Cobb-Douglas preferences and pollution in a bilateral oligopoly market*

(Preliminary version, do not cite)

Anicet B. KABRE

EconomiX-CNRS UMR 7235
Université Paris-Nanterre
Bureau G209
200 Avenue de la République
92001 Nanterre Cedex.
Mail: kabre.anicet@parisnanterre.fr
Tel: +33 (0) 1 40 97 59 22

Abstract

In this note, we introduce pollution and examine its effects in a finite bilateral oligopoly model where agents have asymmetric Cobb-Douglas preferences. We define two strategic equilibria: the Stackelberg-Cournot equilibrium with pollution (SCEP) and the Cournot equilibrium with pollution (CEP). While the supplied quantities of the polluting and the non-polluting good depend on the preferences of all economic agents in the case of symmetric preferences, we show that when preferences are asymmetric, *i)* at both equilibria, each polluter's equilibrium supply depends only on the non-polluters' preferences for the non-polluting good; *ii)* at the CEP, the polluters' level of emissions is more sensitive to non-polluters preferences for the non-polluting good compared to their own preferences for this good; *iii)* at the SCEP, when polluters have higher preferences for the non-polluting good, the follower's level of emissions is more sensitive to polluters preferences compared to those of the non-polluters, whereas the leader's emissions level is more sensitive to non-polluters preferences.

Keywords: Bilateral oligopoly; pollution; Cobb-Douglas preferences.
JEL Classification: D43, D51, Q52

*I'm grateful to Professors Ludovic A. Julien and Louis De Mesnard for their remarks. All remaining deficiencies are my own.

1 Introduction

In this paper, we investigate the consequences of asymmetric Cobb-Douglas preferences on equilibrium strategies and pollution emissions level, in a bilateral oligopoly model with Stackelberg competition. To this end, we extend Julien and Tricou (2012)'s bilateral oligopoly model based on (Gabszewicz and Michel, 1997) by introducing a polluting good and assuming asymmetric preferences. We study two models of bilateral oligopoly equilibrium in an exchange economy with production and pollution emissions. In the Stackelberg-Cournot model, we assume that a Stackelberg leader and one follower produce a non-polluting good while other followers supply a polluting good. By contrast, in the Cournot equilibrium model, all firms set their decisions simultaneously.

Studies dealing with pollution mostly focuses on partial equilibrium and are devoted to pollution permit market. In a seminal paper, Hahn (1984) pioneered the analysis of strategic interactions in pollution permit markets. He considers two different scenarios. In the first, it is assumed that firms sell their products on perfectly competitive markets. In the second scenario, one dominant firm is assumed to face a competitive fringe. In this case, it is shown that the permit market is cost-effective if the dominant firm's initial endowment of permits is such that he chooses not to trade. Westskog (1996) extends Hahn (1984)'s model by considering several dominant firms and a competitive fringe in the permit market. In line with Hahn (1984), he finds that the permit market is cost-effective if the dominant firms endowments of permits are such that they don't need to be exchanged. By contrast, Stavins (1995) shows that the permit market is no-longer cost-effective when exchanges are allowed. All the above mentioned studies share the assumption that the dominant firm behaves non-strategically in the final product market. Montero (2009) relaxed that assumption by allowing firms to compete on both the permit markets and the product market. More recently, Dickson and Mackenzie (2018)'s studied strategic trade in pollution permit market where firms decide endogenously to be buyers or sellers. They investigate the interplay between market power in the product market and the permit market equilibrium, and examine the effect of increased demand in the product market. They show that there is a unique equilibrium in which trade in permits takes place.

This market structure has several applications in ecology. Consider a polluting company (for example Mac Donald, a fish processing company) owned by several shareholders who have different market powers. These shareholders consume another good (meat or fish) used as an input to produce the final good (burgers or canned fish). All agents of the economy consume the two goods. In such a context, we describe how the agents' preferences for the non-polluting good would affect strategies and, show how to act effectively on these preferences to reduce pollution, and thus preserve the environment.

