1-a)Y} + (1-q)\frac{\alpha}{x_d + (1-a)\phi Y} = \lambda \tag{70}$$

$$q\frac{1-\alpha}{x_n + kaY} + (1-q)\frac{1-\alpha}{x_n + ka\phi Y} = \lambda \tag{71}$$

$$qY\left[\frac{\alpha}{x_d + (1-a)Y} - k\frac{1-\alpha}{x_n + kaY}\right] + (1-q)\phi Y\left[\frac{\alpha}{x_d + (1-a)\phi Y} - k\frac{1-\alpha}{x_n + ka\phi Y}\right] = 0 \tag{72}$$

$$-q\left[(1-a)\frac{\alpha}{x_d + (1-a)Y} + ka\frac{1-\alpha}{x_n + kaY}\right] - (1-q)\phi\left[(1-a)\frac{\alpha}{x_d + (1-a)\phi Y} + ka\frac{1-\alpha}{x_n + ka\phi Y}\right] = \dot{\mu} - \rho \tag{73}$$

The three first equations allow to compute x_d, x_n and a as functions of Y and λ. Notice that algebraic expressions of these variables are impossible to obtain.

This phase starts with no storage, at date $\underline{T}_a$ defined in Eq. (68). Necessary condition for existence: $k\overline{\phi} - \phi > 0$ i.e. $\phi < \tilde{\phi}_0$. a is increasing along this phase.

At the end of this phase (date $\underline{T}_b$) fossil consumption at day stops. From Eq. (70), date $\underline{T}_b$ is characterized by:

$$\lambda(\underline{T}_b) = \frac{\alpha}{(1 - a(\underline{T}_b))\underline{\phi} Y(\underline{T}_b)} \tag{74}$$

Then Eqs. (72) and (71) yield:

$$\lambda(\underline{T}_b) = \frac{qk(1-\phi)}{\overline{\phi} - k\phi}\frac{1-\alpha}{x_n(\underline{T}_b) + ka(\underline{T}_b)Y(\underline{T}_b)} \tag{75}$$

Again, we only have implicit expressions for $\lambda(\underline{T}_b)$.

A necessary condition for the existence of $\underline{T}_b$ is $\overline{\phi} - k\phi > 0$, which is always true (see Figure 12).

B.1.3 Full storage

This case with full storage cannot appear under our assumption that initial solar capacity is nil (note that it is true as well in the variability only case). Indeed, intuitively, full storage is likely to occur if on the one hand the preference for night-electricity is higher that the preference for day electricity that is if agents prefer consuming electricity off-peak, and if on the other hand initial solar capacity is large, so that solar electricity is produced at day in excess of the consumption needs. But it will never be optimal to build solar capacity to obtain this kind of solution at another time than the initial one, because of the loss of electricity attached to the storage technology.

Relying on these arguments, we exclude in the following the case of full storage.

B.2 Fossil at night only

This or these phases are intermediate. We have here $x_d = 0$, $x_n > 0$, $\omega_d > 0$, $\omega_n = 0$ and the FOC read:

$$\frac{\alpha}{(1-a)\underline{\phi}Y} = \lambda - \omega_d \tag{76}$$

$$q\frac{1-\alpha}{x_n + kaY} + (1-q)\frac{1-\alpha}{x_n + ka\phi Y} = \lambda \tag{77}$$

$$\frac{\alpha}{1-a} - kY\left[q\frac{1-\alpha}{x_n + kaY} + (1-q)\phi\frac{1-\alpha}{x_n + ka\phi Y}\right] = \underline{\omega}_a - \overline{\omega}_a \tag{78}$$

$$-\frac{\alpha}{Y} - ka\left[q\frac{1-\alpha}{x_n + kaY} + (1-q)\phi\frac{1-\alpha}{x_n + ka\phi Y}\right] = \dot{\mu} - \rho\mu \tag{79}$$

to which we add Eqs. (58) and (59).

B.2.1 No storage

In the case of no storage, the FOC become:

$$\frac{\alpha}{\underline{\phi}Y} = \lambda - \omega_d \tag{80}$$

$$\frac{1-\alpha}{x_n} = \lambda \tag{81}$$

$$\alpha - k\overline{\phi}Y\lambda = \underline{\omega}_a \tag{82}$$

$$-\frac{\alpha}{Y} = \dot{\mu} - \rho\mu \tag{83}$$

Eqs. (59) and (83) yield a saddle-path dynamic system in (μ, Y). The steady state values of μ and Y are:

$$\mu^* = c_1 \tag{84}$$

$$Y^{**} = \frac{\alpha}{\rho c_1} \tag{85}$$

According to (80) and (82), we must have $\frac{\alpha}{\underline{\phi}Y} \leq \lambda \leq \frac{\alpha}{k\bar{\phi}Y}$. Hence this phase begins at $\underline{T}$ (see Eq. (67)), and it ends at T_i defined by:

$$\lambda(T_i) = \frac{\alpha}{k\bar{\phi}Y(T_i)} \tag{86}$$

The existence of this phase requires that:

$$k\bar{\phi} < \underline{\phi} \Longleftrightarrow \phi > \tilde{\phi} \tag{87}$$

See Figure 12. ϕ must be high enough, meaning that the cloud problem is not too severe. If this condition is not satisfied this phase does not exist, implying that the phase with fossil night and day and no storage necessarily ends at date $\underline{T}_a$. This solves the problem of the boundaries of validity of $\underline{T}$ and $\underline{T}_a$.