Following Hahn (1984),Westskog (1996) and Montero (2009), we deal with market power effects when firms emit pollution. The oligopoly models with a finite number of traders were introduced by Gabszewicz and Michel (1997) and pursued by Bloch and Ghosal (1997), Bloch and Ferrer (2001), Dickson and Hartley (2008), Julien and Tricou (2012). In these models, both sides of the market are linked by a price mechanism. This mechanism was developed by Shapley and Shubik (1977) and refined by Sahi and Yao (1989) and Amir et al. (1990). Our model is closely related to Julien and Tricou (2012)'s who assumes that all traders have the same preferences which can be represented by the same log linear utility function. We relax that assumption by considering that agents located on both sides of the market exhibit different preferences which are adequately captured by Cobb-Douglas utility functions. The simple model we develop here allows for the investigation of the role of preferences on production strategies and pollution emissions in two scenarios that differ in terms of symmetric market power (Cournot equilibrium with pollution, namely CEP) and asymmetric market power (Stackelberg-Cournot equilibrium with pollution, namely SCEP). [1] We show that: 1) at the SCEP and the CEP, the strategic supply of each polluter only depends on non-polluters preferences while the emissions level depends on the preferences of all agents; 2) at the CEP, the polluters' emissions level is more sensitive to non-polluters preferences for the non-polluting good compared to their own preferences for this good; 3) at the SCEP, when polluters have a higher preference for the non-polluting good than non-polluters, the leader's emissions (the follower's) is more sensitive to non-polluters (polluters) preferences for the non-polluting good compared to those of the non-polluters.

This article is structured as follows. The next section outlines the model. In sections 3 and 4, we present and analyze the SCEP and CEP respectively. Section 5 provides a comparison between the SCEP and the CEP. We conclude in Section 6.

2 The model

Let us consider an exchange economy with a productive sector. It consists in $n+2$ traders of two types (indexed respectively by $i = 1, 2$ and $j = 1, ...n$) and two divisible commodities (1 et 2). Good 2 which is not produced, is used as an input to produce good 1. p_1 denotes the price of good 1 in terms of good 2 so that good 2 is assumed to be a numeraire; i.e $p_2 = 1$. Pollution results from the processing of good 2. The preferences of each agent are captured by the following utility functions:

$$U(x_1^i, x_2^i) = \alpha \ln x_1^i + (1-\alpha) \ln x_2^i, \quad \alpha \in (0,1) \quad \forall i = 1, 2 \tag{1}$$
$$U(x_1^j, x_2^j) = \Omega \ln x_1^j + (1-\Omega) \ln x_2^j, \quad \Omega \in (0,1) \quad \forall j = 1, ..n \tag{2}$$

Following Gabszewicz and Michel (1997), Julien and Tricou (2012), the initial endowments in good 1 and 2 for both types of agents are respectively given by:

$$w^i = (0, 0), \qquad \forall i = 1, 2 \tag{3}$$
$$w^j = \left(0, \frac{1}{n}\right), \qquad \forall j = 1, ..., n \tag{4}$$

As in Gabszewicz and Michel (1997), an oligopolist must produce to consume. By using z^i quantity of good 2 an oligopolist produces a quantity y^i of good 1 according to the linear technology:

$$y^i = \frac{1}{\beta^i} z^i, \quad \beta^i > 0, \quad \forall i = 1, 2 \tag{5}$$

Following Crettez et al. (2014), the use of an amount z^i of the polluting input generates

[1] Environmental externalities and issues related to the existence and uniqueness of oligopolistic equilibrium in general equilibrium models (due to problems raised in Gabszewicz (2002), Julien (2017)) are beyond the scope of this paper.

a quantity of emissions:

$$e^i = \frac{1}{\gamma^i} z^i, \quad \gamma^i > 0, \quad \forall i = 1, 2 \tag{6}$$

where γ^i measures the magnitude of the pollution. From the last two equations, we express the production y^i of good 1 in terms of the emissions e^i and obtain:

$$y^i = \frac{\gamma^i}{\beta^i} e^i, \quad \forall i = 1, 2 \tag{7}$$

Traders try to manipulate the market price through their strategic supply. Let q^i denote the strategy of agent i; e^i their emissions level and b^j the strategy of agents j, the strategy sets for the supply of both oligopolists are:

$$s^i = \left\{ (q^i, e^i) \in \mathbb{R}_+^2 | 0 \leq q^i \leq \frac{\gamma^i}{\beta^i} e^i \right\}, \quad \forall i = 1, 2$$

$$s^j = \left\{ b^j \in \mathbb{R}_+ | 0 \leq b^j \leq \frac{1}{n} \right\}, \quad \forall j = 1, ..., n$$

Market price is then given by:

$$p_1 = \frac{\sum_{j=1}^n b^j}{\sum_{i=1}^2 q^i} = \frac{B}{Q} \tag{8}$$

Individual allocations are given by:

$$(x_1^i, x_2^i) = \left(y^i - q^i ; \frac{B}{q^i + q^{-i}} q^i - \gamma^i e^i \right), \quad i = 1, 2 \tag{9}$$

$$(x_1^j, x_2^j) = \left(\frac{Q}{b^j + B^{-j}} b^j ; \frac{1}{n} - b^j \right), \quad j = 1, ..., n \tag{10}$$

and yield the following indirect utility levels:

$$V^i(q^i, q^{-i}, B) = \alpha \ln \left(y^i - q^i \right) + (1 - \alpha) \ln \left(\frac{B}{q^i + q^{-i}} q^i - \gamma^i e^i \right), \quad i = 1, 2 \tag{11}$$

$$V^j(Q, b^j, B^{-j}) = \Omega \ln \left(\frac{Q}{b^j + B^{-j}} b^j \right) + (1 - \Omega) \ln \left(\frac{1}{n} - b^j \right) \quad j = 1, ..., n, \tag{12}$$

where $b = (b^1, b^2, ..., b^n)$ is the vector of equilibrium strategies of traders j and $q = (q^1, q^2)$ is the vector of equilibrium strategies of traders i.

3 The Stackelberg-Cournot equilibrium with pollution

In this game, polluting firms compete "*à la Stackelberg*"; Agent 1 behaves as a Stackelberg leader with respect to the $(n + 1)$ remaining agents. The game consists in two stages. In the first, the leader solves the following program:

$$\arg\max_{q^1, e^1} \quad \alpha \ln \left(\frac{\gamma^1 e^1}{\beta^1} - q^1 \right) + (1 - \alpha) \ln \left(\frac{\sum_{j=1}^n g^j(q^1)}{q^1 + f(q^1)} q^1 - \gamma^1 e^1 \right) \tag{13}$$

At the second stage, the $n+1$ follower simultaneously solve the following problems:

$$\arg\max_{q^2,e^2} \quad \alpha \ln\left(\frac{\gamma^2 e^2}{\beta^2} - q^2\right) + (1-\alpha)\ln\left(\frac{B}{q^1+q^2}q^2 - \gamma^2 e^2\right), \forall q^1 \tag{14}$$

$$\arg\max_{b^j} \quad \Omega \ln\left(\frac{q^1+q^2}{b^j+B^{-j}}b^j\right) + (1-\Omega)\ln\left(\frac{1}{n} - b^j\right), \forall q^1 \quad j=1,...,n,. \tag{15}$$

Proposition 1 *: The solution of the SCEP is given by the following equilibrium strategy profiles $(\tilde{q}^1, \tilde{q}^2, b)$ and emissions level $(\tilde{e}^1, \tilde{e}^2)$:*

$$\tilde{q}^1 = \frac{\Omega\beta^2}{4(\beta^1)^2}\phi \tag{16}$$

$$\tilde{q}^2 = \frac{\Omega\phi}{4(\beta^1)^2}\left[2\beta^1 - \beta^2\right] \tag{17}$$

$$b^j = \frac{\Omega(n-1)}{n(n-\Omega)} \quad \forall j = 1,...,n \tag{18}$$

$$\tilde{e}^1 = \frac{\Omega(1+\alpha)}{4\gamma^1}\frac{\beta^2}{\beta^1}\phi \tag{19}$$

$$\tilde{e}^2 = \frac{\Omega\alpha\phi}{2\gamma^2}\left[\left(2-\frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}} + \frac{1-\alpha}{2\alpha}\frac{\beta^2}{\beta^1}\left(2-\frac{\beta^2}{\beta^1}\right)\right], \tag{20}$$

where $\phi = \frac{n-1}{n-\Omega}$.