Day and night electricity consumption are given by:

$$e_d^u = Y, \qquad e_d^l = \phi Y, \qquad e_n^u = e_n^l = x_n = \frac{1-\alpha}{\lambda} \tag{88}$$

B.2.2 Interior storage

In the case of an interior solution on a, Eq. (78) yields:

$$\frac{\alpha}{1-a} = kY\left[q\frac{1-\alpha}{x_n + kaY} + (1-q)\phi\frac{1-\alpha}{x_n + ka\phi Y}\right] \tag{89}$$

Eqs. (77) and (89) implicitly give a and x_n as functions of Y and λ.

Eq. (79) reads:

$$-\frac{\alpha}{(1-a)Y} = \dot{\mu} - \rho\mu \tag{90}$$

At the beginning of this phase there are two possibilities: either fossil fuel night only and $a = 0$ or fossil fuel night and day and $a > 0$.

In case this phase starts with $a = 0$, from Eq. (89), $a \geq 0$ requires $\frac{\alpha}{k\bar{\phi}Y} \leq \lambda$, which shows that this phase begins at T_i (see Eq (86)).

In case this phase starts with $a > 0$ when we stop using fossil during day, Eq. (76) requires that $\frac{\alpha}{(1-a)\underline{\phi}Y} \leq \lambda$ which shows that this phase begins at $\underline{T}_b$ (see Eq. (74)).

The date at which this phase ends is given by the fact that fossil fuel consumption at night becomes nil. Then Eq. (89) shows that $a = 1 - \alpha$ at the end of this phase, and Eq. (77)

that $\lambda = \frac{1}{k\underline{\phi}Y}$, showing that this date is $\overline{T}$ given by:

$$\lambda(\overline{T}) = \frac{1}{k\underline{\phi}Y(\overline{T})} \tag{91}$$

Day and night electricity consumption are given by:

$$e_d = \frac{\alpha}{k\lambda}, \qquad e_n = \frac{1-\alpha}{\lambda} \tag{92}$$

B.3 Fossil at day only

Quite intuitively, this case could occur if initial solar capacity was very large, so that it is optimal to start with full storage at the beginning of the planning horizon, and make storage decrease in time. It cannot occur under our assumption of nil initial solar capacity.

B.4 No fossil

This phase is necessarily the last one, and it always exists. More precisely, it exists as such as soon as $\phi > 0$. For $\phi = 0$ fossil consumption asymptotically decreases towards zero.

We have $x_d = 0$, $x_n = 0$, $\omega_d > 0$, $\omega_n > 0$ $\forall t \geq \overline{T}$.

The FOC then read:

$$\frac{\alpha}{(1-a)\underline{\phi}Y} = \lambda - \omega_d \tag{93}$$

$$\frac{1-\alpha}{ka\underline{\phi}Y} = \lambda - \omega_n \tag{94}$$

$$\frac{a-(1-\alpha)}{a(1-a)} = \underline{\omega}_a - \overline{\omega}_a \tag{95}$$

$$\dot{\mu} - \rho\mu = -\frac{1}{Y} \tag{96}$$

to which we have to add Eqs. (58) and (59).

The no storage ($a = 0$) or complete storage ($a = 1$) cases cannot occur because the marginal utility of consumption at night or day would become infinite. Hence the solution is an interior solution on a, with $\underline{\omega}_a = \overline{\omega}_a = 0$. Eq. (95) yields:

$$a^* = 1 - \alpha \tag{97}$$

There is a constant rate of storage all along this phase, depending on the weight of night-electricity in utility.

Eqs. (59) and (96) yield a saddle-path dynamic system in (μ, Y). The values of μ and Y at the steady state are:

$$\mu^* = c_1 \tag{98}$$

$$Y^* = \frac{1}{\rho c_1} \tag{99}$$

Finally, Eqs (93) and (94) imply $\frac{1}{\underline{\phi} Y} \leq \lambda$ and $\frac{1}{k\underline{\phi} Y} \leq \lambda$. The second condition is more stringent than the first one. It shows that this phase begins at date $\overline{T}$, which is also the date at which the ceiling is reached.

Day and night electricity consumption are given by:

$$e_d^u = \alpha Y, \qquad e_d^l = \alpha \phi Y, \qquad e_n^u = (1-\alpha)kY, \qquad e_n^l = (1-\alpha)k\phi Y \tag{100}$$

B.5 Respect of the carbon budget

The model is completed by specifying that the total quantity of fossil fuel burned cannot exceed the carbon budget:

$$\overline{X} = \int_0^{\underline{T}} (x_n(t) + x_d(t))dt + \int_{\underline{T}}^{T_i} x_n(t)dt + \int_{T_i}^{\overline{T}} x_n(t)dt \tag{101}$$

C A polar case: intermittency with no sun at all in the bad case

Two very different optimal solutions are possible.