From these equilibrium strategies given in proposition 1, the market price is:

$$\tilde{p}_1 = 2\beta^1 \tag{21}$$

The individual allocations are:

$$(\tilde{x}_1^1, \tilde{x}_2^1) = \left[\frac{\alpha\Omega\beta^2}{4(\beta^1)^2}\phi; \frac{\Omega(1-\alpha)\beta^2}{4\beta^1}\phi\right] \tag{22}$$

$$(\tilde{x}_1^2, \tilde{x}_2^2) = \left[\tilde{x}_1^2, \tilde{x}_2^2\right] \tag{23}$$

$$\tilde{x}_1^2 = \frac{\Omega\phi}{2}\left[\frac{\alpha}{\beta^2}\sqrt{2-\frac{\beta^2}{\beta^1}} + \frac{1-\alpha}{2(\beta^1)^2}\left(2-\alpha\frac{\beta^2}{\beta^1}\right)\right] \tag{24}$$

$$\tilde{x}_2^2 = \frac{\Omega(1-\alpha)}{2}\phi\left[\sqrt{2-\frac{\beta^2}{\beta^1}} - \frac{\beta^2}{2\beta^1}\left(2-\frac{\beta^2}{\beta^1}\right)\right] \tag{25}$$

$$(\tilde{x}_1^j, \tilde{x}_2^j) = \left(\frac{\Omega\phi}{2n\beta^1}, \frac{(1-\Omega)\phi}{n}\right) \quad \forall j = 1,...,n. \tag{26}$$

Finally, equilibrium utility levels write as:

$$\tilde{U}^1 = \alpha \ln \frac{\alpha}{\beta^1(1-\alpha)} + \ln \frac{\Omega \beta^2 \phi}{4\beta^1} + \ln(1-\alpha) \tag{27}$$

$$\tilde{U}^2 = \alpha \ln \left[\tilde{x}_1^2\right] + (1-\alpha) \ln \left[\tilde{x}_2^2\right] \tag{28}$$

$$\tilde{U}^j = \Omega \ln \left(\frac{\Omega \phi}{2n\beta^1}\right) + (1-\Omega) \ln \frac{1-\Omega\phi}{n}, \quad j = 1, ..., n. \tag{29}$$

We remark that: $\frac{\partial \tilde{e}^1}{\partial \Omega} = \frac{n}{n-\Omega} \frac{\alpha+1}{4\gamma^1} \frac{\beta^2}{\beta^1} \phi$; $\quad \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\Omega}{4\gamma^1} \frac{\beta^2}{\beta^1} \phi$; $\quad \frac{\partial \tilde{e}^2}{\partial \alpha} = \frac{\Omega \phi}{2\gamma^2} \sqrt{2 - \frac{\beta^2}{\beta^1}} \left[1 - \frac{\beta^2}{2\beta^1} \sqrt{2 - \frac{\beta^2}{\beta^1}}\right]$; $\frac{\partial \tilde{e}^2}{\partial \Omega} = \frac{n}{n-\Omega} \frac{\alpha \phi}{2\gamma^2} \sqrt{2 - \frac{\beta^2}{\beta^1}} \left[1 + \frac{1-\alpha}{\alpha} \frac{\beta^2}{2\beta^1} \sqrt{2 - \frac{\beta^2}{\beta^1}}\right]$.

While partial equilibrium models with pollution usually assume an exogenous (often linear) market demand function (Hahn (1984), Montero (2009), Chen and Hobbs (2005), Sanin and Zanaj (2011), Sanin and Zanaj (2012)), the market demand in our model is endogenous and depends on the preferences of agents.

Proposition 2 *: In the SCEP, marginal variations of the leader and the follower emission levels are more sensitive to the non-polluters' preferences for the non-polluting good than to the polluters' ones.*

Proof: The emissions level of polluters increase with their preferences for the product good, but this increase remains low compared with what we would have obtained if non-polluters preferences varied in the same proportions. Indeed, the differences of the marginal variations are $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\tilde{e}^1}{\Omega(n-\Omega)(1+\alpha)}[n + n(\alpha - \Omega) + \Omega^2] \quad > 0$ and $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega} = \frac{\phi}{2\gamma^2} \sqrt{2 - \frac{\beta^2}{\beta^1}} \left[(\Omega - \alpha\frac{n}{n-\Omega}) - \frac{\beta^2 \Omega}{2\beta^1} \sqrt{2 - \frac{\beta^2}{\beta^1}} [\Omega + \frac{n}{n-\Omega} \frac{1-\alpha}{2}]\right] < 0, \quad$ if $\Omega < \alpha$. However, when α is high, the value obtained from $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega}$ is greater than that obtained with a small value of α, and the follower marginal variation of emissions become more and more sensitive to polluters preferences compared to those of the non-polluters. If $\alpha > \Omega$, $\alpha - \Omega > 0$, $(\alpha - \Omega) < 1, \Rightarrow n + n(\alpha - \Omega) + \Omega^2 > 0$, so $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} > 0$. Morever,the emission elasticities resulting from a variation of preferences yield the following expressions:

$$\varepsilon_{\tilde{e}^1/\alpha} = \frac{\alpha}{1+\alpha} \quad < 1$$

$$\varepsilon_{\tilde{e}^2/\alpha} = \frac{1 - \frac{\beta^2}{2\beta^1}\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}}{1 + \frac{1-\alpha}{\alpha}\frac{\beta^2}{2\beta^1}\left(2 - \frac{\beta^2}{\beta^1}\right)^{\frac{1}{2}}} \quad < 1$$

$$\varepsilon_{\tilde{e}^1/\Omega} = \varepsilon_{\tilde{e}^2/\Omega} = \left(1 + \frac{\Omega}{n-\Omega}\right) \quad > 1$$

4 The Cournot equilibrium with pollution

Here, polluters compete "*à la Cournot*". The Cournot equilibrium is obtained as the solution of the following system of simultaneous optimization programs:

$$\arg\max_{q^1,e^1} \quad \alpha \ln\left(\frac{\gamma^1 e^1}{\beta^1} - q^1\right) + (1-\alpha)\ln\left(\frac{B}{q^1+q^2}q^1 - \gamma^1 e^1\right), i=1 \tag{30}$$

$$\arg\max_{q^2,e^2} \quad \alpha \ln\left(\frac{\gamma^2 e^2}{\beta^2} - q^2\right) + (1-\alpha)\ln\left(\frac{B}{q^1+q^2}q^2 - \gamma^2 e^2\right), i=2 \tag{31}$$

$$\arg\max_{b^j} \quad \Omega \ln\left(\frac{q^1+q^2}{b^j+B^{-j}}b^j\right) + (1-\Omega)\ln\left(\frac{1}{n} - b^j\right), j=1,...,n. \tag{32}$$

Proposition 3 *: The solution of the CEP is given by the following strategy profiles* $(\widehat{q}^1, \widehat{q}^2, b)$ *and emissions level* $(\widehat{e}^1, \widehat{e}^2)$*:*

$$\widehat{q}^1 = \frac{\Omega\beta^2}{(\beta^1+\beta^2)^2}\phi \tag{33}$$

$$\widehat{q}^2 = \frac{\Omega\beta^1}{(\beta^1+\beta^2)^2}\phi \tag{34}$$

$$b^j = \frac{\Omega(n-1)}{n(n-\Omega)} = \frac{\Omega}{n}\phi \quad \forall j=1,...,n \tag{35}$$

$$\widehat{e}^1 = \frac{\Omega}{\gamma^1}\frac{\beta^2(\alpha\beta^2+\beta^1)}{(\beta^1+\beta^2)^2}\phi \tag{36}$$

$$\widehat{e}^2 = \frac{\Omega}{\gamma^2}\frac{\beta^1(\alpha\beta^1+\beta^2)}{(\beta^1+\beta^2)^2}\phi, \tag{37}$$

where $\phi = \frac{n-1}{n-\Omega}$.

From those strategies, we deduce the market price $p_1 = \frac{\sum b^j}{\sum q^i}$ which is:

$$\widehat{p}_1 = \beta^1 + \beta^2 \tag{38}$$

Individual allocations are

$$\widehat{x}^1 = (\widehat{x}^1_1, \widehat{x}^1_2) = \left[\frac{\alpha\Omega}{\beta^1}\left(\frac{\beta^2}{\beta^1+\beta^2}\right)^2\phi; \Omega(1-\alpha)\left(\frac{\beta^2}{\beta^1+\beta^2}\right)^2\phi\right] \tag{39}$$

$$\widehat{x}^2 = (\widehat{x}^2_1, \widehat{x}^2_2) = \left[\frac{\alpha\Omega}{\beta^2}\left(\frac{\beta^1}{\beta^1+\beta^2}\right)^2\phi; \Omega(1-\alpha)\left(\frac{\beta^1}{\beta^1+\beta^2}\right)^2\phi\right] \tag{40}$$

$$\widehat{x}^j = (\widehat{x}^j_1, \widehat{x}^j_2) = \left[\frac{\Omega\phi}{n(\beta^1+\beta^2)}; \frac{(1-\Omega)\phi}{n}\right]. \tag{41}$$