If ϕ is high enough, the optimal solution when intermittency is taken into account is very close to the one we get when it is not. The succession of phases is the same. Only the switching dates differ. At the limit, when $\phi \to 1$, we obtain exactly the variability-only solution.

If ϕ is small we get a very different solution, characterized by a reluctance of the planner to abandon fossil at day in case of the occurrence of the bad event, which is here actually very bad, as we have explained above. We illustrate this solution in the limit case where $\phi = 0$.

C.1 Case $\phi = 0$, no storage, fossil night and day

$$q\frac{\alpha}{x_d + Y} + (1-q)\frac{\alpha}{x_d} = \lambda \tag{102}$$

which gives implicitly x_d as a function of Y and λ. It is a quadratic equation which positive root is:

$$x_d = \frac{Y}{2}\left[\left(\frac{\alpha}{\lambda Y} - 1\right) + \left(\left(\frac{\alpha}{\lambda Y} - 1\right)^2 + 4\frac{\alpha(1-q)}{\lambda Y}\right)^{\frac{1}{2}}\right] \tag{103}$$

$$\lambda(\underline{T}_a) = \frac{1}{k}\frac{\alpha}{x_d(\underline{T}_a) + Y(\underline{T}_a)}$$

$$\dot{\mu} - \rho\mu = -q\frac{\alpha}{x_d + Y} = -q\frac{\alpha}{\frac{Y}{2}\left[\left(\frac{\alpha}{\lambda Y} + 1\right) + \left(\left(\frac{\alpha}{\lambda Y} - 1\right)^2 + 4\frac{\alpha(1-q)}{\lambda Y}\right)^{\frac{1}{2}}\right]} \tag{104}$$

Let $X = \frac{\alpha}{\lambda Y}$. Eq. (103) reads:

$$x_d = \frac{Y}{2}\left[(X-1) + \left((X-1)^2 + 4(1-q)X\right)^{\frac{1}{2}}\right]$$

At date $\underline{T}_a$ we have:

$$x_d + Y = \frac{\alpha}{k\lambda} = \frac{XY}{k} \Longrightarrow x_d = \left(\frac{X}{k} - 1\right)Y$$

Therefore, eliminating x_d between these two last equations:

$$\left(\frac{X}{k} - 1\right)Y = \frac{Y}{2}\left[(X-1) + \left((X-1)^2 + 4(1-q)X\right)^{\frac{1}{2}}\right]$$

i.e.

$$2\left(\frac{X}{k} - 1\right) = (X-1) + \left((X-1)^2 + 4(1-q)X\right)^{\frac{1}{2}}$$

i.e.

$$\left(\frac{2}{k} - 1\right)X - 1 = \left((X-1)^2 + 4(1-q)X\right)^{\frac{1}{2}}$$

which yields:

$$\left(\frac{2}{k} - 1\right)^2 X^2 + 1 - 2\left(\frac{2}{k} - 1\right)X = (X-1)^2 + 4(1-q)X$$

i.e.

$$\left(\frac{2}{k} - 1\right)^2 X^2 + 1 - 2\left(\frac{2}{k} - 1\right)X = (X-1)^2 + 4(1-q)X = X^2 + 1 - 2X + 4(1-q)X$$

i.e.

$$\left[\left(\frac{2}{k} - 1\right)^2 - 1\right]X^2 = \left[-2 + 4(1-q) + 2\left(\frac{2}{k} - 1\right)\right]X = 4\left[\frac{1}{k} - q\right]X$$

i.e.

$$\left(\frac{2}{k}-2\right)\frac{2}{k}X = 4\left[\frac{1}{k}-q\right]$$

i.e.

$$X = \frac{k(1-kq)}{1-k}$$

Therefore:

$$\lambda(\underline{T}_a) = \frac{1-k}{k(1-kq)}\frac{\alpha}{Y(\underline{T}_a)} \tag{105}$$

C.2 Case $\phi = 0$, interior storage, fossil night and day

$$\frac{\alpha}{x_d+(1-a)Y} = k\frac{1-\alpha}{x_n+kaY} \tag{106}$$

$$q\frac{\alpha}{x_d+(1-a)Y}+(1-q)\frac{\alpha}{x_d} = \lambda \tag{107}$$

$$q\frac{1-\alpha}{x_n+kaY}+(1-q)\frac{1-\alpha}{x_n} = \lambda \tag{108}$$

$$\dot{\mu}-\rho\mu = -q\left[(1-a)\frac{\alpha}{x_d+(1-a)Y}+ka\frac{1-\alpha}{x_n+kaY}\right] \tag{109}$$

This phase starts with no storage at date $\underline{T}_a$. It never ends: in the long run, x_d and x_n tend asymptotically to zero, and, as shown by the first equation, a tends to a^*.