and yield the following utility levels

$$\widehat{U}^1 = 2\ln\left(\frac{\beta^2}{\beta^1+\beta^2}\right) + (1-\alpha)\ln(1-\alpha) + \ln\alpha\Omega\phi - \alpha\ln\beta^1 \tag{42}$$

$$\widehat{U}^2 = 2\ln\left(\frac{\beta^1}{\beta^1+\beta^2}\right) + (1-\alpha)\ln(1-\alpha) + \ln\alpha\Omega\phi - \alpha\ln\beta^2 \tag{43}$$

$$\widehat{U}^j = \Omega\ln\left[\frac{\Omega\phi}{(\beta^1+\beta^2)^n}\right] + (1-\Omega)\ln(1-\Omega\phi) + (\Omega-1)\ln n. \tag{44}$$

The elasticity are given by:

$$\varepsilon_{\widehat{e}^1/\alpha} = \frac{\alpha\beta^2}{\alpha\beta^2+\beta^1} \quad < 1$$
$$\varepsilon_{\widehat{e}^1/\Omega} = \varepsilon_{\widehat{e}^2/\Omega} = \frac{n}{n-\Omega} \quad < 1$$

5 Comparison of equilibrium outcomes

We now proceed with the comparison of equilibrium outcomes in the SCEP and CEP. The strategic supplies of goods 1 and 2 exclusively depend on non-polluters preferences while emissions level depend on the preferences of all agents (polluters and non-polluters).

Proposition 4 *: In the SCEP and CEP, when preferences are asymmetric, the quantities supplied by agents only depend on the non-polluters preferences. But, when preferences are symmetric, equilibrium quantities depend on the preferences of all agents.*

Proof: If preferences are asymmetric, $\frac{\partial \widehat{q}^i}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^{-i}}{(\beta^i+\beta^{-i})^2} > 0$; $\frac{\partial \tilde{q}^1}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^2}{4(\beta^1)^2}\phi > 0$; $\frac{\partial \tilde{q}^2}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\phi}{4(\beta^1)^2}\left[2\beta^1-\beta^2\right] > 0$; $\frac{\partial \widehat{q}^i}{\partial\alpha} = \frac{\partial \tilde{q}^1}{\partial\alpha} = \frac{\partial \tilde{q}^2}{\partial\alpha} = 0$; $\frac{\partial b^j}{\partial\alpha} = 0$. However, when all agents have the same utility function $U(x_1^h, x_2^h) = \alpha\ln x_1^h + (1-\alpha)\ln x_2^h, \quad \alpha \in (0,1) \quad \forall i = 1,2, \quad \forall j = 1,2,....n$, the strategic supplies are given by $\widehat{q}^1 = \frac{\alpha\beta^2}{(\beta^1+\beta^2)^2}\phi; \quad \widehat{q}^2 = \frac{\alpha\beta^1}{(\beta^1+\beta^2)^2}\phi$; $b^j = \frac{\alpha(n-1)}{n(n-\alpha)} \quad j = 1,...,n$.

We now focus on the effect of a change in agent preferences on the level of emissions. Indeed we have: $\frac{\partial \widehat{e}^i}{\partial\Omega} = \frac{n}{n-\Omega}\frac{\beta^{-i}(\alpha\beta^{-i}+\beta^i)}{\gamma^i(\beta^{-i}+\beta^i)^2}\phi \quad > 0$ and $\frac{\partial \widehat{e}^i}{\partial\alpha} = \frac{1}{\gamma^i}\frac{\Omega(\beta^{-i})^2}{(\beta^{-i}+\beta^i)^2}\phi \quad > 0$. The emissions level of a relatively inefficient firm (with a high value for β) is less sensitive to a variation in preferences compared to that of a relatively efficient firm: indeed, if $\beta^{-i} < \beta^i$, $\frac{1}{\gamma^i}\frac{\Omega(\beta^{-i})^2}{(\beta^{-i}+\beta^i)^2}\phi < \frac{1}{\gamma^{-i}}\frac{\Omega(\beta^i)^2}{(\beta^{-i}+\beta^i)^2}\phi$ and $\frac{1}{\gamma^{-i}}\frac{\beta^{-i}(\alpha\beta^{-i}+\beta^i)}{(\beta^{-i}+\beta^i)^2}\phi < \frac{1}{\gamma^i}\frac{\beta^i(\alpha\beta^i+\beta^{-i})}{(\beta^{-i}+\beta^i)^2}\phi$. Then, the difference in the variation of the emissions level resulting from a variation of the preferences is represented by:

$$\frac{\partial \widehat{e}^i}{\partial\alpha} - \frac{\partial \widehat{e}^i}{\partial\Omega} = \frac{\beta^{-i}\phi}{\gamma^i(n-\Omega)(\beta^i+\beta^{-i})^2}\left[\beta^{-i}(n(\Omega-\alpha)-\Omega^2) - n\beta^i\right] \tag{45}$$

Proposition 5 *: At both equilibria, if $\alpha > \Omega$, polluters' level of emissions remain more sensitive to non-polluters preferences compared with their own preferences. However, at the CEP, the polluter is more sensitive to their own preferences rather than non-polluters preferences if $\frac{\beta^{-i}}{\beta^i} > \frac{n}{n(\Omega-\alpha)-\Omega^2}$.*

Proof: $\forall \beta^i$, if $\alpha > \Omega$, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} < 0$. In addition, solving $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} > 0$, needs $\beta^{-i}(n(\Omega - \alpha) - \Omega^2) - n\beta^i > 0$ ie $\frac{\beta^{-i}}{\beta^i} > \frac{n}{n(\Omega-\alpha)-\Omega^2}$; the ratio of marginal costs remaining positive and $\frac{1}{n(\Omega-\alpha)-\Omega^2}$ being negative ($\alpha > \Omega$).

Remark 1: When $\alpha > \Omega$, this condition is sufficient to yield $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} < 0$. This indicates that a higher preference of polluters for their own good, their level of emissions increases more with non-polluters preferences compared to the increase observed if their own preferences varied.

Proposition 6 *: An increase in non-polluters preferences for the non-polluting good affects more the polluter's marginal variation of emissions if their market power is sufficiently high. However, when their market power decreases, polluters preferences have no impact.*

Proof: At the SCEP, if $\alpha > \Omega$, we have $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\tilde{e}^1}{\Omega(n-\Omega)(1+\alpha)}[n + n(\alpha - \Omega) + \Omega^2] \quad > 0$ and $\frac{\partial \tilde{e}^2}{\partial \alpha} - \frac{\partial \tilde{e}^2}{\partial \Omega} = \frac{\phi}{2\gamma^2}\sqrt{2 - \frac{\beta^2}{\beta^1}}\left[(\Omega - \alpha\frac{n}{n-\Omega}) - \frac{\beta^2\Omega}{2\beta^1}\sqrt{2 - \frac{\beta^2}{\beta^1}}[\Omega + \frac{n}{n-\Omega}\frac{1-\alpha}{2}]\right] < 0$. However with $\alpha > \Omega$, at the CEP we get $\frac{\partial \widehat{e}^2}{\partial \alpha} - \frac{\partial \widehat{e}^2}{\partial \Omega} = \frac{\beta^1\phi}{\gamma^2(n-\Omega)(\beta^2+\beta^1)^2}\left[\beta^1(n(\Omega - \alpha) - \Omega^2) - n\beta^2\right] > 0$ because $\frac{\beta^1}{\beta^2}$ is always positive.

Remark 2: A polluter is more sensitive to its own preferences compared to those of non-polluters if its market power is below a certain threshold. From this threshold, their emissions becomes more sensitive to non-polluters preferences. Moreover, at the CEP, the impact on polluters' emissions level resulting from a variation of their preferences may be identical to that which would be recorded if the preferences of non-polluters varied in the same proportion. However, if the same firm competed at the SCEP as a leader, this coincidence could not be observed. Indeed, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} = 0$ if $\frac{\beta^i}{\beta^{-i}} = \frac{n(\Omega-\alpha)-\Omega^2}{n}$ while $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} \neq 0$ because the value recommended for α is greater than 1, i.e $\alpha = 1 + \Omega - \frac{\Omega^2}{n} < 0$. The ratio of marginal costs remaining positive, $\frac{\partial \widehat{e}^i}{\partial \alpha} - \frac{\partial \widehat{e}^i}{\partial \Omega} = 0$ if and only if $\Omega > \alpha$.

Finally, we determine the conditions under which the two equilibria coincide.

Proposition 7 *: When $\beta^1 = \beta^2 = \beta$ and $\gamma^i = \gamma$, the SCEP coincides with the CEP.*

Remark: Morever, $\frac{\partial \tilde{e}^1}{\partial \Omega} - \frac{\partial \tilde{e}^1}{\partial \alpha} = \frac{\partial \tilde{e}^2}{\partial \Omega} - \frac{\partial \tilde{e}^2}{\partial \alpha} = \frac{\partial \widehat{e}^i}{\partial \Omega} - \frac{\partial \widehat{e}^i}{\partial \alpha} = \frac{\phi}{4\gamma(n-\Omega)}[n + n(\alpha - \Omega) + \Omega^2] > 0$.

Proof: If $\beta^i = \beta$ and $\gamma^i = \gamma$, we get $\tilde{q}^1 = \tilde{q}^2 = \widehat{q}^i = \frac{\Omega}{4\beta}\phi$ and $\tilde{e}^1 = \tilde{e}^2 = \widehat{e}^i = \frac{\Omega(\alpha+1)}{4\gamma}\phi$.

6 Conclusion

We set out to examine the consequences of asymmetric preferences on emissions when polluters have asymmetric and symmetric market power. Two results might be emphasized. First, the supply of each polluter depends on the non-polluters' preferences. Second, when the polluter's market power increases, each polluter becomes more sensitive to the preferences of the non-polluters.

References

Amir, R., Sahi, S., Shubik, M., and Yao, S. (1990). A strategic market game with complete markets. *Journal of Economic Theory*, 51(1):126–143.

Bloch, F. and Ferrer, H. (2001). Strategic complements and substitutes in bilateral oligopolies. *Economics Letters*, 70:83–87.

Bloch, F. and Ghosal, S. (1997). Stable Trading Structures in Bilateral Oligopolies. *Journal of Economic Theory*, 74:368–384.

Chen, Y. and Hobbs, B. F. (2005). An Oligopolistic Power Market Model with Tradable NOx Permits. *IEEE Transactions on Power System*, 20(1):119–129.

Crettez, B., Jouvet, P.-a., and Julien, L. A. (2014). Tax Policy in a Simple General Oligopoly Equilibrium Model with Pollution Permits. *Les Cahiers de la chaire Economie du Climat, Working paper series*, (13).

Dickson, A. and Hartley, R. (2008). The strategic Marshallian cross. *Games and Economic Behavior*, 64(2):514–532.

Dickson, A. and Mackenzie, I. A. (2018). Strategic trade in pollution permits. *Journal of Environmental Economics and Management*, (2009):1–26.

Gabszewicz, J. J. (2002). Strategic Multilateral Exchange: General Equilibrium with Imperfect Competition. (Cheltenham: Edward-Elgar).

Gabszewicz, J. J. and Michel, P. (1997). Oligopoly Equilibria in Exchange Economies. *in Eaton, B.C. and Harris, R.G. (eds). Trade, Technology and Economics. Essays in honour of R.G. Lipsey*, (Cheltenham: Edward-Elgar).

Hahn, R. (1984). Market Power and Transferable Property Rights. *Quarterly Journal of Economics*, 99(4):753–765.

Julien, L. A. (2017). Hierarchical competition and heterogoneous behavior in noncooperative oligopoly markets. *EconomiX Working Paper*, (22):33 pages.

Julien, L. A. and Tricou, F. (2012). Market price mechanisms and stackelberg general equilibria: An example. *Bulletin of Economic Research*, 64(2):239–252.

Montero, J. P. (2009). Market power in pollution permit markets. *Energy Journal*, 30(Special issue 2):115–142.

Sahi, S. and Yao, S. (1989). The non-cooperative equilibria of a trading economy with complete markets and consistent prices. *Journal of Mathematical Economics*, 18(4):325–346.

Sanin, M.-E. and Zanaj, S. (2011). A Note on Clean Technology Adoption and its Influence on Tradeable Emission Permits Prices. *Environmental and Resource Economics*, 48(4):561–567.

Sanin, M.-E. and Zanaj, S. (2012). Clean Technology Adoption Under Cournot Competition. *Strategic Behavior and the Environment*, 2(2):159–172.

Shapley, L. and Shubik, M. (1977). Trade Using One Commodity as a Means of Payment. *Journal of Political Economy*, 85(5):937.

Stavins, R. N. (1995). Transaction Costs and Tradeable Permits. *Journal of Environmental Economics and Management*, (29):133–148.

Westskog, H. (1996). Market Power in a system of tradeable CO2 Quotas. *The energy Journal*, 17(3):85–103.